*Supplement*

# Analysis of Interactions and Nonlinear Effects with Missing Data: A Factored Regression Modeling Approach Using Maximum Likelihood Estimation

**Table of Contents**

Enclosed in this document are the supplemental materials for our article entitled "Analysis of Interactions and Nonlinear Effects with Missing Data: A Factored Regression Modeling Approach Using Maximum Likelihood Estimation".

Supplement A contains additional tables to the results of Study 1 reported in the article. Supplement B contains an additional simulation in which a regression model with quadratic effects was investigated. Supplement C contains additional tables to the results of Study 2 reported in the article. Supplement D contains additional tables to the results of Study 3 reported in the article. Supplement E reports results of an additional simulation which compares the performance of the factored regression model based ML estimation with the latent-moderated-structures estimation method.

## Supplement A: Additional Results for Study 1

The following pages contain the additional results for Simulation Study 1. Tables A1 to A4 show the relative bias, RMSE, and the coverage rates for the two first order effects $\beta_1$ and $\beta_2$, and the interaction effect $\beta_3$ as a function of the missing data (25% or 50%), and the $R^2$ ($R^2 = .25$ or .50) for uncorrelated predictors ($\rho_{xz} = 0$). Tables B5 to B8 show the relative bias, RMSE, and the coverage rates for the two first order effects $\beta_1$ and $\beta_2$, and the interaction effect $\beta_3$ as a function of the missing data (25% or 50%), and the $R^2$ ($R^2 = .25$ or .50) for correlated predictors ($\rho_{xz} = .5$).

Table A1

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Uncorrelated Predictors x and z, 25% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.02 | 0.02 | 0.35 | 0.35 | 94.6 | 94.7 |
| | 250 | 0.01 | 0.01 | 0.21 | 0.21 | 94.6 | 94.8 |
| | 500 | 0.00 | 0.00 | 0.15 | 0.15 | 95.0 | 95.2 |
| | 1,000 | 0.00 | 0.00 | 0.11 | 0.11 | 94.9 | 94.6 |
| $\beta_2$ | 100 | 0.01 | 0.02 | 0.32 | 0.33 | 94.6 | 93.7 |
| | 250 | 0.00 | 0.00 | 0.20 | 0.20 | 95.0 | 94.5 |
| | 500 | 0.00 | 0.00 | 0.14 | 0.14 | 94.4 | 93.7 |
| | 1,000 | 0.00 | 0.00 | 0.10 | 0.10 | 93.6 | 93.4 |
| $\beta_3$ | 100 | 0.00 | 0.01 | 0.36 | 0.37 | 94.4 | 94.4 |
| | 250 | −0.01 | 0.00 | 0.22 | 0.22 | 94.2 | 94.7 |
| | 500 | 0.00 | 0.00 | 0.15 | 0.16 | 93.9 | 94.2 |
| | 1,000 | 0.01 | 0.01 | 0.11 | 0.11 | 94.4 | 94.8 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.01 | 0.01 | 0.37 | 0.37 | 94.9 | 95.0 |
| | 250 | 0.00 | 0.01 | 0.23 | 0.23 | 95.7 | 95.9 |
| | 500 | 0.01 | 0.02 | 0.16 | 0.16 | 95.9 | 95.8 |
| | 1,000 | −0.01 | 0.01 | 0.12 | 0.12 | 94.9 | 94.3 |
| $\beta_2$ | 100 | −0.01 | **−0.13** | 0.33 | 0.36 | 94.2 | 91.3 |
| | 250 | −0.01 | **−0.14** | 0.20 | 0.25 | 94.7 | **86.3** |
| | 500 | 0.00 | **−0.13** | 0.14 | 0.19 | 95.7 | **84.3** |
| | 1,000 | 0.00 | **−0.13** | 0.10 | 0.16 | 95.5 | **74.3** |
| $\beta_3$ | 100 | −0.01 | 0.03 | 0.38 | 0.40 | 93.8 | 94.3 |
| | 250 | 0.00 | 0.03 | 0.24 | 0.25 | 94.3 | 94.5 |
| | 500 | 0.00 | 0.02 | 0.16 | 0.17 | 95.5 | 95.1 |
| | 1,000 | 0.00 | 0.03 | 0.11 | 0.12 | 94.9 | 93.9 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A2

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Uncorrelated Predictors x and z, 25% Missing Data on x, and $R^2$ = .50*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.00 | 0.01 | 0.20 | 0.21 | 94.6 | 93.8 |
| | 250 | 0.01 | 0.01 | 0.13 | 0.13 | 93.8 | 93.6 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.09 | 94.4 | 94.4 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.06 | 95.4 | 95.0 |
| $\beta_2$ | 100 | 0.01 | 0.01 | 0.19 | 0.20 | 94.9 | 93.1 |
| | 250 | −0.01 | −0.01 | 0.12 | 0.13 | 94.2 | 93.6 |
| | 500 | 0.00 | 0.00 | 0.08 | 0.09 | 94.7 | 94.1 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.06 | 94.6 | 93.7 |
| $\beta_3$ | 100 | −0.01 | 0.00 | 0.20 | 0.22 | 95.7 | 94.7 |
| | 250 | 0.00 | 0.01 | 0.12 | 0.13 | 95.9 | 95.1 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.09 | 94.5 | 94.2 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.06 | 95.2 | 95.1 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.00 | 0.00 | 0.20 | 0.21 | 95.1 | 94.7 |
| | 250 | 0.00 | 0.00 | 0.13 | 0.13 | 94.8 | 94.6 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.09 | 94.7 | 94.5 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.07 | 94.6 | 94.1 |
| $\beta_2$ | 100 | −0.01 | **−0.13** | 0.19 | 0.24 | 94.2 | **86.0** |
| | 250 | 0.00 | **−0.13** | 0.11 | 0.18 | 94.8 | **80.3** |
| | 500 | 0.00 | **−0.13** | 0.08 | 0.16 | 95.1 | **63.7** |
| | 1,000 | 0.00 | **−0.13** | 0.06 | 0.15 | 94.8 | **38.8** |
| $\beta_3$ | 100 | 0.00 | 0.02 | 0.22 | 0.23 | 94.2 | 93.1 |
| | 250 | 0.00 | 0.02 | 0.13 | 0.14 | 94.1 | 93.9 |
| | 500 | 0.00 | 0.02 | 0.09 | 0.10 | 95.5 | 94.9 |
| | 1,000 | 0.00 | 0.02 | 0.06 | 0.07 | 95.5 | 94.2 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A3

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Uncorrelated Predictors x and z, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.01 | 0.01 | 0.45 | 0.46 | 91.8 | 92.8 |
| | 250 | 0.01 | 0.02 | 0.26 | 0.27 | 93.7 | 93.9 |
| | 500 | 0.01 | 0.01 | 0.17 | 0.18 | 96.0 | 96.1 |
| | 1,000 | 0.01 | 0.01 | 0.13 | 0.13 | 94.2 | 94.3 |
| $\beta_2$ | 100 | 0.00 | 0.01 | 0.36 | 0.38 | 93.6 | 92.4 |
| | 250 | 0.00 | −0.01 | 0.21 | 0.23 | 95.2 | 93.6 |
| | 500 | 0.00 | 0.00 | 0.14 | 0.15 | 95.3 | 93.0 |
| | 1,000 | 0.00 | 0.00 | 0.10 | 0.11 | 96.0 | 93.4 |
| $\beta_3$ | 100 | −0.03 | 0.00 | 0.45 | 0.51 | **90.7** | 91.2 |
| | 250 | 0.00 | 0.02 | 0.26 | 0.29 | 93.1 | 92.7 |
| | 500 | 0.01 | 0.01 | 0.17 | 0.19 | 95.8 | 95.2 |
| | 1,000 | 0.00 | 0.00 | 0.12 | 0.13 | 95.1 | 93.9 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.02 | −0.01 | 0.47 | 0.50 | 92.6 | 92.6 |
| | 250 | 0.01 | 0.00 | 0.28 | 0.29 | 94.6 | 94.5 |
| | 500 | 0.02 | 0.01 | 0.20 | 0.22 | 93.9 | 93.5 |
| | 1,000 | 0.00 | −0.01 | 0.14 | 0.15 | 94.4 | 94.2 |
| $\beta_2$ | 100 | −0.02 | **−0.24** | 0.33 | 0.44 | 95.9 | **87.7** |
| | 250 | −0.03 | **−0.28** | 0.20 | 0.36 | 95.1 | **75.2** |
| | 500 | 0.00 | **−0.26** | 0.15 | 0.30 | 94.1 | **60.8** |
| | 1,000 | −0.01 | **−0.26** | 0.11 | 0.29 | 95.1 | **30.9** |
| $\beta_3$ | 100 | −0.03 | 0.01 | 0.48 | 0.56 | 91.4 | 92.2 |
| | 250 | 0.01 | 0.04 | 0.26 | 0.30 | 94.5 | 93.8 |
| | 500 | 0.00 | 0.03 | 0.18 | 0.22 | 95.1 | 94.5 |
| | 1,000 | 0.01 | 0.04 | 0.14 | 0.16 | 93.7 | 92.3 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A4

