A. Tenti-model

In this part of the Supplementary Material the mathematical the mathematical background of the Tenti model is provided, which was used to calculate the simulated spectra for spontaneous and coherent RBS. The account given here is largely based on the original work of Boley et al. [19] and subsequent descriptions by Pan [33] and by Gu [27]. In the Tenti models non-degenerate eigenvectors or moments written as linearized approximations of the WCU equation are derived. In the Tenti-S7 model there are seven of such components including a traceless pressure tensor, while in Tenti-S6 there are only six, and the traceless pressure tensor is omitted. These components, written as deviations from equilibrium $h_i(\boldsymbol{r}, \boldsymbol{v}, t)$ and involving the Maxwell distribution function $\phi_M(v)$, are the following:
(1) the deviation of the total number density:

$$n_0 \nu(\boldsymbol{r}, t) = n_0 \sum_i x_i \int \phi_M(v) h_i d^3 v \ , \tag{21}$$

(2) the macroscopic flow velocity, or the momentum:

$$v_0 \boldsymbol{u}(\boldsymbol{r}, t) = \sum_i x_i \int \boldsymbol{v} \phi_M h_i d^3 v \ , \tag{22}$$

(3) the deviation of the translational temperature:

$$T_0 \tau_{\rm tr}(\boldsymbol{r}, t) = \frac{1}{k_B c_{\rm tr}} \sum_i x_i \int (\frac{1}{2} m v^2 - \frac{3}{2} k_B T_0) v \phi_M h_i d^3 v \ , \tag{23}$$

(4) the deviation of the internal temperature

$$T_0 \boldsymbol{\tau}_{\rm int}(\boldsymbol{r}, t) = \frac{1}{k_B c_{\rm int}} \sum_i x_i \int (E_i - \langle E \rangle) \phi_M h_i d^3 v \ , \tag{24}$$

where T_0 is the equilibrium temperature, $\mathrm{c_{tr}}$ and $c_{\rm int}$ are the translational and internal specific heat capacities, and $\langle E \rangle = \sum_i x_i E_i$ stands for the mean energy of all internal states,
(5) the translational flux:

$$n_0 k_B T_0 v_0 \boldsymbol{q}_{\rm tr} = n_0 \sum_i x_i \int (\frac{1}{2} m v^2 - \frac{5}{2} k_B T_0) \boldsymbol{v} \phi_M h_i d^3 v \ , \tag{25}$$

(6) the internal heat flux

$$n_0 k_B T_0 v_0 \boldsymbol{q}_{\rm int} = n_0 \sum_i x_i \int (E_i - \langle E \rangle) \boldsymbol{v} \phi_M h_i d^3 v \ . \tag{26}$$

Insertion of the linearized distribution function $f(\boldsymbol{r}, \boldsymbol{v}, t)$ as given in Eq. (7) into

the full WCU, presented in Eq. (6), yields:

$$\left(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla\right)h_i = n_0\sum_{jgl}x_j\int|\boldsymbol{v}-\boldsymbol{v}_1|\sigma_{ij}^{gl}[h_g(\boldsymbol{v}')+h_l(\boldsymbol{v}_1')-h_i(\boldsymbol{v})-h_j(\boldsymbol{v}_1)]d\Omega\phi_M d^3v_1\ . \tag{27}$$

Using Dirac notations, Eq. (27) can be written in simplified form:

$$(\frac{\partial}{\partial t}+\boldsymbol{v}\cdot\nabla)|h\rangle = n_0\boldsymbol{J}|h\rangle\ , \tag{28}$$

with the column vector $|h\rangle = [h_1, h_2, ...h_n]^T$ and the Hermitian conjugate $\langle h| = [h_1^*, h_2^*, ...h_n^*]$. $\boldsymbol{J}$ is a $N\times N$ matrix, and each element of this matrix is an integral operator. $\boldsymbol{J}$ can be separated into a term for elastic collisions $\boldsymbol{J}'$ with $g=i$ and $l=j$ and a term representing inelastic collisions $\boldsymbol{J}''$ with $g\neq i$ and $l\neq j$, that is, $\boldsymbol{J}=\boldsymbol{J}'+\boldsymbol{J}''$.

