

# Probabilistic Modelling of Parameter Variability for Analysing Grid-Connected LV Feeders with DG

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**Abstract-** There is mounting pressure on utilities worldwide to incorporate renewable energy sources in their distribution of electricity. The inclusion of distributed generation (DG) on feeders connected to the grid presents significant problems when feeder performance must be assessed. Two issues arising from the inclusion of DG are voltage regulation and reverse power feed. There is variability in the generation of power from solar panels or from wind turbines due to daily solar cycles and seasonal (weather related) changes. And then there is the daily and seasonal variability due to the stochastic behaviour of domestic loads. Together these present formidable challenges to the analyst. Frequently the problem presents itself when an existing feeder is required to host randomly located elements of embedded generation. This is different from a design analysis that includes both feeder and optimal location of DG. Clearly with so much variability, the conventional approach of using deterministic methods with ‘average values’ for the input parameters (loads and DG), is less than adequate. For a more realistic approach time-dependent statistical models are required to describe the variability of these kinds of load and generation. Whether the statistical approach of analysing the behaviour of a feeder uses Monte Carlo simulations or a direct probabilistic method, load and generation variability require time-based modelling. This paper presents an approach for modelling both loads and DG as probability Beta probability density functions (PDFs).

**Index Terms**—Distributed Generation, probabilistic feeder performance, grid integration, load modeling, PV models, risk.

## I. INTRODUCTION

The context within which this work is described relates to the urgent pressure from ecological, political and economic institutions to vigorously promote the connection of renewable generation to the utility grids. While these pressures are being exerted utility engineers are faced with the responsibility of preserving the technical integrity of the delivery system. The two objectives are not always compatible due to uncontrollable variability. To address this, Grid Codes are commonly compiled by the countries concerned to regulate the connection of DG (wind, solar panels, micro-hydro etc.) on a feeder. However, there is substantial diversity in the style and practice of the distribution systems used around the world. The international body, Cigré, is currently investigating the effects of these issues through the Study Committee C6.

As stated in the abstract the main issues relating to the connection of DG are: [1]

- Voltage regulation
- Reverse power feed
- Protection
- Stability (including islanding)
- Reliability

These phenomena need to be analysed appropriately to facilitate informed decision making when hosting DG on a feeder. In addition to the listed technical considerations, commercial incentives such as time-of-use tariffs might also be developed. For the purpose of this paper only distribution systems are considered.

These analyses require appropriate inputs and analytical procedures. The inputs are:

- Feeder parameters – conductor type and length
- Load parameters – statistical variation with time
- DG parameters – statistical generation output with time

Most DG is hosted on existing grid circuits that were not specifically designed for that purpose. In general feeder parameters are well known (assuming the routes, lengths and sizes have been recorded at construction). Operating temperature affects resistance and should be included in calculations.

Loads are generally stochastic by nature – particularly those of residential customers. Their statistical variations are significant and should be adequately described in a statistical function. Similarly, renewable energy sources are subject to seasonal and time-of day variations. So they too require statistical functions to describe their generation parameters.

In this paper we examine the variation of both load parameters and the outputs from DG and how these combinations need to be uniquely treated in the analysis of feeder performance. The examples used are chiefly based on research done in the residential sector in South Africa.

## II. LOAD VARIABILITY AND LOAD MODELS

The modelling of loads takes two forms – electrical parameter and statistical parameter models. In the former the electrical characteristics of the load are described (constant impedance, constant current, constant power or combinations). In their work on MV systems, Singh et al [2] discuss the placement of DG is affected by these models. By

contrast, in this work we concentrate on LV systems with residential loads. Several years of load research in South Africa in approximately 40 sites (each with approximately 50 households) has yielded a very large amount of data sampled at 5-minute intervals. For various reasons described in previous publications the load is modelled as ‘constant current’ [3].

