

Updated assessment of the South African kingklip resource that includes catch-at-length data for the one-stock and two-stock hypotheses

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ABSTRACT

This paper updates the kingklip assessments by Brandão and Butterworth (2008a and 2008b) which incorporated catch-at-length data in the assessment models by taking further and updated data into account. Further, the analysis of the two stock hypothesis model is extended to the West coast as well. If the resource is treated as two separate stocks, current (2017 for West coast and 2016 for the South coast) depletion of spawning biomass on the West and South coasts is estimated at 0.64 and 0.85 respectively. A similar status is obtained with the resource treated as a single stock, which yields a current depletion estimate of 0.72. A more pessimistic picture is obtained with the West coast selectivity functions assumed for both one and two stock hypotheses models, especially for the two stock hypothesis for the South coast, which yields a current depletion estimate of 0.43. However the fit to the catch-at-length data from both the surveys and the commercial fishery is worse in this case. The two sensitivity tests investigated for the one stock hypothesis show similar results to those obtained for the base case model.

INTRODUCTION

South African kingklip landings currently constitute a proportion of the by-catch from hake-directed trawls off the West and South coasts. A directed longline fishery for this species had existed since 1983, before it was terminated in 1990. Since that date longline catches have been restricted to by-catch from hake-directed longlining.

This paper updates the assessment of the kingklip resource (which includes catch-at-length data) for the combined South and West coasts that was presented by Brandão and Butterworth (2008a) and for the South coast only by Brandão and Butterworth (2008b). This paper further extends the assessment of the two stock hypothesis to the West coast that had not been carried out previously. Further data inputs available are taken into account as well as updates to the previous data. All available indices of abundance

are taken into account.

DATA

Catch Data

Total annual catches of kingklip for the West and South coasts from hake-directed trawls over the period 1932–2016 and from hake-directed longliners for the periods 1983–2016 are shown in Table 1 and Figure 1. The total annual catches for 2017 are not available for the present analysis; however, to be able to include the information from surveys for 2017 in the West coast, the assumption was made to set the total annual catches for 2017 to be the same as that in 2016.

Survey abundance data

Survey abundance data for each of the West and South coasts from 1986–2017 are used (Table 2). Previously no distinction had been made between surveys that surveyed the coast up to 200m and those that surveyed the coast up to 500m. The choice made here between using abundance indices for the 200 or 500m coastal area was based on selecting the longest series. Thus, for the South coast surveys conducted in May/June (autumn) a 500m coastal area was selected, while for those in Sept/Oct (spring) a 200m coastal area was chosen. In previous assessments of the kingklip resource, abundance indices estimated from surveys carried out on the *Nansen* had been used. These indices are no longer being used.

CPUE data

CPUE abundance data for the years 1983 to 1991 for the trawl and longline fisheries from Punt and Japp (1994) are used (see Table 3). There have been recent attempts to develop CPUE index for kingklip from the fishery data but for the moment this is not ready to be used in the assessment model.

Catch-at-length data

Survey catch-at-length data for each coast are available for most years in which a survey was carried out. Observer commercial catch-at-length data disaggregated by coast and fishery are available in the main from 2000 to 2010 for the longline fishery. Land based catch-at-length data for the trawl fishery is used and is available from 2005 to 2016 for the West coast and from 2008 to 2015 and data in 2001 in the South coast.

THE MODEL

The Age-Structured Production Model (ASPM) used to model a one or a two stock hypothesis, including catch-at-length data, is as in Brandão and Butterworth (2008a). Detailed specification of the model is given in the Appendix.

The biological parameters used in the assessment are those used by Punt and Japp (1994) and as used by Brandão and Butterworth (2008a). These values are listed in Table 4.

As in Brandão and Butterworth (2008a), the different gear types (old or new) used during the surveys are differentiated. Only surveys from the vessels the *Africana* are used.

Initial attempts at fitting the ASPM to include catch-at-length data to the one and the two stock hypotheses of the kingklip resource encountered convergence problems. The best fits attained under these non-convergence situations showed some extreme results when applied to the two stock hypothesis model of the South coast. Given these problems, the base case models for both the one and two stock hypotheses are based on first obtaining the best fit for the one hypothesis model (under non-convergence) and then fixing the selectivity functions estimated from this fit to both the one and the two stock models which will then form the base case models. An alternative model was investigated in which the estimated selectivity

functions from the two stock hypothesis model for the West coast is used as the fixed selectivity functions for the South coast as well as for the one stock hypothesis model. Convergence problems were still encountered in fitting some of the models even with fixed selectivity functions.

The ages at which the selectivity function decreases for older fish for the commercial and the survey selectivities were initially freed in the model to be estimated. However because of convergence problems these parameters have been fixed to values that were based on those used in Brandão and Butterworth (2008a, 2008b), but updated to obtain better fits to the one stock hypothesis model, although without the program indicating full convergence. These values are set to be $a_{ch}^{surv,SC} = 2$ for the survey selectivities and $a_{ch}^{com,c} = 5$ for the commercial selectivities for the South coast and $a_{ch}^{surv,WC} = a_{ch}^{com,WC} = 7.968$ for the survey and commercial selectivities for the West coast (see the Appendix). A sensitivity test was conducted for the base case one stock hypothesis model, in which these ages were estimated.

Another sensitivity test was conducted for base case one stock hypothesis model, in which the steepness parameter was changed from 0.5 to 0.75.

RESULTS

Results for the base case one and two stock hypotheses assessments of kingklip are given in Table 5. The current assessment estimates the depletion (B_{sp}/K_{sp}) at the beginning of 2017 for the South coast at 85% of the pre-exploitation abundance. For the West coast, this depletion is estimated at 64%. For the one stock hypothesis, depletion is estimated at 72%. The q estimates for the surveys are all less than 1, suggesting considerable underestimation of biomass in absolute terms by the swept-area method.

The fits of the model to the various abundance indices available are shown in Figure 2 for the two stock hypothesis model and in Figure 3 for the one stock hypothesis model. The current assessments show that all of the abundance indices for the West coast and the South coast (with the possible exception of the spring survey) suggest a slight recovery for both the one and two stock hypotheses models (Figures 2 and 3).

