

Online Supplement for “A Symmetric Capacity-Constrained
Differentiated Oligopoly Model for the United States Pediatric
Vaccine Market with Linear Demand”

Banafsheh Behzad

Department of Information Systems
California State University, Long Beach
`banafsheh.behzad@csulb.edu`

Sheldon H. Jacobson

Department of Computer Science
University of Illinois at Urbana-Champaign
`shj@illinois.edu`

Matthew J. Robbins

Department of Operational Sciences
Air Force Institute of Technology
`matthew.robbins@afit.edu`

Proof of Theorem 1. Pure strategy equilibria can be found by solving the following system of optimization problems:

$$\begin{aligned} & \underset{p_i \geq 0}{\text{maximize}} && p_i \left(a - bp_i + c \sum_{j=1, j \neq i}^n p_j \right), \\ & \text{subject to} && a - bp_i + c \sum_{j=1, j \neq i}^n p_j \leq k_i, \quad i = 1, 2, \dots, n. \end{aligned} \tag{1}$$

The objective function in (1) maximizes the profit of each manufacturer i , while the constraint implies that the demand of each manufacturer is less than the capacity of the manufacturer. To solve this optimization problem, Lagrange multipliers λ_i are introduced, with the Lagrangian functions

$$\mathcal{L}_i = p_i \left(a - bp_i + c \sum_{j=1, j \neq i}^n p_j \right) - \lambda_i \left(a - bp_i + c \sum_{j=1, j \neq i}^n p_j - k_i \right), \quad i = 1, 2, \dots, n. \tag{2}$$

To maximize \mathcal{L}_i , the Karush-Kuhn-Tucker (KKT) conditions are written as

$$\frac{\partial \mathcal{L}_i}{\partial p_i} = a - 2bp_i + c \sum_{j=1, j \neq i}^n p_j + \lambda_i b = 0, \quad i = 1, 2, \dots, n, \quad (3)$$

$$a - bp_i + c \sum_{j=1, j \neq i}^n p_j \leq k_i, \quad i = 1, 2, \dots, n, \quad (4)$$

$$\lambda_i (a - bp_i + c \sum_{j=1, j \neq i}^n p_j - k_i) = 0, \quad i = 1, 2, \dots, n, \quad (5)$$

$$\lambda_i \geq 0, \quad i = 1, 2, \dots, n.$$

If $(p_1, p_2, \dots, p_n, \lambda_1, \lambda_2, \dots, \lambda_n)$ exists, then (p_1, p_2, \dots, p_n) is a pure strategy equilibrium. From (3), it is known that,

$$p_i = \frac{a + c \sum_{j=1, j \neq i}^n p_j + \lambda_i b}{2b}, \quad i = 1, 2, \dots, n. \quad (6)$$

(a) In (5), if $\lambda_i = 0$, then from (6), $p_i = \frac{a + c \sum_{j=1, j \neq i}^n p_j}{2b}$, $i = 1, 2, \dots, n$. Writing p_i in terms of p_j , $i = 1, 2, \dots, n$, $i \neq j$, results in $p_i = p^B = \frac{a}{2b - (n-1)c}$, the Bertrand-Chamberlin equilibrium. To find $k(\gamma)$, the profit at the Bertrand-Chamberlin equilibrium is computed ($\pi^B = b(p^B)^2$). From the inverse demand function and by setting $q_j = k$, ($i, j = 1, 2, \dots, n$, $i \neq j$), the contingent demand is $q_i = \alpha - \gamma(n-1)k - p_i$. The monopoly profit on the contingent demand q_i is $(\frac{\alpha - \gamma(n-1)k}{2})^2$. From [5], $k(\gamma)$ is computed by setting profits at the candidate Bertrand-Chamberlin equilibrium π^B to be equal to the monopoly profits on the contingent demand, $(\frac{\alpha - \gamma(n-1)k}{2})^2$ (i.e., $\pi^B = (\frac{\alpha - \gamma(n-1)k}{2})^2$).