The load data were statistically analysed and it was found that at any time interval the customers had significant dispersion (standard deviation) and often displayed considerable ‘skewness’ in their probability distributions (PDF). This is contrary to the commonly accepted Gaussian description. It was found that the Beta PDF is the best fit for the South African grouped residential loads [4]. It is a versatile PDF and has been shown to be suitable for feeder design by direct probabilistic methods [4] and in reliability analyses using Monte Carlo simulations [5]. The Beta PDF is useful because (a) it can be negatively and positively skewed (b) it is supported on a finite base, and (c) it can replicate various shapes [6]. It has two shape parameters:  $\alpha$  and  $\beta$  and is constrained by a finite limit, C. This limit can conveniently be chosen as the circuit breaker size for load current data.

A parametric description of the load statistic at a given interval provides the basis for a probabilistic approach to the analysis of the network. This has been the practice in South Africa for dimensioning LV feeder components where the Herman Beta (H-B) method is prescribed by the utilities since 1999 [7].

As examples, consider LV feeders feeding typical residential customers at sites A and B. Figure 1 shows the daily load current profiles for a high-end customer class in at site A in South Africa for typical summer loading and peak winter loading. The power factor is assumed to be unity.

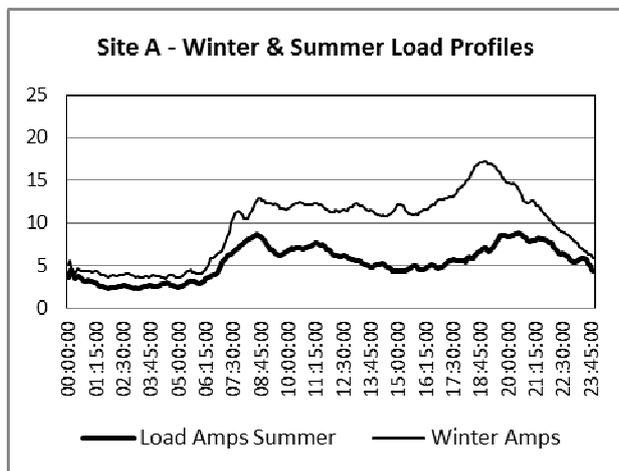


Figure 1 Winter and Summer Load Current Profiles for Site A

The after diversity maximum demand (ADMD) is shown to occur about 19:15 and is characterized by a Beta PDF with  $\alpha = 1.23$  and  $\beta = 4.4$  with  $C = 80$  Amps. The mean is 17.44 Amps and the standard deviation is 12.83 Amps. This is depicted by the curve in Figure 2.

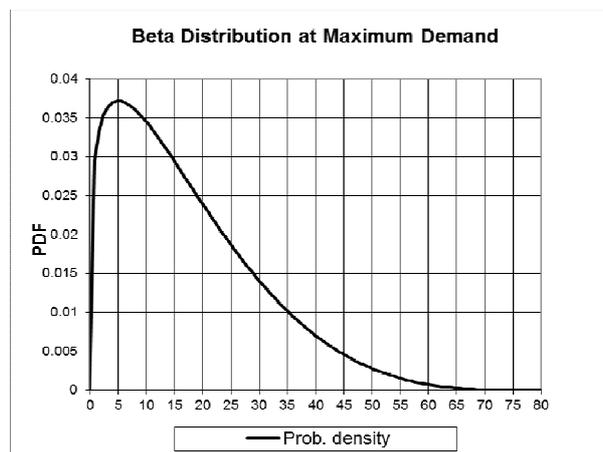


Figure 2 Beta PDF of Load Current at site A during System Maximum Demand

From the shape of the curve it is immediately evident that (a) there is a large variation among the customers and (b) the PDF is very right skewed and quite different from a Gaussian model. These characteristics are accommodated in the H-B method when the Beta parameters are used for dimensioning the feeder with passive loads. The Beta PDF for load modeling has also been used in analyzing the effects of connecting electric vehicles [8]. However, as will be shown, the load profile that produces the maximum demand condition is not necessarily useful for analyzing the feeder performance when DG is added.