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Uncorrelated Predictors x and z, 50% Missing Data on x, and $R^2$ = .50*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.03 | 0.04 | 0.23 | 0.26 | 92.5 | 92.7 |
| | 250 | 0.01 | 0.01 | 0.15 | 0.16 | 93.3 | 92.7 |
| | 500 | 0.00 | 0.01 | 0.10 | 0.11 | 95.0 | 93.7 |
| | 1,000 | 0.00 | 0.00 | 0.07 | 0.08 | 94.0 | 93.7 |
| $\beta_2$ | 100 | 0.00 | 0.00 | 0.21 | 0.24 | 93.9 | 91.9 |
| | 250 | 0.00 | 0.00 | 0.12 | 0.14 | 96.2 | 94.0 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.10 | 94.4 | 93.7 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.07 | 94.1 | 93.3 |
| $\beta_3$ | 100 | 0.01 | 0.03 | 0.25 | 0.28 | 93.6 | 93.0 |
| | 250 | −0.01 | 0.00 | 0.14 | 0.16 | 95.3 | 94.2 |
| | 500 | 0.00 | 0.01 | 0.10 | 0.11 | 95.1 | 94.1 |
| | 1,000 | 0.00 | 0.00 | 0.07 | 0.07 | 94.7 | 94.9 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.02 | −0.02 | 0.26 | 0.28 | 91.0 | 93.0 |
| | 250 | 0.00 | −0.03 | 0.15 | 0.17 | 94.7 | 94.6 |
| | 500 | 0.00 | −0.03 | 0.10 | 0.12 | 94.3 | 94.1 |
| | 1,000 | 0.00 | −0.03 | 0.07 | 0.09 | 95.4 | 93.0 |
| $\beta_2$ | 100 | −0.01 | **−0.25** | 0.21 | 0.35 | 94.1 | **77.0** |
| | 250 | −0.01 | **−0.26** | 0.13 | 0.30 | 93.4 | **52.0** |
| | 500 | 0.00 | **−0.26** | 0.09 | 0.28 | 93.8 | **23.4** |
| | 1,000 | 0.00 | **−0.25** | 0.06 | 0.26 | 95.2 | **4.4** |
| $\beta_3$ | 100 | −0.03 | 0.00 | 0.26 | 0.31 | 91.5 | 92.8 |
| | 250 | 0.00 | 0.01 | 0.15 | 0.18 | 94.0 | 92.4 |
| | 500 | 0.00 | 0.01 | 0.10 | 0.12 | 94.6 | 93.5 |
| | 1,000 | 0.00 | 0.01 | 0.07 | 0.08 | 95.2 | 94.0 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A5

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 25% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.01 | 0.01 | 0.49 | 0.50 | 92.7 | 92.5 |
| | 250 | 0.01 | 0.01 | 0.29 | 0.30 | 95.5 | 94.7 |
| | 500 | −0.01 | −0.01 | 0.21 | 0.22 | 95.0 | 95.1 |
| | 1,000 | −0.01 | −0.01 | 0.14 | 0.15 | 94.8 | 95.4 |
| $\beta_2$ | 100 | −0.02 | −0.02 | 0.45 | 0.46 | 94.9 | 94.3 |
| | 250 | 0.00 | 0.00 | 0.27 | 0.28 | 94.7 | 93.9 |
| | 500 | 0.00 | 0.01 | 0.20 | 0.20 | 95.6 | 94.6 |
| | 1,000 | 0.01 | 0.01 | 0.14 | 0.14 | 94.6 | 94.1 |
| $\beta_3$ | 100 | 0.00 | 0.02 | 0.38 | 0.42 | 94.2 | 93.8 |
| | 250 | 0.00 | 0.01 | 0.23 | 0.24 | 93.4 | 94.3 |
| | 500 | −0.01 | 0.00 | 0.16 | 0.17 | 94.3 | 94.0 |
| | 1,000 | 0.00 | 0.01 | 0.12 | 0.12 | 92.5 | 92.8 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.01 | 0.00 | 0.53 | 0.52 | 93.4 | 93.6 |
| | 250 | 0.01 | 0.00 | 0.33 | 0.33 | 94.7 | 94.8 |
| | 500 | 0.00 | 0.00 | 0.23 | 0.23 | 94.9 | 95.0 |
| | 1,000 | −0.01 | −0.01 | 0.16 | 0.15 | 95.4 | 94.9 |
| $\beta_2$ | 100 | 0.00 | **−0.13** | 0.47 | 0.50 | 93.2 | 92.4 |
| | 250 | −0.01 | **−0.15** | 0.27 | 0.32 | 95.3 | 91.8 |
| | 500 | 0.00 | **−0.15** | 0.20 | 0.26 | 94.3 | **88.0** |
| | 1,000 | 0.01 | **−0.14** | 0.14 | 0.20 | 95.5 | **83.7** |
| $\beta_3$ | 100 | 0.00 | 0.04 | 0.41 | 0.41 | 92.6 | 92.4 |
| | 250 | −0.01 | 0.04 | 0.23 | 0.24 | 94.7 | 94.7 |
| | 500 | 0.00 | 0.05 | 0.17 | 0.18 | 93.4 | 92.4 |
| | 1,000 | 0.00 | 0.05 | 0.11 | 0.13 | 96.3 | 92.3 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A6

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 25% Missing Data on x, and $R^2$ = .50*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.00 | 0.01 | 0.28 | 0.29 | 93.7 | 93.7 |
| | 250 | 0.01 | 0.01 | 0.17 | 0.18 | 94.0 | 94.5 |
| | 500 | 0.00 | 0.00 | 0.12 | 0.12 | 94.5 | 94.1 |
| | 1,000 | 0.00 | 0.00 | 0.08 | 0.09 | 95.6 | 95.9 |
| $\beta_2$ | 100 | −0.01 | −0.01 | 0.27 | 0.28 | 93.7 | 92.6 |
| | 250 | 0.00 | 0.00 | 0.16 | 0.17 | 94.9 | 93.3 |
| | 500 | 0.00 | 0.00 | 0.11 | 0.12 | 95.2 | 93.1 |
| | 1,000 | 0.00 | 0.00 | 0.08 | 0.09 | 95.1 | 94.0 |
| $\beta_3$ | 100 | 0.00 | 0.01 | 0.22 | 0.24 | 95.0 | 94.2 |
| | 250 | 0.01 | 0.01 | 0.14 | 0.14 | 94.7 | 93.4 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.10 | 95.2 | 95.0 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.07 | 96.3 | 95.8 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.01 | −0.01 | 0.30 | 0.30 | 92.6 | 93.6 |
| | 250 | 0.01 | −0.02 | 0.18 | 0.17 | 95.1 | 94.8 |
| | 500 | 0.01 | −0.02 | 0.13 | 0.13 | 93.9 | 93.3 |
| | 1,000 | 0.00 | −0.03 | 0.09 | 0.09 | 95.6 | 94.6 |
| $\beta_2$ | 100 | −0.01 | **−0.14** | 0.26 | 0.31 | 94.4 | **89.9** |
| | 250 | −0.01 | **−0.14** | 0.15 | 0.21 | 96.8 | **87.0** |
| | 500 | −0.01 | **−0.14** | 0.12 | 0.18 | 94.2 | **76.2** |
| | 1,000 | 0.00 | **−0.13** | 0.08 | 0.15 | 96.3 | **66.1** |
| $\beta_3$ | 100 | 0.00 | 0.02 | 0.23 | 0.22 | 93.7 | 93.5 |
| | 250 | 0.00 | 0.01 | 0.13 | 0.13 | 95.8 | 95.0 |
| | 500 | 0.00 | 0.02 | 0.10 | 0.09 | 95.2 | 94.0 |
| | 1,000 | 0.00 | 0.02 | 0.06 | 0.06 | 95.5 | 95.1 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A7