The trajectories with 95% confidence intervals for depletion (B_{sp}/K_{sp}) and the spawning biomass (B_{sp}) for the base case ($h = 0.5$) are shown in Figures 4a–b for the one and the two stock hypothesis models. Both one and two stock hypotheses models show an increasing trend since the end of the 1980's. Figure 4b also shows these results for the one stock hypothesis alternative model in which the selectivity functions are fixed to those estimated for the two stock hypothesis for the West coast. Figure 4c shows the trajectories for the two stock alternative model for the South coast. Results for this alternative model are given in Table 6. Fixing the selectivity functions to those estimated by the two stock hypothesis model for the West coast results in a more pessimistic status estimated for the kingklip resource for both the one or the two stock hypotheses, especially for the South coast.

Figure 5a shows the estimated selectivity curves for the survey and the commercial fishery for the one stock hypothesis base case model. These selectivity functions are then applied to the one and the two stock hypotheses models as fixed parameters. Figure 5b shows the estimated selectivity functions for the two stock hypothesis model for the West coast, which are then used as fixed parameters by the alternative assessment models.

Fits to the averaged catch-at-length distributions for the surveys and the commercial fishery are shown in Figures 6a-b for the one and two stock hypotheses base case models. Figure 7 shows these fits to the two stock hypothesis alternative model for the South coast. These plots show that the averaged catch-at-length distributions for the surveys and the commercial fishery in the South coast are not well fitted by assuming the same selectivity functions as for the West coast.

Figure 8 shows the fishing mortality for the one stock hypothesis base case model.

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Table 1. Yearly catches (in tons) of kinglip taken by the trawl and longline fisheries on the West and South coasts of South Africa.

Year	West coast		South coast		Year	West coast		South coast	
	Trawl	Longline	Trawl	Longline		Trawl	Longline	Trawl	Longline
1932	436	0	164	0	1975	2 600	0	982	0
1933	290	0	110	0	1976	2 519	0	952	0
1934	290	0	110	0	1977	1 953	0	737	0
1935	508	0	192	0	1978	2 551	0	1 759	0
1936	508	0	192	0	1979	3 080	0	1 532	0
1937	508	0	192	0	1980	4 415	0	878	0
1938	508	0	192	0	1981	3 149	0	963	0
1939	508	0	192	0	1982	2 410	0	721	0
1940	508	0	192	0	1983	2 246	842	1 169	200
1941	436	0	164	0	1984	2 558	1881	1 034	1159
1942	436	0	164	0	1985	1 750	1314	1 650	5656
1943	436	0	164	0	1986	2 287	1231	399	7453
1944	436	0	164	0	1987	2 083	1948	392	4504
1945	944	0	356	0	1988	1 519	2091	408	3311
1946	726	0	274	0	1989	1 407	1607	223	2209
1947	798	0	302	0	1990	1 002	557	266	708
1948	1 089	0	411	0	1991	1 271	0	680	0
1949	1 307	0	493	0	1992	1 884	0	676	0
1950	1 379	0	521	0	1993	2 207	0	884	0
1951	1 742	0	658	0	1994	1 445	92	1 560	48
1952	2 032	0	768	0	1995	1 863	65	1 275	48
1953	1 960	0	740	0	1996	1 596	170	1 981	60
1954	1 452	0	548	0	1997	1 972	155	2 128	120
1955	1 669	0	631	0	1998	1 632	53	1 366	87
1956	1 452	0	548	0	1999	2 104	141	1 737	171
1957	1 089	0	411	0	2000	2 166	199	1 465	103
1958	1 234	0	466	0	2001	2 651	183	2 210	57
1959	1 452	0	548	0	2002	2 280	312	2 479	202
1960	1 089	0	411	0	2003	1 870	317	2 558	160
1961	1 524	0	576	0	2004	1 823	266	2 539	141
1962	1 234	0	466	0	2005	1 790	255	1 851	121
1963	1 307	0	493	0	2006	1 476	110	1 322	127
1964	1 016	0	384	0	2007	1 213	105	1 223	85
1965	1 815	0	685	0	2008	1 122	83	1 307	118
1966	2 686	0	1 014	0	2009	1 153	138	958	140
1967	2 323	0	877	0	2010	1 405	199	1 057	149
1968	2 105	0	795	0	2011	1 540	212	891	126
1969	2 105	0	795	0	2012	1 866	270	1 272	112
1970	2 105	0	795	0	2013	1 801	281	1 995	84
1971	3 557	0	1 343	0	2014	1 525	327	1 584	25
1972	3 774	0	1 426	0	2015	1 610	335	1 441	28
1973	4 210	0	1 590	0	2016	1 613	414	1 217	21
1974	2 532	0	956	0	2017	1 613†	414†	1 217†	21†

† Catch data for 2017 assumed to be the same as in 2016.

Table 2. Abundance indices of kingklip in tons together with CVs obtained from surveys (separated by season) for the West and South coasts of South Africa. Values in bold denote abundance estimates obtained using the new rather than the old gear on *Africana*.

Year	West coast				South coast			
	Jan/Feb (summer)		Jul/Aug (winter)		Sep/Oct (spring) (0 – 200 m)		May/Jun (autumn) (0 – 500 m)	
	Index	CV	Index	CV	Index	CV	Index	CV
1985	8 004	0.139	5 603	0.343	—	—	—	—
1986	3 708	0.160	2 279	0.155	2 780	0.239	—	—
1987	2 817	0.193	5 243	0.243	3 416	0.182	—	—
1988	5 538	0.209	1 578	0.251	—	—	6 478	0.455
1989	—	—	1 082	0.337	—	—	—	—
1990	3 994	0.266	1 251	0.471	1 098	0.354	—	—
1991	3 490	0.299	—	—	2 138	0.274	7 499	0.146
1992	7 404	0.190	—	—	1 704	0.216	3 064	0.399
1993	10 176	0.186	—	—	1 135	0.201	8 759	0.393
1994	8 175	0.179	—	—	1 133	0.276	34 989	0.664
1995	7 121	0.263	—	—	1 152	0.427	20 623	0.409
1996	11 222	0.313	—	—	—	—	3 502	0.189
1997	5 878	0.222	—	—	—	—	5 130	0.268
1998	—	—	—	—	—	—	—	—
1999	14 640	0.303	—	—	—	—	11 350	0.611
2000	—	—	—	—	—	—	—	—
2001	—	—	—	—	2 033	0.292	—	—
2002	13 233	0.165	—	—	—	—	—	—
2003	14 039	0.314	—	—	4 291	0.586	8 690	0.745
2004	7 472	0.181	—	—	497	0.360	716	0.346
2005	5 616	0.165	—	—	—	—	7 472	0.886
2006	8 083	0.296	—	—	1 761	0.447	1 297	0.249
2007	5 634	0.259	—	—	939	0.273	3 297	0.475
2008	4 727	0.141	—	—	4 896	0.204	3 066	0.220
2009	10 639	0.189	—	—	—	—	6 072	0.302
2010	13 474	0.137	—	—	—	—	7 347	0.349
2011	15 707	0.166	—	—	—	—	4 879	0.392
2012	7 279	0.169	—	—	—	—	—	—
2013	7 627	0.275	—	—	—	—	—	—
2014	8 724	0.153	—	—	—	—	1 842	0.609
2015	11 419	0.335	—	—	—	—	1 353	0.266
2016	6 502	0.142	—	—	499	0.230	9 256	0.635
2017	5 099	0.288	—	—	—	—	—	—

Table 3. Standardised commercial CPUE indices of relative abundance for kingklip for the trawl and longline fishery for the South and West coasts of South Africa. These data have been obtained from Punt and Japp (1994).