### III. DG VARIABILITY AND DG MODELS

All DG that derives its energy from natural sources such as wind, sunlight or micro hydro will be affected by variability. The variability of wind is often modelled by the Weibull distribution [9] while the Beta PDF has been suggested by Atwa et al for photovoltaic generation (PV) [10]. In this work we will investigate PV generation but the concepts may be applicable to other forms of generation. PV output is affected by:

- Conversion efficiency
- Latitude
- Season
- Orientation to the sun (tilt and tracking)
- Shadows (buildings and trees)
- Clouds
- Temperature
- Cleanliness
- Age of cell

PV output data for two sites close to examples A and B were generated in 2005 [11]. They were derived from irradiation data obtained from the HelioClim satellite using the Albedo 0.2, Perez tilted surface model. A typical roof slope of  $15^\circ$  was used and electrical and aging de-rating factors were applied.

The corresponding PV outputs for site A (33.9S 18.48E) and site B (26.1S 28.05E) are shown in Figures 3 and 4.

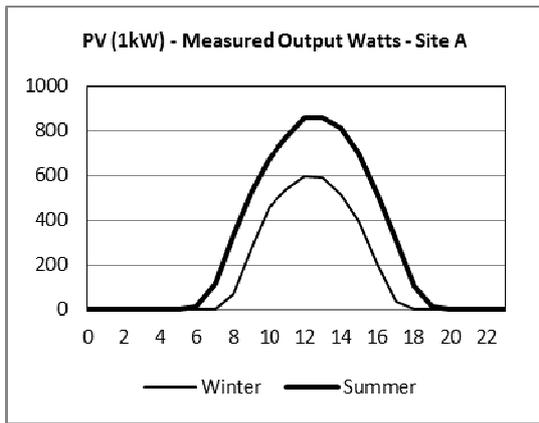


Figure 3 PV outputs for Site A

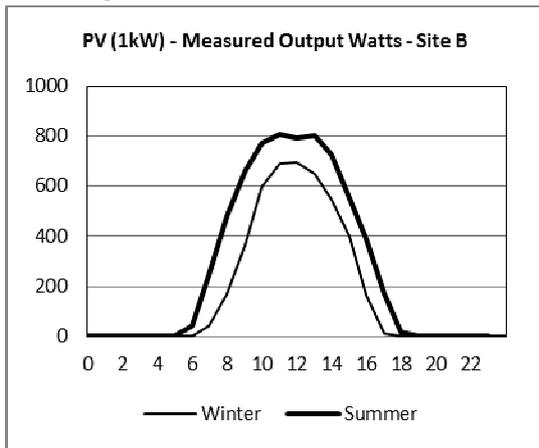


Figure 4 PV outputs for Site B

In a similar way the variability of wind generation may be represented by a bi-modal probability distribution with daily wind pattern and seasonal variation, as suggested in Figure 5 where  $P_c$  is the annual or monthly capacity factor and the PDF at any time of day is characteristic (see also Figure 8)

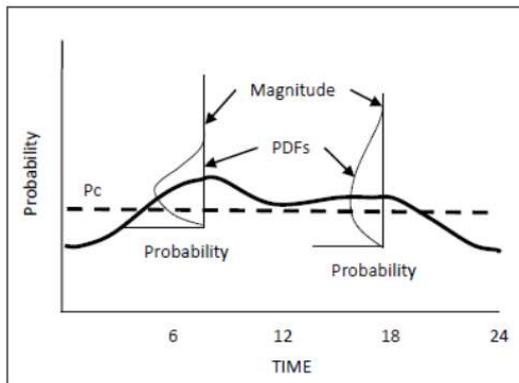


Figure 5 Bi-modal probability distribution of Wind generation

#### IV. COMBINATION OF LOADS AND GENERATORS

The probabilistic analysis of the feeder performance will mainly hinge on (a) voltage regulation and (b) reverse power feed. Assuming that the feeder performs within voltage regulation limits when supplying passive loads, the worst

scenarios to consider are when the load is light and the PV generation is greatest. This could give rise to both phenomena. So, consider a scenario where the typical summer and the mid-winter peak load profiles for a high end customer class are depicted by actual loads measured at site B in Figure 6. Superimposed on these graphs is an idealized PV output with a peak of 3kW (or 13 Amps at 230 V). If it is required to represent this PV output as a Beta PDF in the calculations, we can approximate it as a symmetrical Beta PDF with high  $\alpha$  and  $\beta$  values (low dispersion).