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.05 | **0.06** | 0.60 | 0.62 | 92.6 | 92.0 |
| | 250 | 0.01 | 0.01 | 0.36 | 0.37 | 94.6 | 94.7 |
| | 500 | 0.01 | 0.01 | 0.24 | 0.25 | 95.0 | 94.6 |
| | 1,000 | 0.01 | 0.01 | 0.17 | 0.18 | 95.4 | 95.0 |
| $\beta_2$ | 100 | −0.01 | −0.02 | 0.50 | 0.55 | 93.3 | 92.6 |
| | 250 | 0.00 | 0.00 | 0.31 | 0.33 | 94.1 | 92.6 |
| | 500 | −0.01 | −0.01 | 0.21 | 0.22 | 95.9 | 94.7 |
| | 1,000 | 0.00 | 0.00 | 0.15 | 0.16 | 94.6 | 94.1 |
| $\beta_3$ | 100 | −0.01 | 0.00 | 0.43 | 0.53 | 93.8 | 93.5 |
| | 250 | 0.01 | 0.03 | 0.26 | 0.30 | 95.0 | 96.0 |
| | 500 | 0.00 | 0.01 | 0.17 | 0.21 | 94.4 | 93.2 |
| | 1,000 | 0.00 | 0.00 | 0.12 | 0.14 | 95.2 | 95.3 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.03 | 0.00 | 0.67 | 0.66 | **90.5** | 92.7 |
| | 250 | 0.00 | −0.03 | 0.41 | 0.41 | 93.4 | 94.7 |
| | 500 | 0.00 | −0.03 | 0.29 | 0.28 | 94.1 | 94.9 |
| | 1,000 | 0.00 | −0.02 | 0.20 | 0.20 | 95.1 | 95.1 |
| $\beta_2$ | 100 | −0.03 | **−0.29** | 0.51 | 0.62 | 94.1 | **88.6** |
| | 250 | 0.00 | **−0.27** | 0.30 | 0.43 | 94.5 | **86.1** |
| | 500 | 0.00 | **−0.27** | 0.20 | 0.35 | 95.7 | **79.3** |
| | 1,000 | 0.00 | **−0.28** | 0.15 | 0.32 | 95.3 | **61.9** |
| $\beta_3$ | 100 | 0.05 | **0.07** | 0.48 | 0.54 | 92.2 | 91.2 |
| | 250 | 0.01 | 0.04 | 0.28 | 0.30 | 93.5 | 93.2 |
| | 500 | 0.00 | 0.02 | 0.18 | 0.20 | 94.9 | 93.7 |
| | 1,000 | 0.00 | 0.03 | 0.13 | 0.15 | 94.2 | 92.5 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table A8

*Study 1: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 50% Missing Data on x, and $R^2$ = .50*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 100 | 0.03 | 0.03 | 0.34 | 0.37 | 94.6 | 93.2 |
| | 250 | 0.02 | 0.03 | 0.21 | 0.22 | 93.5 | 92.8 |
| | 500 | 0.01 | 0.01 | 0.14 | 0.15 | 94.6 | 94.5 |
| | 1,000 | 0.00 | 0.00 | 0.10 | 0.11 | 94.6 | 94.2 |
| $\beta_2$ | 100 | 0.00 | 0.01 | 0.30 | 0.35 | 94.1 | 91.1 |
| | 250 | −0.02 | −0.02 | 0.18 | 0.20 | 94.9 | 92.6 |
| | 500 | −0.01 | 0.00 | 0.12 | 0.13 | 95.1 | 93.7 |
| | 1,000 | 0.00 | 0.00 | 0.09 | 0.10 | 94.0 | 93.5 |
| $\beta_3$ | 100 | 0.00 | 0.04 | 0.25 | 0.30 | 92.9 | 92.7 |
| | 250 | −0.01 | 0.00 | 0.15 | 0.17 | 94.5 | 94.6 |
| | 500 | 0.00 | 0.01 | 0.10 | 0.12 | 94.8 | 94.8 |
| | 1,000 | 0.00 | 0.00 | 0.07 | 0.08 | 94.6 | 94.0 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 100 | 0.03 | −0.04 | 0.37 | 0.37 | 92.3 | 93.0 |
| | 250 | 0.01 | **−0.06** | 0.23 | 0.23 | 93.2 | 92.9 |
| | 500 | 0.01 | **−0.06** | 0.16 | 0.17 | 93.3 | 91.6 |
| | 1,000 | 0.00 | **−0.07** | 0.10 | 0.13 | 95.1 | **90.5** |
| $\beta_2$ | 100 | −0.04 | **−0.29** | 0.29 | 0.42 | 94.5 | **81.9** |
| | 250 | −0.01 | **−0.26** | 0.17 | 0.33 | 95.3 | **70.6** |
| | 500 | −0.01 | **−0.27** | 0.12 | 0.30 | 93.3 | **47.0** |
| | 1,000 | 0.00 | **−0.25** | 0.08 | 0.27 | 94.7 | **22.0** |
| $\beta_3$ | 100 | −0.01 | −0.01 | 0.26 | 0.27 | 92.8 | 94.3 |
| | 250 | 0.00 | −0.01 | 0.16 | 0.15 | 92.8 | 93.4 |
| | 500 | 0.00 | −0.01 | 0.11 | 0.10 | 94.7 | 94.2 |
| | 1,000 | 0.00 | −0.01 | 0.07 | 0.07 | 95.0 | 94.1 |

*Note.* FRM = factored regression modeling approach; JAV = maximum likelihood estimation with assumption of normally distributed interaction effect *xz*; RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

## Supplement B: Additional Simulation for Nonlinear Effects with a Continuous Predictor

In this simulation study, we investigated how the different missing data approaches performed for a multiple regression analysis with linear and quadratic effects for the predictor $x$. Again, we assumed that data were missing on the predictor variable $x$. Paralleling the analysis of interaction effects, the nonlinearly transformed incomplete predictor variable (i.e., $x^2$) could be included as just another variable in the JAV approach (Seaman et al., 2012; von Hippel, 2009). We also implemented the factored regression modeling approach (FRM) in which the regression model with a quadratic effect was the substantive model of interest.

### Method

**Simulation model and conditions.** The population model used to generate the data was the regression model with linear and quadratic effects as given by Equation 2. The predictor $x$ was assumed to be normally distributed with zero mean and variance of one. The regression coefficients were set to $\beta_0 = 4$, $\beta_1 = -4$, and $\beta_2 = 1$. These coefficients reflect a moderately strong curvilinear relationship between $y$ and $x$ (see Bartlett et al., 2015). The residuals followed a normal distribution, and the standard deviations were chosen such that the explained variance ($R^2$) was equal to .25 or .50. This resulted in the following effect sizes for the quadratic effect (proportion of variance in $y$ explained after taking into account the linear effect of $x$): .25 (for $R^2 = .25$) and .50 (for $R^2 = .50$). The sample sizes were set to $N = 100$, 250, 500, and 1,000.

Missing data on the predictor $x$ were generated as in Study 1 (see Equation 19). We again assumed that the data were MCAR (missingness on $x$ was independent of $y$; i.e., $\rho_{mis} = 0$) or MAR (missingness on $x$ depended on $y$; $\rho_{mis} = .7$) and set the missing data rates to 25% and 50%. The same specifications as in Study 1 were used to implement the two missing data approaches.

For each of $2 \times 4 \times 2 \times 2 = 32$ conditions, 1,000 data sets were simulated, and the missing values were treated with both different missing data approaches. Again, we calculated the relative bias, RMSE, and coverage rate to evaluate the quality of the parameter estimates.

### Results

Table B1 shows the relative bias, RMSE, and the coverage rates for the regression intercept $\beta_0$, the linear effect $\beta_1$, and the quadratic effect $\beta_2$ as a function of the sample size ($N = 100$, 200, 500, and 1,000) and the missing data mechanism (MCAR or MAR) for a missing data rate of 25% and $R^2 = .25$. Table B2 shows the relative bias, RMSE, and the coverage rates for a missing data rate of 25% and $R^2 = .50$. Table B3 shows the relative bias, RMSE, and the coverage rates for a missing data rate of 50% and $R^2 = .25$. Table B4 shows the relative bias, RMSE, and the coverage rates for a missing data rate of 50% and $R^2 = .50$.