(b) In (5), setting the expression in parentheses equal to zero results in system of equations $a - bp_i + c \sum_{j=1, j \neq i}^n p_j - k = 0$, $i = 1, 2, \dots, n$. From (6) and by solving the system of equations for λ_i ,

$$\lambda_i = \lambda = \frac{k(2b - c(n-1)) - ab}{cb(n-1) - b^2} \quad i = 1, 2, \dots, n.. \quad (7)$$

Then for $i = 1, 2, \dots, n$,

$$p_i = p = \frac{a}{2b - (n-1)c} + \frac{k(2b - c(n-1)) - ab}{2b - (n-1)c(c(n-1) - b)} = \alpha - k - k\gamma(n-1) = P(k). \quad (8)$$

In this case, manufacturers are so capacity-constrained (i.e., $k \leq q^C$) that the competitive equilibrium $P(k)$ is obtained [5].

Note that in (5), if $\lambda_i = 0$ for any i and the expression in parentheses be equal to zero for any l , $i, l = 1, 2, \dots, n$ $i \neq l$, then given that the parameter γ is ranging from zero to one and the capacity k is positive, (4) is violated. Therefore, the Bertrand-Chamberlin and the competitive equilibria provide a unique pure strategy equilibrium depending on the value of the capacity. \square

Proof of Theorem 2. By Theorem 1, a pure strategy equilibrium exists if either $k \geq k(\gamma)$ or $k \leq q^C$. From [1], there exists a continuous mixed strategy equilibrium with support on a price interval $[\underline{p}, \bar{p}]$. From the inverse demand function and by setting $q_j = k$, the contingent demand is $q_i = \alpha - \gamma(n-1)k - p_i$, $i, j = 1, 2, \dots, n, j \neq i$. The desirable price is the one which maximizes the profits. Therefore, the supremum of the support should be the monopoly price on the contingent demand of a manufacturer when all the other manufacturers undercut its price (i.e., $\bar{p} = \arg \max_p p(\alpha - \gamma(n-1)k - p)$), and hence, $\bar{\pi} = \bar{p}(\alpha - \gamma(n-1)k - \bar{p})$. The expected profit for the manufacturer at any price $p \in [\underline{p}, \bar{p}]$ must be similar, say $\bar{\pi}$, provided that the other manufacturers use the ϕ . The reason is that, if a manufacturer charges \bar{p} , it knows it will be surely undercut and charging other than \bar{p} will not be optimal. The expected profit when setting price \underline{p} is $\underline{p}k$, and hence, $\underline{p} = \bar{\pi}/k$. For any $p \in [\underline{p}, \bar{p}]$, the expected profit is given by

$$\bar{\pi} = \left(1 - [\phi(p)]^{(n-1)}\right) pk + [\phi(p)]^{(n-1)} p(\alpha - \gamma(n-1)k - p). \quad (9)$$

Equation (9) holds since the probability that all the other manufacturers undercut the price of one manufacturer is $[\phi(p)]^{(n-1)}$ and in this case the profit is equal to $p(\alpha - \gamma(n-1)k - p)$. On the other hand, the probability that one manufacturer undercuts the price of all the other manufacturers is $(1 - [\phi(p)]^{(n-1)})$ and the profit is equal to pk . Solving for $\phi(p)$ in (9), gives the desired result. This proof follows along the lines of [3], [2] and [4]. \square

References

- [1] P. Dasgupta and E. Maskin. The existence of equilibrium in discontinuous economic games, II: Applications. *Review of Economic Studies*, 53:27–41, 1986.
- [2] D. Kreps and J. Scheinkman. Quantity pre-commitment and Bertrand competition yield Cournot outcomes. *Bell Journal of Economics*, 14:326–37, 1983.
- [3] R. Levitan and M. Shubik. Price duopoly and capacity constraints. *International Economic Review*, 13:111–22, 1972.
- [4] X. Vives. Rationing rules and Bertrand-Edgeworth equilibria in large markets. *Economics Letters*, 21:113–116, 1986.
- [5] X. Vives. *Oligopoly Pricing*. MIT Press, Cambridge, 1999.