The eigenvectors and eigenvalues for the elastic collision operator $\boldsymbol{J}'$ have been studied by Wang Chang et al. [60]:

$$\begin{aligned}\boldsymbol{J}'|\Psi_{s0}\rangle &= (2k/m)^{1/2}\lambda_s|\Psi_{s0}\rangle\\ \boldsymbol{J}'|\Psi_{sn}\rangle &= (2k/m)^{1/2}\mu_s|\Psi_{sn}\rangle\ (1\leq n\leq N-1)\ ,\end{aligned} \tag{29}$$

where λ_s, μ_s are the eigenvalues. The eigenvectors Ψ_{sn} of $\boldsymbol{J}'$ are orthogonal to one another, as $\langle\Psi_{sn}|\Psi_{s'n'}\rangle=\delta_{ss'}$, and they span the entire Hilbert space. The eigenvectors Ψ_{sn} are given by:

$$|\Psi_{sn}\rangle = \begin{pmatrix}\Phi_{rlm}P_n(\varepsilon_1)\\ \Phi_{rlm}P_n(\varepsilon_2)\\ \cdot\\ \cdot\\ \cdot\end{pmatrix}\ , \tag{30}$$

where $\varepsilon_i = E_i/(k_BT_0)$ is the dimensionless energy, and $P_n(\varepsilon_i)$ is a polynomial of the internal energy given by:

$$P_0(\varepsilon_i)=1,\ P_1(\varepsilon_i)=\frac{\varepsilon_i-\langle\varepsilon\rangle}{\sqrt{\langle(\varepsilon-\langle\varepsilon\rangle)^2\rangle}}$$

and in general

$$P_n(\varepsilon_i)=K_n\left[\varepsilon_iP_{n-1}-\sum_{m=0}^{n-1}\langle\varepsilon P_{n-1}P_m\rangle P_m(\varepsilon_i)\right],\ 1<n<N\ ,$$

with K_n chosen to normalize $\langle P_n^2\rangle = 1$. Φ_{rlm} is the eigenfunction of the collision operator for an atomic gas with a Maxwell force law, or a molecular gas with only one internal energy state, which can be further expressed as:

$$\Phi_{rlm}=\sqrt{\frac{2\pi^{3/2}r!}{(r+l+1/2)!}}S_{l+1/2}^{(r)}(c^2)c^lY_{lm}(\hat{c})\ , \tag{31}$$

where $S_n^{(m)}(x)$ is the Sonine polynomial and $Y_{lm}(\theta,\psi)$ is the spherical harmonic for the direction angles θ,ψ of the dimensionless velocity $\boldsymbol{c}$. It is clear in the above equations that s can be written as (rlm), so Φ_{sn} is rewritten as Φ_{rlmn}.

Using the eigenvectors of $\boldsymbol{J}'$, the elastic collision operator can be diagonalized by inserting the completeness identity. Following the method of Gross and Jackson [61], all of the eigenvalues except for a particular set are set equal to a common constant, namely:

$$\begin{aligned}\boldsymbol{J}'|h\rangle &= \sum_r J'_r|\Psi_r\rangle\langle\Psi_r|h\rangle \\ &\approx \sum_{r<R} J'_r|\Psi_r\rangle\langle\Psi_r|h\rangle + J'_R\sum_{r\geq R}|\Psi_r\rangle\langle\Psi_r|h\rangle \\ &= \sum_{r<R}(J'_r-J'_R)|\Psi_r\rangle\langle\Psi_r|h\rangle + J'_R|h\rangle\ , \end{aligned} \tag{32}$$

where Ψ_r means Ψ_{rlmn}, J'_r is the eigenvalue of Ψ_r and the common constant J'_R is the eigenvalue for all Ψ_r with $r\geq R$. In the above equation the completeness condition $\sum_r|\Psi_r\rangle\langle\Psi_r|=1$ is used.

Tenti et al. [20] have chosen six non-degenerate eigenvectors with clear physical meanings: $\Psi_{000,0}$ the fraction of particles in different internal states, $\Psi_{01m,0}$ the momentum, $\Psi_{100,0}$ the translational energy, $\Psi_{11m,0}$ the translational heat flux, and $\Psi_{000,1}$ the internal energy, $\Psi_{01m,1}$ the internal heat flux which are linked to Eqs. (21)$-$ (26). The corresponding eigenvalues are J'_{000}, J'_{010}, J'_{100}, J'_{110}, J'_{001}, J'_{011}, denoting in J'_{rln} with

$$J'_{rln}=\frac{1}{2l+1}\sum_m J'_{rlm,n}\ .$$

The other eigenvectors are degenerated and are assumed to have the same eigenvalue J'_{020}. Hence, the resulting elastic collision model is represented by:

$$\begin{aligned}(J'h)_i = &-J'_{020}\left[\nu_i+2\boldsymbol{c}\cdot\boldsymbol{u}+(c^2-\frac{3}{2})\tau_{\rm tr}\right]+(-J'_{020}+J'_{110})\frac{4}{5}(c^2-\frac{5}{2})\boldsymbol{c}\cdot\boldsymbol{q}_{\rm tr}\\ &+(-J'_{020}+J'_{011})\frac{\varepsilon_i-\langle\varepsilon\rangle}{c_{\rm int}}2\boldsymbol{c}\cdot\boldsymbol{q}_{\rm int}+J'_{020}h_i\ .\end{aligned} \tag{33}$$