As expected, for MCAR data, both approaches produced approximately unbiased estimates, with the exception that the FRM approach provided slightly negatively biased estimates for the regression intercept when the sample size was small ($N = 100$). However, when the data in $x$ were MAR, the JAV approach produced negatively biased estimates of the linear and quadratic effects of $x$. The bias for the JAV approach did not depend on the sample size.

The results for RMSE showed that the FRM approach required a relatively large sample size ($N \geq 500$) to be more accurate overall than the JAV approach; with smaller sample sizes, none of the approaches were consistently superior to the other approaches.

The FRM approach provided acceptable coverage rates with MCAR and MAR data. By contrast, the coverage rates produced by the JAV approaches were too small when the data were MAR. Especially with larger sample sizes ($N \geq 500$), the coverage rates for the linear and quadratic effects ($\beta_1$ and $\beta_2$) were clearly below the nominal 95% level.

### Summary and Discussion

Consistent with previous results, the simulation showed that the JAV approach, which simply treated the nonlinear transformation of the predictor as another variable, produced biased estimates of quadratic effects when the data for the predictor were MAR (Bartlett et al., 2015; Seaman et al., 2012). Further simulations with a large sample size (and 50% missing data in $x$) showed again that the estimated bias for the JAV approach depended on the missing data mechanism and the explained variance $R^2$. By contrast, the FRM approach, which accommodates the substantive analysis model, provided approximately unbiased estimates with accurate standard errors. In this simulation, we focused on a quadratic relationship between the predictor and the outcome variable. However, the substantive model could easily be extended to include other nonlinear transformations such as higher order polynomials (i.e., cubic) and fractional polynomials or B-splines (Fox, 2016).

Table B1

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with a Quadratic Effect as a Function of Sample Size for 25% Missing Data on $x$, and $R^2 = .25$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | 0.01 | 0.95 | 0.97 | 93.6 | 92.7 |
| | 250 | 0.00 | 0.01 | 0.56 | 0.57 | 94.6 | 94.6 |
| | 500 | 0.00 | 0.00 | 0.39 | 0.39 | 95.3 | 94.8 |
| | 1,000 | 0.00 | 0.00 | 0.27 | 0.27 | 95.2 | 94.9 |
| $\beta_1$ | 100 | 0.00 | 0.01 | 0.91 | 0.94 | 94.0 | 93.8 |
| | 250 | 0.00 | 0.01 | 0.55 | 0.56 | 94.0 | 93.9 |
| | 500 | 0.00 | 0.00 | 0.38 | 0.39 | 94.9 | 94.4 |
| | 1,000 | 0.00 | 0.00 | 0.26 | 0.26 | 96.4 | 95.3 |
| $\beta_2$ | 100 | 0.00 | 0.00 | 0.21 | 0.22 | 94.2 | 93.7 |
| | 250 | 0.00 | 0.00 | 0.13 | 0.13 | 94.7 | 93.7 |
| | 500 | 0.00 | 0.00 | 0.09 | 0.09 | 93.9 | 93.3 |
| | 1,000 | 0.00 | 0.00 | 0.06 | 0.06 | 96.0 | 95.1 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | 0.02 | 0.87 | 0.87 | 96.0 | 94.6 |
| | 250 | 0.00 | 0.01 | 0.54 | 0.54 | 94.9 | 93.8 |
| | 500 | 0.00 | 0.01 | 0.38 | 0.38 | 94.4 | 92.5 |
| | 1,000 | 0.00 | 0.01 | 0.27 | 0.27 | 94.7 | 93.5 |
| $\beta_1$ | 100 | 0.01 | −0.01 | 0.86 | 0.82 | 95.0 | 94.6 |
| | 250 | −0.01 | −0.02 | 0.51 | 0.49 | 94.8 | 93.9 |
| | 500 | 0.00 | −0.02 | 0.36 | 0.34 | 94.7 | 92.7 |
| | 1,000 | 0.00 | −0.01 | 0.25 | 0.24 | 94.2 | 92.9 |
| $\beta_2$ | 100 | 0.01 | −0.01 | 0.21 | 0.20 | 94.2 | 94.2 |
| | 250 | −0.01 | −0.02 | 0.12 | 0.11 | 94.9 | 94.4 |
| | 500 | 0.00 | −0.02 | 0.08 | 0.08 | 94.7 | 94.1 |
| | 1,000 | 0.00 | −0.01 | 0.06 | 0.06 | 94.2 | 93.0 |

*Note.* FRM = factored regression modeling approach; JAV = Maximum likelihood estimation with assumption of normally distributed quadratic effect $x^2$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $N$ = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table B2

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with a Quadratic Effect as a Function of Sample Size for 25% Missing Data on $x$, and $R^2 = .50$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_0$ | 100 | 0.00 | 0.01 | 0.55 | 0.58 | 93.7 | 91.8 |
| | 250 | 0.00 | 0.00 | 0.32 | 0.33 | 95.6 | 94.3 |
| | 500 | 0.00 | 0.00 | 0.23 | 0.24 | 94.8 | 93.2 |
| | 1,000 | 0.00 | 0.00 | 0.16 | 0.17 | 94.1 | 93.2 |
| $\beta_1$ | 100 | 0.00 | 0.01 | 0.52 | 0.57 | 94.4 | 92.3 |
| | 250 | 0.00 | 0.00 | 0.31 | 0.32 | 94.9 | 93.7 |
| | 500 | 0.00 | 0.00 | 0.21 | 0.23 | 95.3 | 93.3 |
| | 1,000 | 0.00 | 0.00 | 0.15 | 0.16 | 94.3 | 92.5 |
| $\beta_2$ | 100 | 0.00 | 0.01 | 0.13 | 0.14 | 94.6 | 93.0 |
| | 250 | 0.00 | 0.00 | 0.07 | 0.08 | 94.7 | 92.5 |
| | 500 | 0.00 | 0.00 | 0.05 | 0.05 | 96.2 | 93.6 |
| | 1,000 | 0.00 | 0.00 | 0.04 | 0.04 | 94.7 | 91.6 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_0$ | 100 | 0.00 | −0.02 | 0.50 | 0.49 | 94.6 | 93.9 |
| | 250 | 0.00 | −0.01 | 0.31 | 0.30 | 93.8 | 92.9 |
| | 500 | 0.00 | −0.02 | 0.21 | 0.21 | 94.8 | 91.9 |
| | 1,000 | 0.00 | −0.01 | 0.15 | 0.15 | 95.1 | 91.6 |
| $\beta_1$ | 100 | 0.00 | **−0.06** | 0.48 | 0.50 | 95.9 | **89.8** |
| | 250 | 0.00 | −0.05 | 0.30 | 0.34 | 94.6 | **86.0** |
| | 500 | 0.00 | **−0.06** | 0.20 | 0.29 | 95.5 | **73.6** |
| | 1,000 | 0.00 | **−0.06** | 0.14 | 0.25 | 95.2 | **55.9** |
| $\beta_2$ | 100 | 0.00 | **−0.06** | 0.11 | 0.12 | 95.3 | **89.6** |
| | 250 | 0.00 | −0.05 | 0.07 | 0.08 | 94.5 | **82.8** |
| | 500 | 0.00 | **−0.06** | 0.05 | 0.07 | 95.8 | **71.3** |
| | 1,000 | 0.00 | **−0.06** | 0.03 | 0.06 | 95.1 | **50.1** |

*Note.* FRM = factored regression modeling approach; JAV = Maximum likelihood estimation with assumption of normally distributed quadratic effect $x^2$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $N$ = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table B3