Year	West coast		South coast	
	Trawl	Longline	Trawl	Longline
1983	1.786		1.294	
1984	2.147	2.253	1.230	2.276
1985	2.193	1.302	1.250	3.082
1986	1.829	1.394	1.190	3.113
1987	1.530	1.300	0.906	2.397
1988	1.420	1.294	0.826	2.202
1989	0.897	1.234	0.763	1.551
1990	0.720	1.000	0.520	1.000
1991	1.000		1.000	

Table 4. Biological parameters values for kingklip for the West and South coasts of South Africa. Note that for simplicity, maturity is assumed to be knife-edge in age. These values are as used by Punt and Japp (1994).

Parameter	West coast	South coast	Coasts combined
Natural mortality M (yr^{-1})	0.2	0.2	0.2
von Bertalanffy growth			
L_{∞} (cm)	129.2	136.0	132.6
κ (yr^{-1})	0.141	0.142	0.142
t_0 (yr)	-0.32	0.22	0.05
Weight (in gm) length relationship			
e ($\text{g}\cdot\text{cm}^{-1}$)	0.00083	0.00162	0.00132
f	3.41	3.26	3.31
Age of "plus group" (yr)	30	30	30
Age at maturity (yr)	5	5	5
Steepness parameter (h)	0.5	0.5	0.5

Table 5. Estimated model parameters for the **base case** model ($h = 0.5$, selectivity functions estimated from the **one stock hypothesis** model) for the one and two stock hypotheses. The “current” year refers to the most recent year of the assessment (i.e. 2017 for the one stock and West coast two stock models and 2016 for the two stock model for the South coast). 95% confidence intervals calculated from the Hessian matrix are shown for some parameters.

Parameter estimates	Two stock hypothesis		One stock hypothesis
	West coast	South coast	Coasts combined
-ln L: Total	294.1	569.3	862.1
-ln L: Survey (summer)	26.40	—	25.21
-ln L: Survey (winter)	5.284	—	4.800
-ln L: Survey (spring)	—	19.78	19.67
-ln L: Survey (autumn)	—	35.45	35.87
-ln L: CPUE (trawl) WC/SC	-4.450	-7.582	-4.648 / -7.832
-ln L: CPUE (longline) WC/SC	-7.477	-3.385	-8.062 / -3.460
-ln L: Survey length (summer)	125.9	—	126.5
-ln L: Survey length (winter)	57.56	—	57.7
-ln L: Survey length (spring)	—	85.21	85.67
-ln L: Survey length (autumn)	—	316.0	316.9
-ln L: trawl length WC/SC	51.96	75.35	52.00 / 74.92
-ln L: longline length WC/SC	38.92	48.47	38.50 / 48.30
K^{sp}	100 180 (69 949; 130 411)	201 389 (176 689; 226 089)	232 294 (155 564; 309 024)
$B_{current}^{sp}$	64 296 (29 698; 98 894)	172 009 (144 159; 199 859)	167 672 (86 534; 248 810)
$B_{current}^{sp} / K^{sp}$	0.642 (0.490; 1.000)	0.854 (0.695; 1.013)	0.722 (0.611; 0.833)
$K_{exp}^{trawl} (WC)$	30 945	—	71 756
$K_{exp}^{trawl} (SC)$	—	43 617	50 310
$B_{current}^{exp, trawl} (WC / SC)$	23 397	39 394	57 665 / 41 400
$B_{current}^{exp, trawl} / K^{exp} (WC / SC)$	0.756	0.903	0.804 / 0.793
$B_{current}^{exp, longline} / K^{exp} (WC / SC)$	0.716	0.883	0.774 / 0.793
$q_{survey}^{summer} (old gear)$	0.351	—	0.139
$q_{survey}^{summer} (new gear)$	0.340	—	0.139
$q_{survey}^{winter} (old gear)$	0.076	—	0.029
$q_{survey}^{spring} (new gear)$	—	0.067	0.066
$q_{survey}^{spring} (old gear)$	—	0.075	0.074
$q_{survey}^{autumn} (old gear)$	—	0.045	0.047
$q_{survey}^{autumn} (new gear)$	—	0.023	0.023
$\sigma_{CPUE}^{trawl} (WC)$	0.370	—	0.362
$\sigma_{CPUE}^{trawl} (SC)$	—	0.261	0.254
$\sigma_{CPUE}^{longline} (WC)$	0.208	—	0.192
$\sigma_{CPUE}^{longline} (SC)$	—	0.374	0.370
B_{MSY}^{sp}	45 092	91 377	105 399
$MSYL / K^{sp}$	0.450	0.454	0.454

Table 6. Estimated model parameters for the **alternative** models ($h = 0.5$, selectivity functions estimated from the **two stock hypothesis** model for West coast) for the one and two stock hypotheses. The “current” year refers to the most recent year of the assessment (i.e. 2017 for the one stock and West coast two stock models and 2016 for the two stock model for the South coast). 95% confidence intervals calculated from the Hessian matrix are shown for some parameters.