For the inelastic operator $\boldsymbol{J}''$, Boley et al. [19] used the model developed by Hanson and Morse [62] which inserts the completeness identity twice into $J''|h\rangle$ as:

$$\boldsymbol{J}''|h\rangle=\sum_{sn}|\Psi_{sn}\rangle\langle\Psi_{sn}|\sum_{s'n'}J''|\Psi_{s'n'}\rangle\langle\Psi_{s'n'}|h\rangle$$

and puts $\langle\Psi_{sn}|\boldsymbol{J}''|\Psi_{s'n'}\rangle = C\times\delta_{ss'}\delta_{nn'}$, with C a constant beyond a certain range of s, n. Following the same notations and language for the elastic operator, the inelastic operator $\boldsymbol{J}''|h\rangle$ can be expressed as:

$$\boldsymbol{J}''|h\rangle=\sum_{rlmn,r'n'}(J''^{rln}_{r'l'n'}-J''_{020}\delta_{rr'}\delta_{nn'})|\Psi_{rlmn}\rangle\langle\Psi_{r'lmn'}|h\rangle+J''_{020}h\ , \tag{34}$$

where

$$J''^{rln}_{r'l'n'} = \frac{1}{2l+1}\sum_m \langle \Psi_{rlmn}|\boldsymbol{J}''|\Psi_{r'l'mn'}\rangle$$

and $J''_{rln} = J''^{rln}_{rln}$ for simplicity. Then the inelastic collision model is

$$\begin{aligned}(J''h)_i = &-J''_{020}\left[\nu + 2\boldsymbol{c}\cdot\boldsymbol{u} + \left(c^2 - \frac{3}{2} + \varepsilon_i - \langle\varepsilon\rangle\right)\frac{c_{\rm tr}\tau_{\rm tr} + c_{\rm int}\tau_{\rm int}}{c_{\rm tr}+c_{\rm int}}\right] \\ &+ \left(J''_{100} - \frac{c_{\rm int}}{c_{\rm tr}+c_{\rm int}}J''_{020}\right)\left[c^2 - \frac{3}{2} - \frac{3}{2c_{\rm int}}(\varepsilon_i - \langle\varepsilon\rangle)\right](\tau_{\rm tr} - \tau_{\rm int}) \\ &+ (-J''_{020} + J''_{011})\frac{\varepsilon_i - \langle\varepsilon\rangle}{c_{\rm int}} 2\boldsymbol{c}\cdot\boldsymbol{q}_{\rm int} \\ &- J''^{110}_{011}\frac{2\sqrt{2}}{\sqrt{5c_{\rm int}}}\left[(\varepsilon_i - \langle\varepsilon\rangle)\boldsymbol{c}\cdot\boldsymbol{q}_{\rm tr} + \boldsymbol{c}(c^2 - \frac{5}{2})\cdot\boldsymbol{q}_{\rm int}\right] \\ &+ (J''_{110} - J''_{020})\frac{4}{5}\boldsymbol{c}(c^2 - \frac{5}{2})\cdot\boldsymbol{q}_{\rm tr} + J''_{020}h_i\ .\end{aligned} \tag{35}$$

Therefore, the entire collision operator can be denoted as $\boldsymbol{J}^{(6)}$ with $\boldsymbol{J}^{(6)} = \boldsymbol{J}' + \boldsymbol{J}''$, where (6) stands for the 6 moments, and the model is referred to as the Tenti-S6 model. As a result, the linearized WCU equation (28) is written as:

$$(\frac{\partial}{\partial t} + v\cdot\nabla)|h\rangle = n_0\boldsymbol{J}^{(6)}|h\rangle\ . \tag{36}$$