#### (a) Individual In-feed

Clearly, the analysis should include the determination of the cross-over points indicated on the summer load profile. In this case the morning cross-over occurs at 09:30 and at 15:30 in the afternoon. At this stage a customer with PV facility starts to generate into the feeder and change the voltage regulation along it.

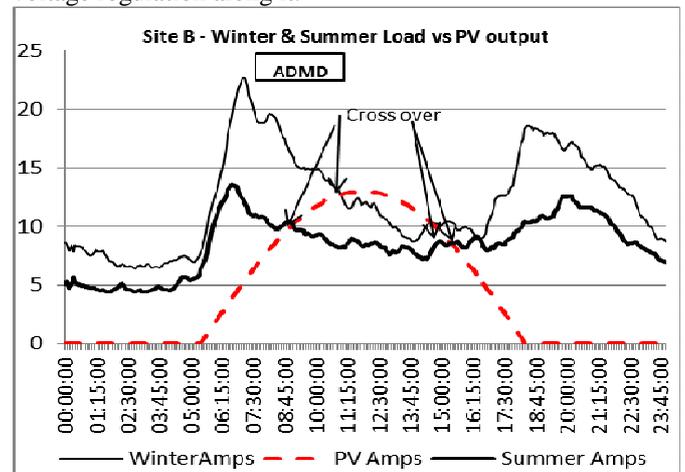


Figure 6 winter and summer load profiles with added DG

Now, let us examine the distribution of load currents at these points by their Beta parameters with  $C = 60$  Amps, as shown in Table I.

TABLE I Statistical parameters of loads at Cross-over points (Site B)

Time	Mean	Std Dev	$\alpha$	$\beta$
09:30	10.04	8.41	1.02	7.81
15:30	16.26	8.23	2.58	6.93

Note the large dispersion and the skewness of the two distributions illustrated in Figure 7 (a symmetrical distribution occurs when  $\alpha = \beta$ ).

The PV outputs were generated for hourly intervals. If we analyse these data at the approximate cross-over intervals indicated in Figure 6 we obtain the following statistical parameters for Site B.

TABLE II Site B Statistical parameters of PV outputs

Time	Mean	Std Dev	$\alpha$	$\beta$
09:00	121.4	8.7	8.7	7.09
15:00	463	106.3	9.73	11.26

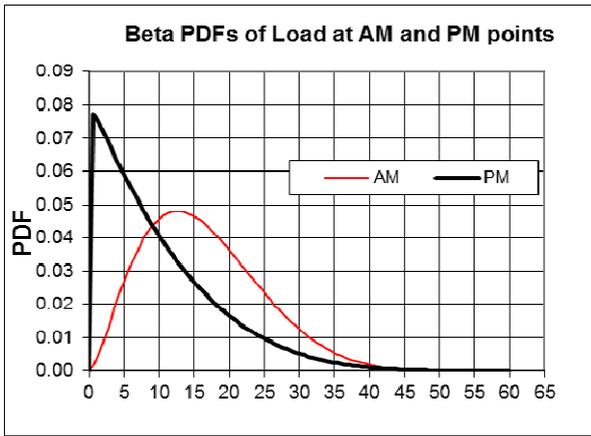


Figure 7 Beta PDF graphs of the Loads at the cross-over intervals (Site B)