Parameter estimates	Two stock hypothesis		One stock hypothesis
	West coast	South coast	Coasts combined
-ln L: Total	287.2	746.0	1063.5
-ln L: Survey (summer)	26.16	—	26.81
-ln L: Survey (winter)	5.154	—	3.901
-ln L: Survey (spring)	—	23.37	20.57
-ln L: Survey (autumn)	—	31.28	32.84
-ln L: CPUE (trawl) WC/SC	-4.654	-9.726	-5.768 / -9.653
-ln L: CPUE (longline) WC/SC	-7.818	-2.777	-8.743 / -4.208
-ln L: Survey length (summer)	125.8	—	130.5
-ln L: Survey length (winter)	51.04	—	51.45
-ln L: Survey length (spring)	—	175.2	206.4
-ln L: Survey length (autumn)	—	350.3	366.1
-ln L: trawl length WC/SC	52.21	98.56	57.77 / 96.18
-ln L: longline length WC/SC	39.28	79.47	42.55 / 56.82
K^{sp}	89 932 (72 103; 107 761)	60 060 (53 818; 66 303)	145 155 (130 581; 159 729)
$B_{current}^{sp}$	52 669 (30 588; 74 750)	25 617 (16 116; 35 119)	73 780 (54 078; 93 482)
$B_{current}^{sp} / K^{sp}$	0.586 (0.456; 0.715)	0.427 (0.313; 0.540)	0.508 (0.424; 0.593)
$K_{exp}^{trawl} (WC)$	29 688	—	47 918
$K_{exp}^{trawl} (SC)$	—	19 827	47 918
$B_{current}^{exp, trawl} (WC / SC)$	20 961	10 915	30 703 / 30 703
$B_{current}^{exp, trawl} / K_{exp}^{trawl} (WC / SC)$	0.706	0.551	0.641 / 0.641
$B_{current}^{exp, longline} / K_{exp}^{trawl} (WC / SC)$	0.664	0.501	0.593 / 0.593
$q_{survey}^{summer} (old gear)$	0.410	—	0.262
$q_{survey}^{summer} (new gear)$	0.395	—	0.268
$q_{survey}^{winter} (old gear)$	0.110	—	0.073
$q_{survey}^{spring} (new gear)$	—	0.156	0.063
$q_{survey}^{spring} (old gear)$	—	0.194	0.073
$q_{survey}^{autumn} (old gear)$	—	0.354	0.137
$q_{survey}^{autumn} (new gear)$	—	0.186	0.068
$\sigma_{CPUE}^{trawl} (WC)$	0.362	—	0.320
$\sigma_{CPUE}^{trawl} (SC)$	—	0.206	0.208
$\sigma_{CPUE}^{longline} (WC)$	0.199	—	0.174
$\sigma_{CPUE}^{longline} (SC)$	—	0.408	0.333
B_{MSY}^{sp}	40 306	26 918	65 056
$MSYL / K^{sp}$	0.448	0.448	0.448

Table 7. Estimated model parameters for **sensitivity tests** for the **one stock hypothesis base case model**: 1) the steepness parameter is set at $h = 0.75$ and 2) ages at which the selectivity functions decrease for older fish is estimated. For comparison, the results for the one stock hypothesis base case model are also given here. The “current” year refers to the most recent year of the assessment (i.e. 2017). 95% confidence intervals calculated from the Hessian matrix are shown for some parameters.

Parameter estimates	Base case ($h = 0.5$)	$h = 0.75$	Estimate $a_{ch}^{surv/com,WC/SC}$
-ln L: Total	862.1	861.8	862.0
-ln L: Survey (summer)	25.21	22.80	25.90
-ln L: Survey (winter)	4.800	4.925	4.891
-ln L: Survey (spring)	19.67	20.45	19.67
-ln L: Survey (autumn)	35.87	36.55	35.84
-ln L: CPUE (trawl) WC/SC	-4.648 / -7.832	-4.529 / -7.683	-4.611 / -7.748
-ln L: CPUE (longline) WC/SC	-8.062 / -3.460	-7.961 / -3.401	-7.990 / -3.452
-ln L: Survey length (summer)	126.5	126.7	125.9
-ln L: Survey length (winter)	57.70	58.00	57.60
-ln L: Survey length (spring)	85.67	85.58	85.60
-ln L: Survey length (autumn)	316.9	316.8	316.7
-ln L: trawl length WC/SC	52.00 / 74.92	52.00 / 74.82	52.00 / 74.94
-ln L: longline length WC/SC	38.50 / 48.30	38.5 / 48.32	38.50 / 48.28
K^{sp}	232 294 (155 564; 309 024)	221 766	245 289 (130 306; 360 272)
$B_{current}^{sp}$	167 672 (86 534; 248 810)	174 725	181 304 (61 419; 301 189)
$B_{current}^{sp} / K^{sp}$	0.722 (0.611; 0.833)	0.788	0.739 (0.597; 0.881)
$K_{exp}^{trawl} (WC)$	71 756	68 504	75 689
$K_{exp}^{trawl} (SC)$	50 310	48 030	52 021
$B_{current}^{exp,trawl} (WC / SC)$	57 665 / 41 400	59 612 / 42 739	61 811 / 43 433
$B_{current}^{exp,trawl} / K_{exp}^{trawl} (WC / SC)$	0.804 / 0.793	0.870 / 0.890	0.817 / 0.835
$B_{current}^{exp,longline} / K_{exp}^{trawl} (WC / SC)$	0.774 / 0.793	0.840 / 0.860	0.789 / 0.807
q_{survey}^{summer} (old gear)	0.139	0.135	0.133
q_{survey}^{summer} (new gear)	0.139	0.135	0.134
q_{survey}^{winter} (old gear)	0.029	0.029	0.027
q_{survey}^{spring} (new gear)	0.066	0.064	0.063
q_{survey}^{spring} (old gear)	0.074	0.072	0.070
q_{survey}^{autumn} (old gear)	0.047	0.046	0.043
q_{survey}^{autumn} (new gear)	0.023	0.022	0.021
$\sigma_{CPUE}^{trawl} (WC)$	0.362	0.367	0.363
$\sigma_{CPUE}^{trawl} (SC)$	0.254	0.258	0.256
$\sigma_{CPUE}^{longline} (WC)$	0.192	0.195	0.194
$\sigma_{CPUE}^{longline} (SC)$	0.370	0.373	0.370
B_{MSY}^{sp}	105 399	89 521	111 262
$MSYL / K^{sp}$	0.454	0.404	0.454