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with a Quadratic Effect as a Function of Sample Size for 50% Missing Data on $x$, and $R^2 = .25$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | 0.03 | 1.20 | 1.27 | 93.4 | 91.8 |
| | 250 | 0.00 | 0.00 | 0.65 | 0.68 | 95.6 | 94.4 |
| | 500 | 0.00 | 0.00 | 0.46 | 0.47 | 95.1 | 94.2 |
| | 1,000 | 0.00 | 0.00 | 0.32 | 0.33 | 95.8 | 94.5 |
| $\beta_1$ | 100 | 0.01 | 0.03 | 1.14 | 1.23 | 93.5 | 92.8 |
| | 250 | 0.00 | 0.01 | 0.62 | 0.66 | 95.5 | 94.0 |
| | 500 | 0.00 | 0.00 | 0.45 | 0.47 | 94.9 | 93.9 |
| | 1,000 | 0.00 | 0.00 | 0.30 | 0.31 | 94.7 | 94.4 |
| $\beta_2$ | 100 | 0.01 | 0.03 | 0.27 | 0.29 | 93.7 | 92.6 |
| | 250 | 0.00 | 0.01 | 0.15 | 0.16 | 95.8 | 94.1 |
| | 500 | 0.00 | 0.00 | 0.11 | 0.11 | 94.1 | 93.5 |
| | 1,000 | 0.00 | 0.00 | 0.07 | 0.07 | 95.5 | 93.8 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | 0.01 | 1.10 | 1.06 | 93.7 | **90.8** |
| | 250 | 0.00 | 0.00 | 0.62 | 0.59 | 94.7 | 93.7 |
| | 500 | 0.01 | 0.01 | 0.44 | 0.42 | 94.6 | 93.5 |
| | 1,000 | 0.00 | 0.01 | 0.30 | 0.29 | 94.5 | 92.8 |
| $\beta_1$ | 100 | 0.01 | **−0.08** | 1.05 | 0.96 | 94.0 | 92.4 |
| | 250 | 0.00 | **−0.07** | 0.56 | 0.57 | 94.7 | **89.9** |
| | 500 | 0.01 | **−0.07** | 0.41 | 0.44 | 94.8 | **85.8** |
| | 1,000 | 0.00 | **−0.07** | 0.28 | 0.36 | 94.4 | **75.6** |
| $\beta_2$ | 100 | 0.00 | **−0.08** | 0.24 | 0.22 | 94.4 | 93.4 |
| | 250 | 0.00 | **−0.07** | 0.13 | 0.13 | 94.3 | **89.3** |
| | 500 | 0.00 | **−0.07** | 0.09 | 0.10 | 94.2 | **84.9** |
| | 1,000 | 0.00 | **−0.07** | 0.06 | 0.09 | 95.0 | **72.6** |

*Note.* FRM = factored regression modeling approach; JAV = Maximum likelihood estimation with assumption of normally distributed quadratic effect $x^2$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $N$ = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table B4

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with a Quadratic Effect as a Function of Sample Size for 50% Missing Data on $x$, and $R^2 = .50$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | JAV | FRM | JAV | FRM | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | 0.03 | 0.68 | 0.76 | 94.0 | 91.0 |
| | 250 | 0.00 | 0.01 | 0.40 | 0.44 | 93.4 | **90.1** |
| | 500 | 0.00 | 0.01 | 0.28 | 0.31 | 93.6 | **89.6** |
| | 1,000 | 0.00 | 0.00 | 0.20 | 0.21 | 94.1 | **90.8** |
| $\beta_1$ | 100 | 0.01 | 0.03 | 0.65 | 0.74 | 94.2 | 91.0 |
| | 250 | 0.00 | 0.02 | 0.37 | 0.42 | 94.3 | 91.8 |
| | 500 | 0.00 | 0.01 | 0.25 | 0.29 | 94.5 | **88.7** |
| | 1,000 | 0.00 | 0.00 | 0.18 | 0.21 | 94.2 | **88.8** |
| $\beta_2$ | 100 | 0.01 | 0.03 | 0.15 | 0.18 | 93.0 | **90.5** |
| | 250 | 0.01 | 0.02 | 0.09 | 0.10 | 94.6 | 91.2 |
| | 500 | 0.00 | 0.01 | 0.06 | 0.07 | 94.4 | **89.3** |
| | 1,000 | 0.00 | 0.00 | 0.04 | 0.05 | 94.7 | **89.0** |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_0$ | 100 | 0.01 | −0.03 | 0.56 | 0.53 | 93.8 | 92.0 |
| | 250 | 0.00 | −0.04 | 0.33 | 0.34 | 93.8 | **90.1** |
| | 500 | 0.00 | −0.04 | 0.23 | 0.26 | 95.2 | **86.9** |
| | 1,000 | 0.00 | −0.04 | 0.16 | 0.22 | 95.5 | **78.0** |
| $\beta_1$ | 100 | 0.00 | **−0.12** | 0.52 | 0.66 | 95.3 | **74.1** |
| | 250 | 0.00 | **−0.12** | 0.30 | 0.56 | 95.0 | **49.8** |
| | 500 | 0.00 | **−0.12** | 0.21 | 0.53 | 95.4 | **21.1** |
| | 1,000 | 0.00 | **−0.13** | 0.14 | 0.52 | 95.6 | **2.1** |
| $\beta_2$ | 100 | 0.00 | **−0.12** | 0.12 | 0.16 | 94.6 | **73.8** |
| | 250 | 0.00 | **−0.12** | 0.07 | 0.14 | 94.6 | **44.5** |
| | 500 | 0.00 | **−0.12** | 0.05 | 0.13 | 95.2 | **16.1** |
| | 1,000 | 0.00 | **−0.13** | 0.03 | 0.13 | 95.0 | **0.9** |

*Note.* FRM = factored regression modeling approach; JAV = Maximum likelihood estimation with assumption of normally distributed quadratic effect $x^2$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $N$ = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

## Supplement C: Additional Results for Study 2

The following pages contain the additional results for Simulation Study 2. Table C1 shows the relative bias, RMSE, and the coverage rates for the first order effects $\beta_1$ and $\beta_2$ and the interaction effect $\beta_3$ as a function of the probability $p$ that the dichotomous predictor $x = 1$ for a sample size of $N = 500$, a missing data rate of 50% and $R^2 = .25$. Table C2 shows the relative bias, RMSE, and the coverage rates for a sample size of $N = 1,000$, a missing data rate of 50% and $R^2 = .25$.

