Supplementary Materials

Data Generating Process in Section 5.1

With a given set of bandwidths from small to large, we calibrate an MGWR model using randomly drawn response variable and covariates. The resulting parameter estimates will be used as known processes to reconstruct the final response variable used for performance testing. The detailed steps used in this study are described as follows:

1) Make a gridded coordinate system of size 200 by 200, which has a total of 40,000 locations.
2) Randomly draw a temporary fake response variable \( y \) for each of the grid cells from a normal distribution with 0 as its mean and 1 as its variance.
3) Draw \( k \) covariates \( X_{1...k} \) from a multivariate normal distribution with 0 as the mean and 1 as the variance.
4) Run an MGWR model with an adaptive (nearest neighbors) bi-square kernel using \( y \) as the response variable, \( X_{1...k} \) as the covariates and with a set of predefined adaptive bandwidths to be \([n/k, 2n/k, 3n/k, ..., nk/k]\). A set of parameter surfaces can then be obtained. These sets of parameter estimates will have varying degrees of smoothness but their values will be close to zero. To make the estimates have a reasonable range, we multiply them by 100 and add a constant of 3 to get to the set of the parameter estimates \( \beta_{l...k} \). Figure S1 shows the parameter surfaces for 15 generated known parameter surfaces. We can see that spatial variation in processes gradually decreases as the bandwidth increases.
5) Redraw \( X^* \) but this time we add a small amount of multicollinearity to the covariates pairs (a correlation coefficient of 0.3). This is simply to more accurately represent real-world situations.
6) Generate a final response variable \( y^* \) from the covariates in 5) and parameter surfaces in 4) with added random noise drawn from \( N(0,0.5) \).
7) The resulting \( y^* \) and \( X^*_{1...k} \) are the final synthetic dataset. Figure S2 shows the recovered parameter estimates surfaces from MGWR using this final testing dataset.
Figure S1. Synthesized known parameter surfaces with varying scales from local to global.

Figure S2. Recovered parameter estimates surfaces from MGWR using the final testing dataset.