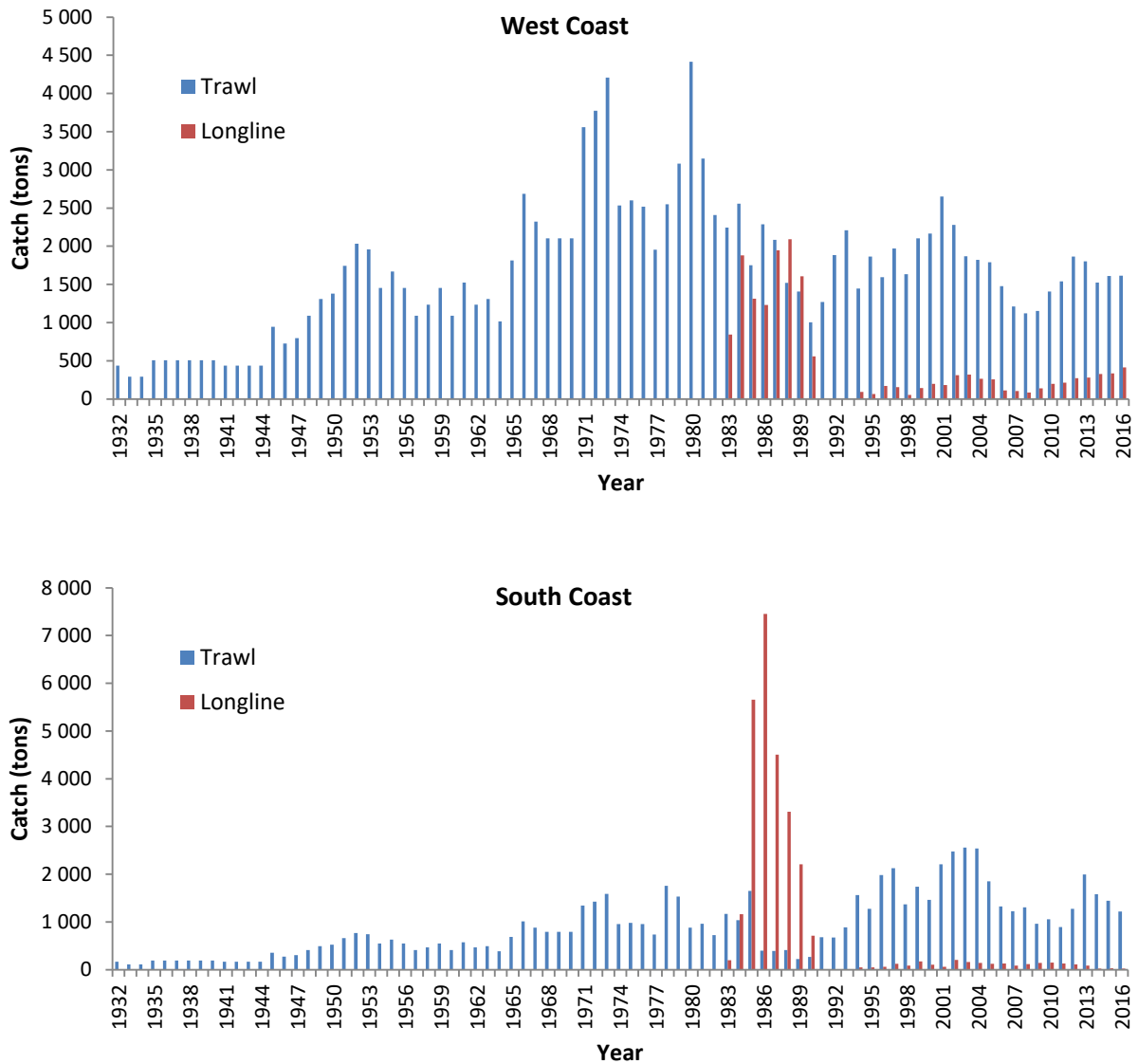


Figure 1. Historical catches of kingklip in the West and South coasts of South Africa separated by gear type (i.e. trawl or longline).