By taking the Fourier-transform and multiplying both sides by $I(\omega, y, c_z) = 1/(\omega/kv_0 - i\cdot n_0 J_{020}/kv_0 - c_z)$, the kinetic equation for $|h\rangle$ in the frequency domain is obtained as:

$$\begin{aligned}\frac{qv_0}{in_0}|h\rangle = I(\omega,y,c_z)\{&-J'_{020}|\nu\rangle - J''_{020}|\Psi_{000,0}\rangle - J_{020}\sqrt{2}u_z|\Psi_{010,0}\rangle \\ &+ \left[J_{100}\sqrt{3/2}\tau_{\rm int} + (J_{020} - J_{100})\sqrt{3/2}\tau_{\rm tr}\right]|\Psi_{100,0}\rangle \\ &+ \left[-J_{100}\frac{3}{2\sqrt{c_{\rm int}}}(\tau_{\rm tr} - \tau_{\rm int}) - J''_{020}\sqrt{c_{\rm int}}\tau_{\rm int}\right]|\Psi_{000,1}\rangle \\ &+ \left[(-J_{020} + J_{011})\sqrt{2/c_{\rm int}}q_{{\rm int},z} - J^{110}_{011}\frac{2}{\sqrt{5}}q_{{\rm tr},z}\right]|\Psi_{010,1}\rangle \\ &+ \left[-(J_{110} - J_{020})\frac{2}{\sqrt{5}}q_{{\rm tr},z} + J^{110}_{011}\sqrt{2/c_{\rm int}}q_{{\rm int},z}\right]|\Psi_{110,0}\rangle + \frac{1}{kn_0}|\Psi_{000,0}\rangle\}\ ,\end{aligned} \tag{37}$$

where the density perturbation $|\nu\rangle = [\nu_1, \nu_2, ...]^T$ and z the direction of the acoustic wave. Note that $\nu, u_z, q_{{\rm tr},z}, \tau_{\rm tr}, \tau_{\rm int}, q_{{\rm int},z}$ are in frequency domain, e.g. ν is short for $\nu(k,\omega)$.

Taking the inner product of this equation by sandwiching with $|\Psi_{000,0}\rangle = [1, 1, ...]^T$, the first of the six equations of the gas-dynamic parameters is obtained - the mass

equation:

$$\begin{aligned}\frac{kv_0}{in_0}\langle\Psi_{000,0}|h\rangle = &-J'_{020}\langle\Psi_{000,0}|I|\nu\rangle - J''_{020}\langle\Psi_{000,0}|I|\Psi_{000,0}\rangle - J_{020}\sqrt{2}u_z\langle\Psi_{000,0}|I|\Psi_{010,0}\rangle \\ &+ \left[J_{100}\sqrt{3/2}\tau_{\rm int} + (J_{020}-J_{100})\sqrt{3/2}\tau_{\rm tr}\right]\langle\Psi_{000,0}|I|\Psi_{100,0}\rangle \\ &+ \left[-J_{100}\frac{3}{2\sqrt{c_{\rm int}}}(\tau_{\rm tr}-\tau_{\rm int}) - J''_{020}\sqrt{c_{\rm int}}\tau_{\rm int}\right]\langle\Psi_{000,0}|I|\Psi_{000,1}\rangle \\ &+ \left[(-J_{020}+J_{011})\sqrt{2/c_{\rm int}}q_{{\rm int},z} - J^{110}_{011}\frac{2}{\sqrt{5}}q_{{\rm tr},z}\right]\langle\Psi_{000,0}|I|\Psi_{010,1}\rangle \\ &+ \left[-(J_{110}-J_{020})\frac{2}{\sqrt{5}}q_{{\rm tr},z} + J^{110}_{011}\sqrt{2/c_{\rm int}}q_{{\rm int},z}\right]\langle\Psi_{000,0}|I|\Psi_{110,0}\rangle \\ &+ \frac{1}{kn_0}\langle\Psi_{000,0}|I|\Psi_{000,0}\rangle\ .\end{aligned} \tag{38}$$