Table C1

*Study 2: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with an Interaction Effect and a Dichotomous Predictor x as a Function of Probability (p) x = 1 for Uncorrelated Predictors x and z, a Sample Size of N = 500, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | | RMSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parm. | *p* | FRM | FRM* | JAV | FRM | FRM* | JAV | FRM | FRM* | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | | | | |
| $\beta_1$ | .1 | 0.02 | 0.02 | 0.03 | 0.44 | 0.45 | 0.44 | 94.3 | 95.1 | 94.6 |
| | .2 | 0.00 | 0.00 | 0.00 | 0.37 | 0.37 | 0.38 | 93.9 | 94.3 | 94.7 |
| | .5 | 0.00 | 0.00 | 0.00 | 0.35 | 0.35 | 0.36 | 94.4 | 94.5 | 94.4 |
| | .8 | 0.03 | 0.03 | 0.03 | 0.52 | 0.52 | 0.52 | 95.2 | 95.4 | 95.2 |
| | .9 | 0.00 | 0.01 | 0.02 | 0.75 | 0.75 | 0.75 | 94.9 | 94.9 | 95.2 |
| $\beta_2$ | .1 | 0.00 | 0.00 | 0.00 | 0.10 | 0.11 | 0.11 | 94.2 | 94.0 | 93.6 |
| | .2 | 0.00 | 0.00 | 0.00 | 0.13 | 0.13 | 0.13 | 94.3 | 94.7 | 94.3 |
| | .5 | 0.00 | 0.00 | −0.01 | 0.22 | 0.22 | 0.23 | 95.5 | 95.3 | 95.0 |
| | .8 | 0.00 | 0.00 | −0.01 | 0.44 | 0.44 | 0.46 | 93.3 | 93.1 | 94.3 |
| | .9 | 0.02 | 0.02 | 0.00 | 0.72 | 0.73 | 0.74 | 93.8 | 93.3 | 94.6 |
| $\beta_3$ | .1 | 0.02 | 0.02 | 0.04 | 0.44 | 0.45 | 0.48 | 94.2 | 93.5 | 93.5 |
| | .2 | −0.02 | −0.02 | −0.01 | 0.35 | 0.36 | 0.38 | 95.0 | 95.3 | 94.3 |
| | .5 | 0.00 | 0.00 | 0.01 | 0.34 | 0.34 | 0.36 | 94.0 | 94.5 | 94.9 |
| | .8 | −0.01 | −0.01 | 0.01 | 0.52 | 0.53 | 0.54 | 93.7 | 93.9 | 94.4 |
| | .9 | −0.02 | −0.02 | 0.00 | 0.77 | 0.78 | 0.79 | 94.2 | 94.6 | 94.8 |
| | *Missing at Random (MAR)* | | | | | | | | | |
| $\beta_1$ | .1 | −0.03 | −0.04 | −0.03 | 0.49 | 0.54 | 0.51 | 93.0 | **90.4** | 94.3 |
| | .2 | −0.01 | −0.02 | −0.03 | 0.40 | 0.42 | 0.40 | 94.6 | 93.3 | 94.3 |
| | .5 | 0.00 | 0.00 | −0.01 | 0.42 | 0.42 | 0.42 | 94.2 | 94.2 | 94.8 |
| | .8 | 0.02 | 0.03 | 0.01 | 0.63 | 0.62 | 0.62 | 93.7 | 94.2 | 95.1 |
| | .9 | 0.00 | 0.04 | −0.02 | 0.89 | 0.89 | 0.87 | 93.0 | 93.6 | 95.7 |
| $\beta_2$ | .1 | −0.01 | 0.00 | −0.05 | 0.11 | 0.10 | 0.13 | 94.7 | 95.0 | 91.7 |
| | .2 | −0.01 | 0.00 | **−0.08** | 0.13 | 0.13 | 0.17 | 94.3 | 94.8 | **88.4** |
| | .5 | 0.00 | 0.00 | **−0.11** | 0.24 | 0.24 | 0.29 | 92.6 | 93.2 | **90.9** |
| | .8 | 0.04 | 0.02 | **−0.07** | 0.50 | 0.50 | 0.54 | 93.4 | 95.1 | 95.7 |
| | .9 | **0.08** | 0.05 | **−0.08** | 0.86 | 0.95 | 0.95 | **88.3** | **90.6** | 94.3 |
| $\beta_3$ | .1 | 0.01 | 0.00 | **0.10** | 0.48 | 0.52 | 0.55 | 91.8 | **89.6** | 92.3 |
| | .2 | 0.01 | −0.02 | **0.09** | 0.38 | 0.40 | 0.44 | 93.7 | 92.6 | 92.6 |
| | .5 | −0.01 | −0.03 | 0.05 | 0.41 | 0.40 | 0.43 | 93.1 | 92.8 | 93.1 |
| | .8 | −0.05 | −0.03 | 0.03 | 0.61 | 0.60 | 0.62 | 92.4 | 94.6 | 95.4 |
| | .9 | **−0.10** | **−0.07** | 0.05 | 0.94 | 1.03 | 1.00 | **87.8** | **90.1** | 93.6 |

*Note.* FRM = factored regression modeling approach; FRM* = factored regression modeling with assumption of normally distributed dichotomous predictor $x$; JAV = Maximum likelihood estimation with assumptions of normally distributed dichotomous predictor $x$ and interaction effect $xz$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $p$ = Probability $x = 1$ for dichotomous covariate $x$. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table C2

*Study 2: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with an Interaction Effect and a Dichotomous Predictor x as a Function of Probability (p) x = 1 for Uncorrelated Predictors x and z, a Sample Size of N = 1,000, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | | RMSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parm. | *p* | FRM | FRM* | JAV | FRM | FRM* | JAV | FRM | FRM* | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | | | | |
| $\beta_1$ | .1 | 0.00 | 0.00 | 0.00 | 0.32 | 0.32 | 0.32 | 93.0 | 93.3 | 94.2 |
| | .2 | 0.00 | 0.00 | 0.00 | 0.25 | 0.25 | 0.26 | 94.8 | 94.0 | 95.1 |
| | .5 | 0.01 | 0.02 | 0.02 | 0.24 | 0.24 | 0.24 | 96.6 | 96.3 | 96.8 |
| | .8 | 0.01 | 0.01 | 0.01 | 0.35 | 0.35 | 0.36 | 95.9 | 95.8 | 95.9 |
| | .9 | 0.01 | 0.01 | 0.02 | 0.51 | 0.51 | 0.51 | 95.2 | 94.9 | 94.9 |
| $\beta_2$ | .1 | 0.00 | 0.00 | 0.00 | 0.07 | 0.07 | 0.07 | 96.3 | 96.3 | 95.8 |
| | .2 | 0.00 | 0.00 | 0.00 | 0.09 | 0.09 | 0.09 | 95.5 | 95.4 | 94.0 |
| | .5 | 0.00 | 0.00 | 0.00 | 0.15 | 0.15 | 0.16 | 95.2 | 95.3 | 95.3 |
| | .8 | 0.00 | 0.00 | 0.00 | 0.32 | 0.32 | 0.32 | 93.3 | 92.9 | 95.0 |
| | .9 | 0.01 | 0.01 | 0.00 | 0.46 | 0.46 | 0.48 | 93.3 | 93.5 | 94.0 |
| $\beta_3$ | .1 | −0.02 | −0.01 | −0.01 | 0.30 | 0.30 | 0.32 | 94.3 | 94.9 | 94.3 |
| | .2 | 0.00 | 0.00 | 0.01 | 0.25 | 0.25 | 0.26 | 94.4 | 94.3 | 94.9 |
| | .5 | 0.00 | 0.00 | 0.00 | 0.25 | 0.25 | 0.26 | 94.0 | 94.0 | 95.2 |
| | .8 | 0.00 | −0.01 | 0.00 | 0.38 | 0.38 | 0.39 | 92.9 | 93.0 | 94.0 |
| | .9 | −0.01 | −0.01 | 0.00 | 0.50 | 0.50 | 0.52 | 93.5 | 94.2 | 94.0 |
| | *Missing at Random (MAR)* | | | | | | | | | |
| $\beta_1$ | .1 | 0.00 | −0.01 | 0.01 | 0.34 | 0.37 | 0.34 | 93.8 | 92.4 | 94.2 |
| | .2 | 0.01 | −0.01 | 0.00 | 0.28 | 0.29 | 0.28 | 95.5 | 95.0 | 95.6 |
| | .5 | 0.00 | −0.01 | −0.01 | 0.30 | 0.29 | 0.30 | 94.9 | 94.7 | 95.0 |
| | .8 | −0.04 | −0.03 | −0.05 | 0.47 | 0.46 | 0.46 | 92.2 | 93.1 | 94.1 |
| | .9 | 0.01 | 0.02 | 0.01 | 0.65 | 0.64 | 0.63 | 94.3 | 94.8 | 95.2 |
| $\beta_2$ | .1 | 0.00 | 0.01 | −0.05 | 0.08 | 0.08 | 0.10 | 93.9 | 93.8 | **87.2** |
| | .2 | 0.00 | 0.01 | **−0.08** | 0.09 | 0.09 | 0.13 | 95.8 | 94.9 | **88.0** |
| | .5 | 0.00 | 0.01 | **−0.11** | 0.17 | 0.17 | 0.23 | 94.1 | 93.7 | **88.9** |
| | .8 | 0.00 | −0.02 | **−0.11** | 0.36 | 0.37 | 0.41 | 93.0 | 94.2 | 94.0 |
| | .9 | **0.09** | 0.05 | −0.01 | 0.58 | 0.59 | 0.62 | 93.0 | 93.2 | 93.9 |
| $\beta_3$ | .1 | −0.01 | −0.03 | **0.07** | 0.32 | 0.35 | 0.37 | 93.7 | 92.7 | 93.6 |
| | .2 | 0.00 | −0.03 | **0.07** | 0.27 | 0.27 | 0.30 | 93.8 | 92.9 | 93.0 |
| | .5 | −0.01 | −0.02 | 0.05 | 0.28 | 0.28 | 0.31 | 92.7 | 92.9 | 91.7 |
| | .8 | −0.02 | 0.01 | **0.07** | 0.44 | 0.44 | 0.47 | 92.5 | 94.1 | 94.2 |
| | .9 | **−0.10** | **−0.07** | −0.02 | 0.64 | 0.64 | 0.67 | 92.2 | 93.4 | 94.1 |

*Note.* FRM = factored regression modeling approach; FRM* = factored regression modeling with assumption of normally distributed dichotomous predictor $x$; JAV = Maximum likelihood estimation with assumptions of normally distributed dichotomous predictor $x$ and interaction effect $xz$; RMSE = root mean square error; Parm. = parameter (regression coefficient); $p$ = Probability $x = 1$ for dichotomous covariate $x$. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

## Supplement D: Additional Results for Study 3

The following pages contain the additional results for Simulation Study 3. The panels of Table Dshow the relative bias, RMSE, and coverage rates for the first order effects $\beta_1$ and $\beta_2$ and the interaction effect $\beta_3$. The degrees of freedom of the scaled chi square distribution of the predictor $x$, the sample size $N$, the missing data rate, and $R^2$ were varied in the simulations. Table D1 shows the results for $N = 500$, Table D2 shows the results for $N = 1,000$, and Table D3 shows the results for $N = 2,000$.