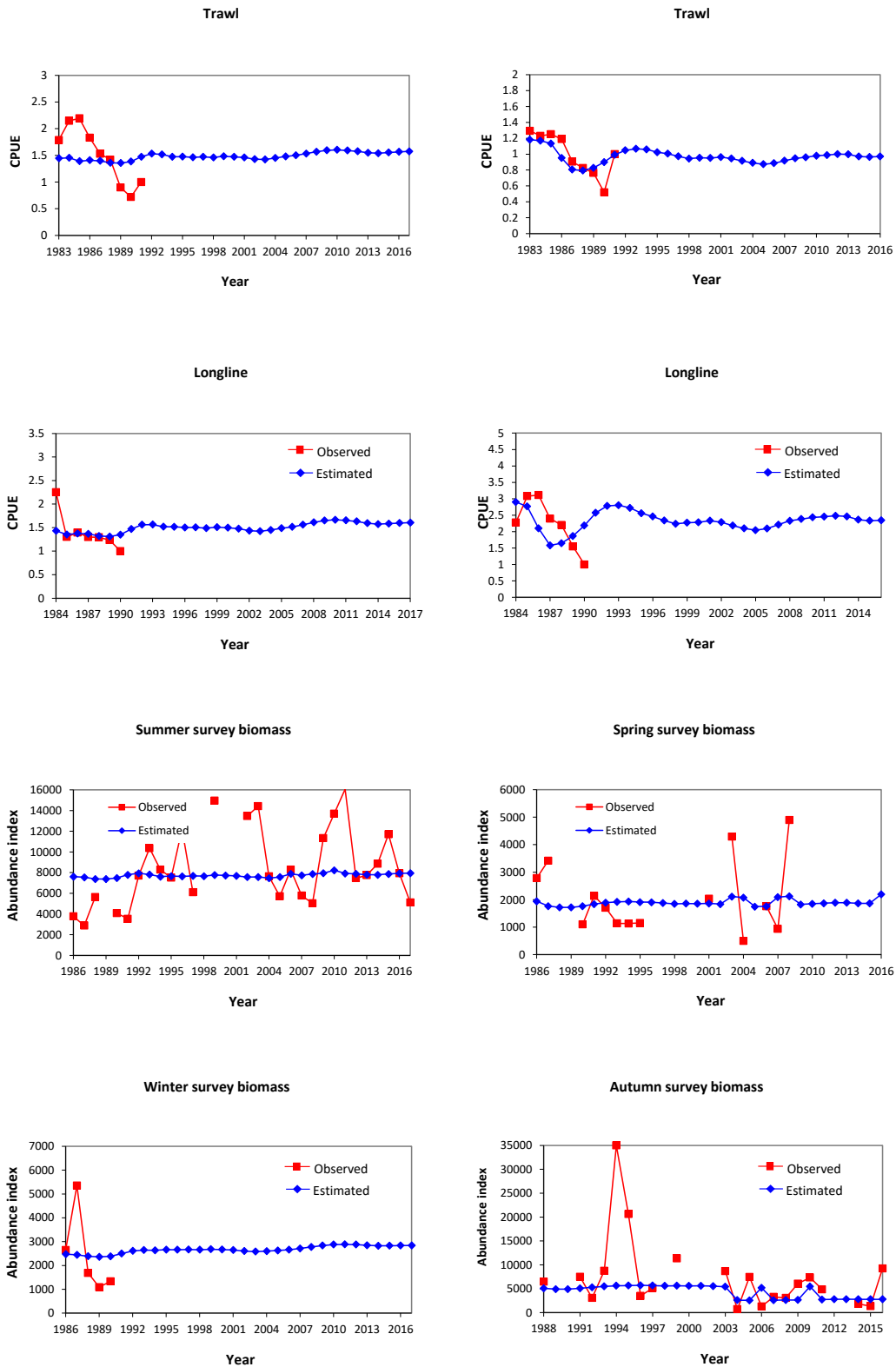


Figure 2. Observed and estimated trend of abundance indices for the **two stock hypothesis base case model** ($h = 0.5$, selectivity functions fixed to those estimated by the one stock model) fitted to data up to 2017 for the West coast (left hand side) and up to 2016 for the South coast (right hand side) of South Africa. Fluctuations shown for the estimated survey biomass are a consequence of differing q 's for the different vessels/gears used from year to year (the default is the old gear).

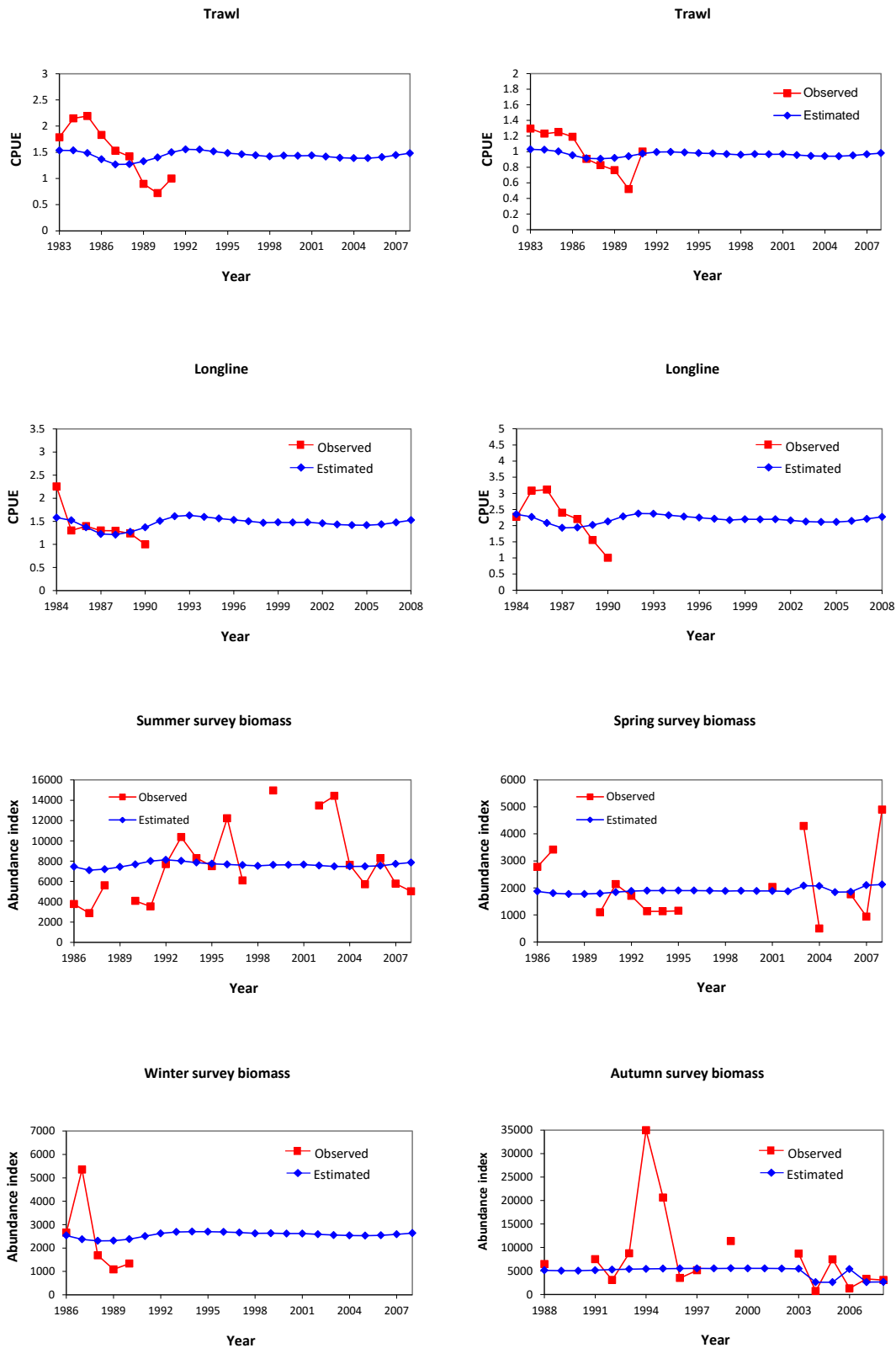


Figure 3. Observed and estimated trend of abundance indices for the **one stock hypothesis base case model** ($h = 0.5$, selectivity functions fixed to those estimated by the one stock model) fitted to data up to 2017 for the West coast (left hand side) and the South coast (right hand side) of South Africa. Fluctuations shown for the estimated survey biomass are a consequence of differing q 's for the different gears used from year to year (the default is the old gear).