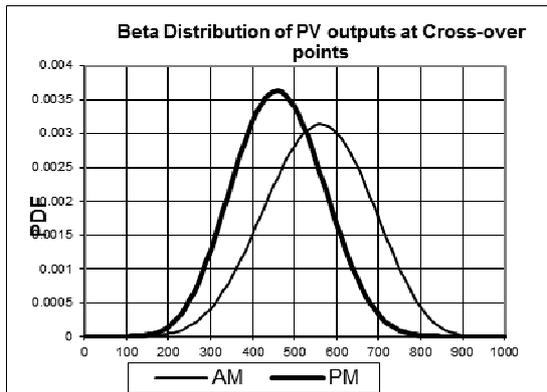


Figure 8 Beta PDFs of PV output at the two Cross-over intervals (Site B)

The two PDFs at the cross-over points are illustrated graphically in Figure 8 and show that the morning curve is left skewed and the afternoon is slightly right skewed,

#### (b) Total In-feed

Suppose the after-diversity-demand is  $ADD_L$ , the total number of customers is  $N_L$ , the after-diversity-generation is  $ADD_G$  and the total number of DG installations is  $N_G$ , then when

$$ADD_L \times N_L \leq ADD_G \times N_G$$

the feeder becomes balanced, with possible islanding conditions developing. This is the threshold at which reverse power supply into the MV system occurs.

#### V. FEEDER PERFORMANCE ASSESSMENT APPROACHES

The most common method of analysing the performance of a feeder both residential loads and DG is to assume average values and to perform deterministic load flows. In the previous two sections we have shown how variable both loads and DG can be. The variability in loads has been recognised and a variety of methods were employed in the past to allow for this in voltage regulation calculations. One approach used to include variability is to investigate different scenarios. One of the difficulties often mentioned is the lack

of appropriate load data. When these data are available statistical models can be developed. The most common model used is Gaussian with the mean and the standard deviation as parameters. This has limitations and loads are more appropriately modelled using the Beta PDF. With either the Gaussian or the Beta model a probabilistic approach is required to perform the analysis. Generally, this is achieved using Monte Carlo simulations (MCS) where the sampling is done from the statistical description [12]. The sampling is usually extensive, typically more than 1000 repetitions.

In passive circuits (without DG) the H-B method has proved to be very useful [13]. This is an analytical method that uses statistical moments to determine the Beta statistical parameters of the output voltage. The process is linear and does not require iterations. Since the output is a range of values described by a Beta PDF it is convenient to assign a level of risk (conversely confidence level) to extract a single value for decision making. It is extensively used in South Africa to dimension feeder conductors in LV distribution design.

When both the loads and the DG units are described by Beta PDFs the two models can be incorporated into the H-B method to analyse the voltage regulation on a feeder in a probabilistic manner. We note that we can express the statistical properties of both load and PV generation at the cross-over points where they intersect as Beta PDFs. It should also be noted that a significant data base should be available from which to derive the parameters. When extreme conditions are to be analysed where PV generation is high and load is (probabilistically) low, the PV generation can be modelled as having very little dispersion. That is, it is regarded as approaching a deterministic value and can be represented by equivalent high alpha and beta values that are equal. This technique is used in the H-B method when fixed loads are to be represented [14]. Preliminary work has been done on incorporating DG into feeder performance evaluation using the H-B approach [15].

#### VI. CONCLUSIONS AND FUTURE WORK

The paper describes the variability of load and DG parameters using examples from loads measured in South Africa as applicable to LV feeders. The general approach can be extended to MV systems with some care. At LV, the loads are assumed to be at unity power factor but this has to be reviewed at MV level. Variations in load magnitude are generally of greater consequence than variations in PF.

Using a probabilistic approach means that input variables expressed as PDFs translate to outputs that may also be described as probability distributions. This means that if a singular result such as probable voltage level is desired, a confidence level (conversely a risk level) needs to be assigned to the output PDF. This is done in the H-B methods. An alternative analysis would examine, for example, what the risk would be of violating regulatory limits with a given number of DGs on a particular feeder.

Future work in South Africa is aimed at compiling a Grid Connection Code for the local utility using probabilistic methods as described in this paper.

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