Table D1

*Study 3: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with an Interaction Effect and a (Scaled) Chi-Square Distributed Predictor x and Uncorrelated Predictors x and z as a Function of the Degrees of Freedom, a Sample Size of N = 500, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | | RMSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parm. | *df* | FRM | FRM* | JAV | FRM | FRM* | JAV | FRM | FRM* | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | | | | |
| $\beta_1$ | 1 | 0.00 | 0.02 | 0.01 | 0.24 | 0.25 | 0.25 | 94.8 | 94.8 | 94.2 |
| | 2 | −0.01 | 0.01 | 0.01 | 0.26 | 0.27 | 0.28 | 94.7 | 94.4 | 94.6 |
| $\beta_2$ | 1 | 0.00 | 0.00 | 0.00 | 0.24 | 0.24 | 0.26 | 95.1 | 95.0 | 95.2 |
| | 2 | 0.05 | 0.01 | 0.00 | 0.32 | 0.33 | 0.34 | 94.5 | 95.1 | 94.7 |
| $\beta_3$ | 1 | −0.01 | 0.01 | 0.01 | 0.25 | 0.25 | 0.27 | 94.3 | 93.2 | 94.1 |
| | 2 | −0.03 | −0.01 | 0.00 | 0.26 | 0.27 | 0.28 | 95.5 | 94.9 | 95.2 |
| | *Missing at Random (MAR)* | | | | | | | | | |
| $\beta_1$ | 1 | −0.01 | −0.02 | −0.02 | 0.25 | 0.28 | 0.25 | 94.3 | 91.4 | 93.6 |
| | 2 | **0.08** | −0.03 | −0.02 | 0.31 | 0.31 | 0.29 | 92.7 | 92.3 | 93.7 |
| $\beta_2$ | 1 | −0.03 | **0.08** | **−0.22** | 0.24 | 0.26 | 0.36 | 94.1 | 91.8 | **85.3** |
| | 2 | 0.02 | **0.06** | **−0.25** | 0.33 | 0.34 | 0.46 | 94.3 | 93.3 | **88.2** |
| $\beta_3$ | 1 | 0.01 | **−0.06** | 0.05 | 0.24 | 0.27 | 0.28 | 95.1 | 91.8 | 93.0 |
| | 2 | −0.02 | −0.03 | **0.06** | 0.27 | 0.28 | 0.31 | 94.7 | 94.2 | 93.6 |

*Note.* FRM = factored regression modeling approach; FRM* = factored regression modeling approach with assumption of normally distributed non-normal predictor $x$; JAV = Maximum likelihood estimation with assumptions of normally distributed non-normal predictor $x$ and interaction effect $xz$; RMSE = root mean square error; Parm. = parameter (regression coefficient); *df* = Degrees of freedom. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table D2

*Study 3: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with an Interaction Effect and a (Scaled) Chi-Square Distributed Predictor x and Uncorrelated Predictors x and z as a Function of the Degrees of Freedom, a Sample Size of N = 1,000, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | | RMSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parm. | *df* | FRM | FRM* | JAV | FRM | FRM* | JAV | FRM | FRM* | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | | | | |
| $\beta_1$ | 1 | −0.01 | 0.00 | 0.00 | 0.16 | 0.17 | 0.17 | 95.5 | 94.8 | 95.3 |
| | 2 | −0.01 | 0.01 | 0.01 | 0.18 | 0.19 | 0.19 | 94.8 | 94.3 | 94.8 |
| | 5 | 0.01 | 0.01 | 0.00 | 0.23 | 0.23 | 0.24 | 92.9 | 92.9 | 92.4 |
| | 10 | 0.01 | 0.01 | 0.02 | 0.28 | 0.28 | 0.29 | 94.0 | 93.8 | 93.6 |
| $\beta_2$ | 1 | 0.00 | 0.00 | 0.00 | 0.17 | 0.17 | 0.18 | 94.7 | 95.3 | 94.8 |
| | 2 | 0.04 | 0.00 | 0.00 | 0.22 | 0.22 | 0.23 | 95.3 | 95.4 | 94.6 |
| | 5 | 0.03 | 0.02 | 0.02 | 0.36 | 0.37 | 0.39 | 96.3 | 96.1 | 96.2 |
| | 10 | 0.00 | −0.01 | −0.03 | 0.59 | 0.59 | 0.63 | 95.0 | 95.1 | 95.8 |
| $\beta_3$ | 1 | −0.01 | 0.00 | 0.01 | 0.17 | 0.17 | 0.18 | 94.7 | 93.8 | 94.6 |
| | 2 | −0.03 | −0.01 | 0.00 | 0.18 | 0.18 | 0.19 | 94.8 | 94.3 | 94.4 |
| | 5 | −0.02 | −0.01 | −0.02 | 0.21 | 0.21 | 0.22 | 96.1 | 96.3 | 96.9 |
| | 10 | 0.00 | 0.00 | 0.02 | 0.25 | 0.25 | 0.27 | 95.5 | 96.0 | 95.9 |
| | *Missing at Random (MAR)* | | | | | | | | | |
| $\beta_1$ | 1 | −0.03 | −0.03 | −0.03 | 0.17 | 0.19 | 0.17 | 94.5 | 93.6 | 94.7 |
| | 2 | **0.08** | −0.03 | −0.01 | 0.22 | 0.21 | 0.20 | 91.7 | 93.8 | 94.5 |
| | 5 | 0.01 | −0.02 | 0.00 | 0.26 | 0.27 | 0.25 | 93.2 | 92.7 | 94.2 |
| | 10 | −0.01 | −0.03 | −0.02 | 0.31 | 0.31 | 0.31 | 94.4 | 94.0 | 94.8 |
| $\beta_2$ | 1 | −0.03 | **0.08** | **−0.22** | 0.18 | 0.20 | 0.30 | 93.8 | **90.0** | **76.4** |
| | 2 | 0.03 | **0.06** | **−0.25** | 0.23 | 0.24 | 0.36 | 94.4 | 93.8 | **82.3** |
| | 5 | 0.00 | 0.04 | **−0.26** | 0.40 | 0.40 | 0.53 | 94.5 | 94.1 | **90.9** |
| | 10 | 0.00 | 0.03 | **−0.31** | 0.67 | 0.67 | 0.84 | 94.6 | 95.2 | 92.0 |
| $\beta_3$ | 1 | 0.01 | **−0.06** | 0.04 | 0.17 | 0.19 | 0.20 | 94.8 | 92.3 | 92.4 |
| | 2 | −0.03 | −0.04 | 0.05 | 0.20 | 0.20 | 0.21 | 94.6 | 94.3 | 92.8 |
| | 5 | −0.01 | −0.02 | 0.04 | 0.23 | 0.23 | 0.26 | 95.1 | 94.8 | 95.1 |
| | 10 | 0.00 | −0.01 | **0.07** | 0.29 | 0.29 | 0.33 | 94.9 | 94.7 | 93.8 |

*Note.* FRM = factored regression modeling approach; FRM* = factored regression modeling approach with assumption of normally distributed non-normal predictor $x$; JAV = Maximum likelihood estimation with assumptions of normally distributed non-normal predictor $x$ and interaction effect $xz$; RMSE = root mean square error; Parm. = parameter (regression coefficient); *df* = Degrees of freedom. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table D3