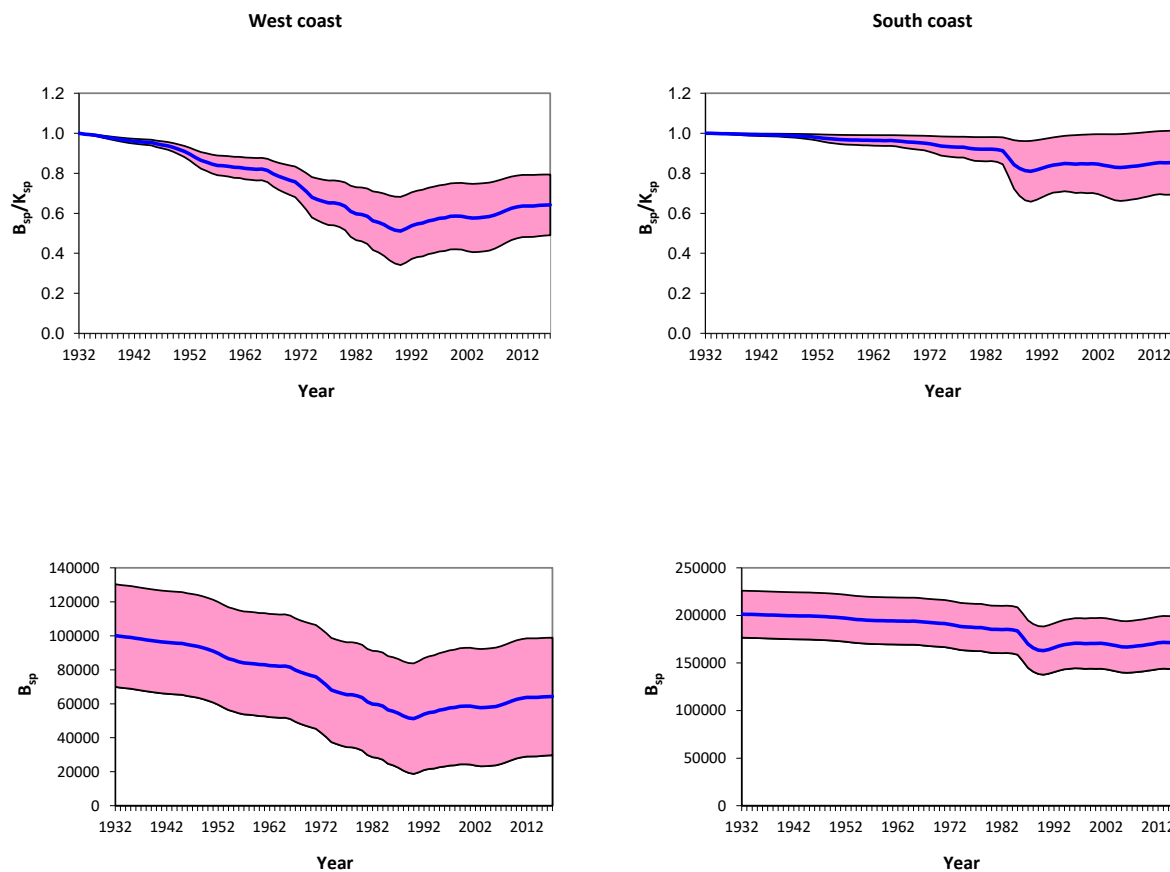


Figure 4a. Trajectories of spawning stock depletion (B_{sp}/K_{sp}) and spawning biomass (B_{sp}) estimates for kingklip for the West (left hand side) and South (right hand side) coasts of South Africa for the **two stock hypothesis base case model** ($h = 0.5$). The shaded area represents the 95% confidence intervals calculated from the Hessian matrix.

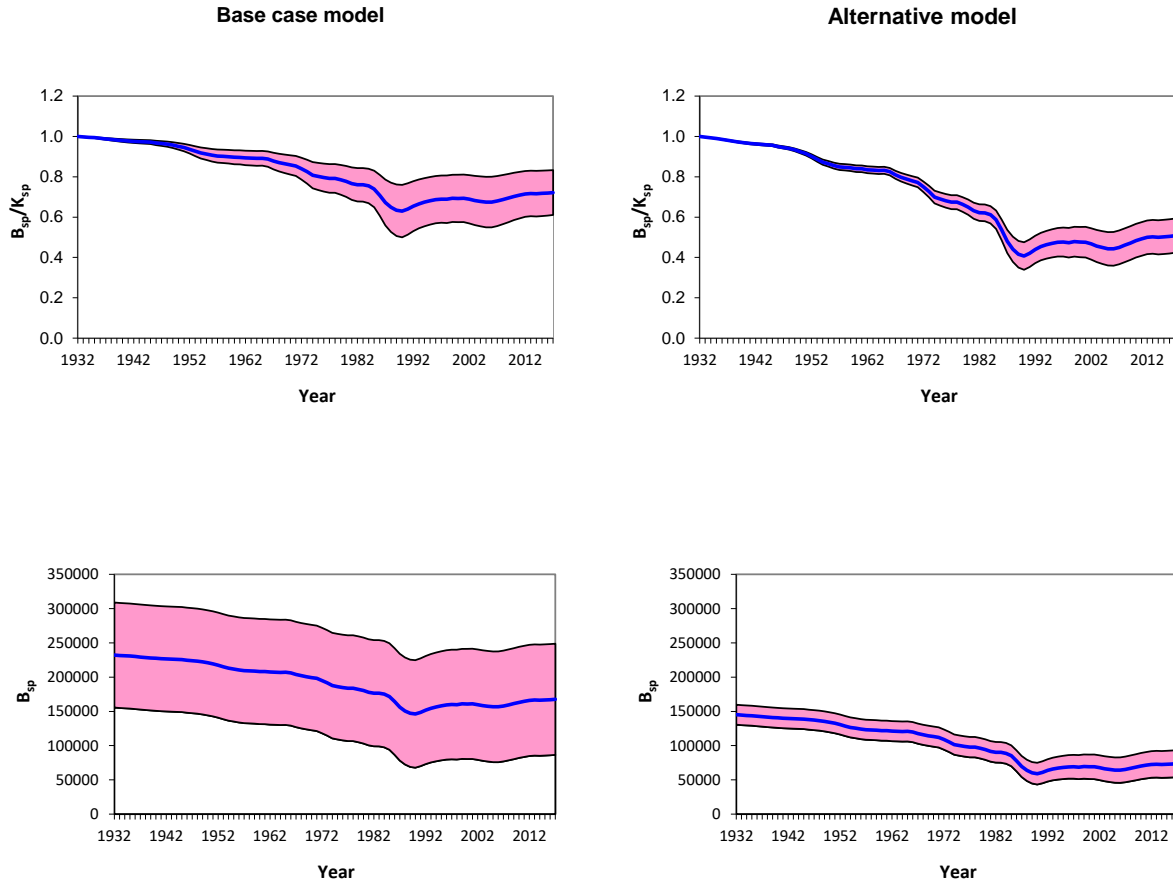


Figure 4b. Trajectories of spawning stock depletion (B_{sp}/K_{sp}) and spawning biomass (B_{sp}) estimates for kingklip of South Africa for the **one stock hypothesis base case (left) and alternative (right) models**. The shaded area represents the 95% confidence intervals calculated from the Hessian matrix.