Some of the inner products has already been calculated as: $\langle\Phi_{000,0}|h\rangle = \nu$, $\langle\Phi_{000,0}|\nu\rangle = \nu I^{00}_{00}$, $\langle\Phi_{000,0}|\Phi_{000,1}\rangle = 0$, $\langle\Phi_{000,0}|\Phi_{010,1}\rangle = 0$ which are detailed in Pan [33]. Defining the symbol $I^{rl}_{r'l'}$:

$$I^{rl}_{r'l'} = \langle\Phi_{rl0,0}|I|\Phi_{r'l'0,0}\rangle\ , \tag{39}$$

which can be expressed in terms of the plasma dispersion function, the first linear equation can be written as:

$$\begin{aligned}&(-J_{020}I^{00}_{00} - \frac{k\ v_0}{i\ n_0})\bar{\nu} + -J_{020}I^{00}_{01}\sqrt{2}u_z + (J_{020}-J_{100})I^{00}_{10}(2/\sqrt{5})q_{{\rm tr},z} \\ &+ (J_{020}-J_{100})I^{00}_{10}\sqrt{c_{\rm tr}}\tau_{\rm tr} + J^{001}_{100}I^{00}_{10}\sqrt{c_{\rm int}}\tau_{\rm int} + J^{110}_{011}I^{00}_{11}\sqrt{2/c_{\rm int}}q_{{\rm int},z} = -\frac{1}{kn_0}I^{00}_{00}\ .\end{aligned} \tag{40}$$

Similarly, one can work out the other five equations, by taking the inner product of $|\Psi_{010,0}\rangle$, $|\Psi_{100,0}\rangle$, ... with Eq. (37). The full result can be written as a matrix equation in the form of $\boldsymbol{AX} = \boldsymbol{B}$, with:

$$\boldsymbol{A} = \frac{n_0}{kv_0}\begin{bmatrix}
-J_{020}I^{00}_{00} - \frac{k\ v_0}{i\ n_0} & -J_{020}I^{00}_{01} & (J_{020}-J_{110})I^{00}_{11} \\
-J_{020}I^{01}_{00} & -J_{020}I^{01}_{01} - \frac{k\ v_0}{i\ n_0} & (J_{020}-J_{110})I^{01}_{11} \\
-J_{020}I^{11}_{00} & -J_{020}I^{11}_{01} & (J_{020}-J_{110})I^{11}_{11} + \frac{k\ v_0}{i\ n_0} \\
-J_{020}I^{10}_{00} & -J_{020}I^{10}_{01} & (J_{020}-J_{110})I^{10}_{11} \\
0 & 0 & -J^{110}_{011}I^{00}_{01} \\
0 & 0 & -J^{110}_{011}I^{01}_{01}
\end{bmatrix}$$

$$\begin{bmatrix}
(J_{020}-J_{100})I^{00}_{10} & J^{001}_{100}I^{00}_{10} & J^{110}_{011}I^{00}_{11} \\
(J_{020}-J_{100})I^{01}_{10} & J^{001}_{100}I^{01}_{10} & J^{110}_{011}I^{01}_{11} \\
(J_{020}-J_{100})I^{11}_{10} & J^{001}_{100}I^{11}_{10} & J^{110}_{011}I^{11}_{11} \\
(J_{020}-J_{110})I^{10}_{10} + \frac{k\ v_0}{i\ n_0} & J^{001}_{100}I^{10}_{10} & J^{110}_{011}I^{10}_{11} \\
-J^{001}_{100}I^{00}_{00} & (J_{001}-J_{020})I^{00}_{00} - \frac{k\ v_0}{i\ n_0} & (J_{011}-J_{020})I^{00}_{01} \\
-J^{001}_{100}I^{01}_{00} & (J_{001}-J_{020})I^{01}_{00} & (J_{011}-J_{020})I^{01}_{01} - \frac{k\ v_0}{i\ n_0}
\end{bmatrix}$$

$$\boldsymbol{X} = \begin{bmatrix} \nu(\boldsymbol{k},\omega) \\ \sqrt{2}u_z(\boldsymbol{k},\omega) \\ (2/\sqrt{5})q_{\mathrm{tr},z}(\boldsymbol{k},\omega) \\ \sqrt{c_{\mathrm{tr}}}\tau_{\mathrm{tr}}(\boldsymbol{k},\omega) \\ \sqrt{c_{\mathrm{int}}}\tau_{\mathrm{int}}(\boldsymbol{k},\omega) \\ \sqrt{2/c_{\mathrm{int}}}q_{\mathrm{int},z}(\boldsymbol{k},\omega) \end{bmatrix} \quad \text{and} \quad \boldsymbol{B} = -\tfrac{1}{q^2 v_0} \begin{bmatrix} I_{00}^{00} \\ I_{00}^{01} \\ I_{00}^{11} \\ I_{00}^{10} \\ 0 \\ 0 \end{bmatrix} . \tag{41}$$