*Study 3: Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with an Interaction Effect and a (Scaled) Chi-Square Distributed Predictor x and Uncorrelated Predictors x and z as a Function of the Degrees of Freedom, a Sample Size of N = 2,000, 50% Missing Data on x, and $R^2$ = .25*

| | | Relative bias | | | RMSE | | | Coverage | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parm. | *df* | FRM | FRM* | JAV | FRM | FRM* | JAV | FRM | FRM* | JAV |
| | *Missing Completely at Random (MCAR)* | | | | | | | | | |
| $\beta_1$ | 1 | −0.01 | 0.00 | 0.00 | 0.11 | 0.12 | 0.12 | 95.1 | 94.5 | 94.4 |
| | 2 | −0.01 | 0.00 | 0.00 | 0.13 | 0.13 | 0.13 | 94.4 | 94.7 | 94.3 |
| $\beta_2$ | 1 | 0.00 | 0.00 | 0.00 | 0.12 | 0.12 | 0.13 | 95.2 | 95.5 | 94.6 |
| | 2 | 0.03 | 0.00 | 0.00 | 0.16 | 0.16 | 0.16 | 94.8 | 95.5 | 94.9 |
| $\beta_3$ | 1 | −0.01 | 0.00 | 0.00 | 0.12 | 0.12 | 0.12 | 94.7 | 93.4 | 94.3 |
| | 2 | −0.02 | 0.00 | 0.00 | 0.13 | 0.13 | 0.13 | 95.2 | 94.8 | 95.5 |
| | *Missing at Random (MAR)* | | | | | | | | | |
| $\beta_1$ | 1 | −0.03 | −0.03 | −0.03 | 0.12 | 0.13 | 0.12 | 94.3 | 92.6 | 94.1 |
| | 2 | **0.07** | −0.02 | −0.01 | 0.16 | 0.15 | 0.14 | 91.1 | 93.5 | 94.3 |
| $\beta_2$ | 1 | −0.03 | **0.09** | **−0.23** | 0.12 | 0.15 | 0.27 | 94.3 | **86.7** | **57.6** |
| | 2 | 0.03 | **0.07** | **−0.24** | 0.17 | 0.18 | 0.31 | 93.9 | 92.4 | **72.6** |
| $\beta_3$ | 1 | 0.02 | **−0.06** | 0.05 | 0.11 | 0.13 | 0.13 | 94.9 | 92.2 | 92.5 |
| | 2 | −0.02 | −0.04 | 0.05 | 0.14 | 0.14 | 0.16 | 94.4 | 93.4 | 91.6 |

*Note.* FRM = factored regression modeling approach; FRM* = factored regression modeling approach with assumption of normally distributed non-normal predictor $x$; JAV = Maximum likelihood estimation with assumptions of normally distributed non-normal predictor $x$ and interaction effect $xz$; RMSE = root mean square error; Parm. = parameter (regression coefficient); *df* = Degrees of freedom. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

## Supplement E: Additional Simulation on Comparison of LMS and FRM Method

We conducted a small additional simulation in which we compared the latent-moderated-structures (LMS) approach for modeling latent interactions (Klein & Moosbrugger, 2000) and the FRM approach for selected conditions of Simulation Study 1. The sample sizes were set to $N = 300$, and 1,000, the predictors $x$ and $z$ were assumed to be correlated ($\rho_{xz} = .5$), and $R^2$ was fixed to .5. Missing data on the predictor $x$ were generated as in the main simulation (Study 1), and the missing data rates were set to 25% and 50%. The LMS approach was specified using the Mplus software (Muthén & Muthén, 2012). As the LMS approach in Mplus requires the specification of latent variables (see Enders et al., 2014), we used a trick and specified latent variables with single indicators and set the residual variance to a very small value (see Mplus discussion forum: `http://www.statmodel.com/discussion/messages/11/19781.html?1459126113`).
The Mplus syntax for the LMS approach reads as follows:

```
TITLE: manifest moderation;

DATA:
FILE IS data.dat;

VARIABLE:
NAMES ARE X Z Y ;
usevar are X Z Y;
MISSING = . ;

ANALYSIS:
TYPE = RANDOM;
ALGORITHM = INTEGRATION;

MODEL:
! define single indicator models;
FX BY X@1;
FZ BY Z@1;
FY BY Y@1;
! specify small residual variances which are
! approximately zero;
X@0.01;
Z@0.01;
Y@0.01;
! specify regression model with interaction effect;
FY ON FX*0 FZ*0;
FXxFZ | FX XWITH FZ;
FY ON FXxFZ;
```

Tables E1 and E2 show for the LMS and FRM approaches the estimated relative bias, RMSE, and coverage rates for the two first order effects $\beta_1$ and $\beta_2$, and the interaction effect $\beta_3$. As

can be seen, the two approaches performed very similarly, with the exception that the coverage rates produced by the LMS approach were slightly too low in a few conditions (see Table E2 in the MAR condition).

Table E1

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 25% Missing Data on x, and $R^2 = .5$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | LMS | FRM | LMS | FRM | LMS |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 300 | 0.00 | 0.02 | 0.16 | 0.17 | 94.5 | 92.4 |
| | 1,000 | 0.00 | 0.02 | 0.08 | 0.09 | 95.1 | 93.3 |
| $\beta_2$ | 300 | 0.00 | 0.01 | 0.15 | 0.16 | 94.5 | 94.1 |
| | 1,000 | 0.00 | 0.01 | 0.08 | 0.09 | 95.0 | 93.3 |
| $\beta_3$ | 300 | 0.00 | 0.02 | 0.11 | 0.12 | 95.4 | 94.2 |
| | 1,000 | 0.00 | 0.02 | 0.07 | 0.07 | 94.1 | 93.8 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 300 | 0.00 | 0.00 | 0.17 | 0.18 | 93.2 | 91.0 |
| | 1,000 | 0.00 | 0.01 | 0.09 | 0.10 | 95.2 | 93.1 |
| $\beta_2$ | 300 | −0.01 | 0.01 | 0.15 | 0.17 | 95.0 | 93.2 |
| | 1,000 | 0.00 | 0.02 | 0.08 | 0.09 | 94.5 | 93.8 |
| $\beta_3$ | 300 | 0.00 | 0.02 | 0.12 | 0.13 | 94.5 | 93.0 |
| | 1,000 | 0.00 | 0.02 | 0.07 | 0.07 | 95.2 | 93.4 |

*Note.* FRM = factored regression modeling approach; LMS = Latent-moderated-structures estimation method (in Mplus); RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.

Table E2

*Relative Bias, RMSE, and Coverage for Regression Coefficients of the Linear Regression with Interaction Effects as a Function of Sample Size for Correlated Predictors x and z, 50% Missing Data on x, and $R^2 = .5$*

| | | Relative bias | | RMSE | | Coverage | |
|---|---|---|---|---|---|---|---|
| Parm. | *N* | FRM | LMS | FRM | LMS | FRM | LMS |
| | *Missing Completely at Random (MCAR)* | | | | | | |
| $\beta_1$ | 300 | 0.00 | 0.02 | 0.18 | 0.20 | 94.0 | 91.5 |
| | 1,000 | 0.01 | 0.03 | 0.10 | 0.11 | 94.9 | 91.3 |
| $\beta_2$ | 300 | 0.01 | 0.02 | 0.17 | 0.18 | 94.7 | 93.4 |
| | 1,000 | 0.00 | 0.01 | 0.09 | 0.09 | 95.4 | 94.1 |
| $\beta_3$ | 300 | 0.00 | 0.02 | 0.14 | 0.14 | 93.7 | 92.2 |
| | 1,000 | 0.00 | 0.02 | 0.07 | 0.08 | 94.1 | 92.3 |
| | *Missing at Random (MAR)* | | | | | | |
| $\beta_1$ | 300 | −0.01 | 0.00 | 0.20 | 0.21 | 93.1 | 92.2 |
| | 1,000 | 0.00 | 0.00 | 0.10 | 0.11 | 94.5 | 92.7 |
| $\beta_2$ | 300 | 0.00 | 0.04 | 0.16 | 0.19 | 93.5 | 93.9 |
| | 1,000 | 0.00 | 0.04 | 0.08 | 0.10 | 95.8 | 94.0 |
| $\beta_3$ | 300 | 0.01 | 0.04 | 0.14 | 0.15 | 93.1 | **90.8** |
| | 1,000 | 0.00 | 0.04 | 0.08 | 0.08 | 94.7 | 91.2 |

*Note.* FRM = factored regression modeling approach; LMS = Latent-moderated-structures estimation method (in Mplus); RMSE = root mean square error; Parm. = parameter (regression coefficient); *N* = Sample size. Relative biases greater in magnitude than |.05| and coverage rates smaller than 91% or larger than 98% are written in bold.