The matrix elements of $\boldsymbol{J}$ in Matrix $\boldsymbol{A}$ can be expressed in terms of the transport coefficients: *i.e.* the shear viscosity η_{s}, the bulk viscosity η_{b} and thermal conductivity κ:

$$\begin{aligned} \eta_{\mathrm{s}} &= \frac{k_B T}{J_{020}} \\ \eta_{\mathrm{b}} &= -\frac{2}{3}\left(\frac{c_{\mathrm{int}}}{c_{\mathrm{tr}}+c_{\mathrm{int}}}\right)^2 \frac{k_B T}{J_{100}} \\ \kappa &= -\frac{k_B^2 T}{m}\frac{\frac{5}{2}J_{011}+c_{\mathrm{int}}J_{110}+(10c_{\mathrm{int}})^{1/2}J_{011}^{110}}{J_{011}J_{110}-(J_{011}^{110})^2} . \end{aligned} \tag{42}$$

Together with the identities:

$$\begin{aligned} J_{020} &= -\frac{k v_0 y}{n_0} \\ J_{110} &= -\frac{2k_B T}{3\eta_{\mathrm{s}}} - \frac{5\gamma_{\mathrm{int}}^2 k_B T}{9\eta_{\mathrm{b}}} \\ J_{011}^{110} &= -\frac{\sqrt{5/8c_{\mathrm{int}}}\gamma_{\mathrm{int}}^2 k_B T}{\eta_{\mathrm{b}}} \\ J_{011} &= -\frac{2\gamma_{\mathrm{int}}k_B T}{3(c_{\mathrm{tr}}+c_{\mathrm{int}})\eta_{\mathrm{b}}}\frac{2\eta_{\mathrm{b}}/5\eta_{\mathrm{s}}(c_{\mathrm{tr}}+c_{\mathrm{int}})^2 + c_{\mathrm{int}}(1+c_{\mathrm{int}}/3)+\gamma_{\mathrm{int}}^2 m\kappa/6k_B\eta_{\mathrm{b}}}{-1+4m\kappa/15k_B\eta_{\mathrm{s}}+2\gamma_{\mathrm{int}}^2 m\kappa/9k_B\eta_{\mathrm{b}}} \\ J_{100} &= \sqrt{\frac{2c_{\mathrm{int}}}{3}}J_{100}^{001} = \sqrt{\frac{2c_{\mathrm{int}}}{3}}J_{001}^{100} = \frac{2c_{\mathrm{int}}}{3}J_{001} . \end{aligned} \tag{43}$$

all the elements of J in Eq. (41), can be expressed in terms of these three transport coefficients, the atomic mass m of the particles, the internal specific heat capacity per molecule c_{int}, and $\gamma_{\mathrm{int}} = c_{\mathrm{int}}/(c_{\mathrm{tr}}+c_{\mathrm{int}})$.

The fluctuation characteristics of the gas are contained in the structure factor:

$$S(\boldsymbol{k},\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t}\mathrm{d}t \int G(\boldsymbol{r},t)\; e^{i\boldsymbol{k}\cdot\boldsymbol{r}}\mathrm{d}\boldsymbol{r} , \tag{44}$$

where the structure factor $S(\boldsymbol{k},\omega)$ is the space-time Fourier-transform of $G(\boldsymbol{r},t)$, which equals the classic time-dependent density Van Hove's correlation function [63].

Boley et al. [19] identified the density correlation function as $G(\boldsymbol{r},t) = n_0\nu(\boldsymbol{r},t)$. Then based on Eq. (44), the RB-scattering profile which is proportional to $S(\boldsymbol{k},\omega)$ can be written as:

$$I(\boldsymbol{k},\omega) \propto S(\boldsymbol{k},\omega) = n_0\nu(\boldsymbol{k},\omega) , \tag{45}$$

where $\nu(\boldsymbol{k}, \omega)$ is the Fourier transform of $\nu(\boldsymbol{r}, t)$. That is, the RB-scattering profile is proportional to the real part of the first element of the vector $\boldsymbol{X}$ with $\boldsymbol{X} = \boldsymbol{A}^{-1}\boldsymbol{B}$.

In the literature there exist a number of codes to numerically compute the Rayleigh-Brillouin spectrum from the Tenti models, e.g. in the PhD Thesis of Pan [33]. Recently a Mathlab-code was developed for calculation of RBS profiles [35], which is made available in the Supplementary Material. These codes produce an RB-scattering spectrum based on inputs for λ_i, θ, p, T, η_s, m, c_int, κ and η_b, where the possibility is included to determine a value of a parameter by fitting to an experimental spectrum.