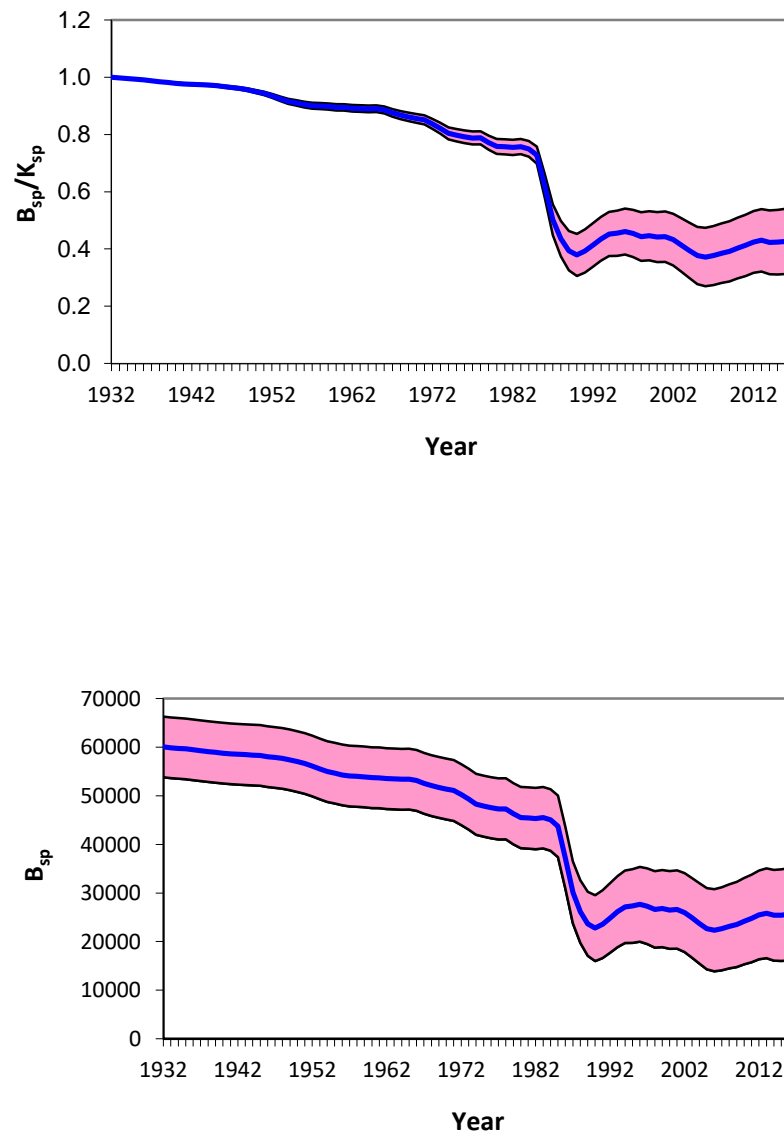


Figure 4c. Trajectories of spawning stock depletion (B_{sp}/K_{sp}) and spawning biomass (B_{sp}) estimates for the South coast kingklip of South Africa for the **two stock hypothesis alternative model**. The shaded area represents the 95% confidence intervals calculated from the Hessian matrix.

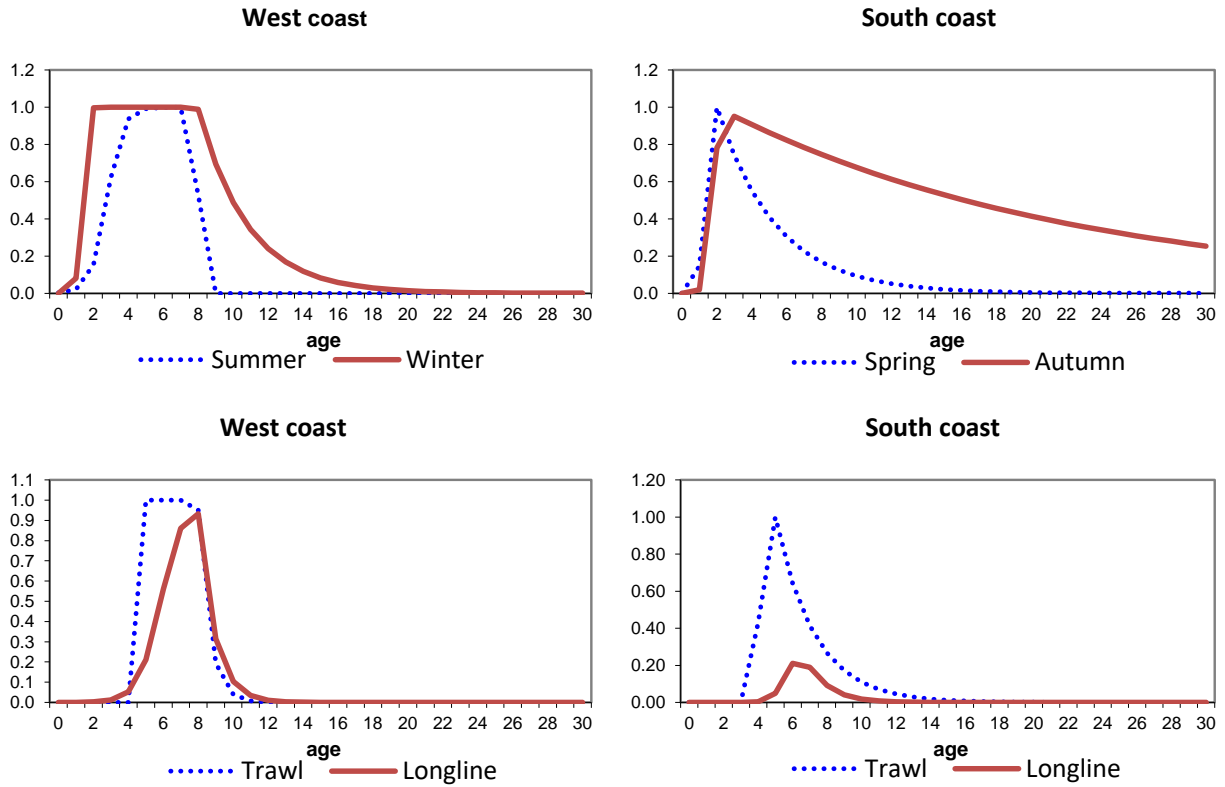


Figure 5a. Estimated selectivity curves for West (left hand side) and South (right hand side) coast surveys (top plots) and for the trawl and longline fishery (bottom plots) for the **one stock hypothesis base case model**.

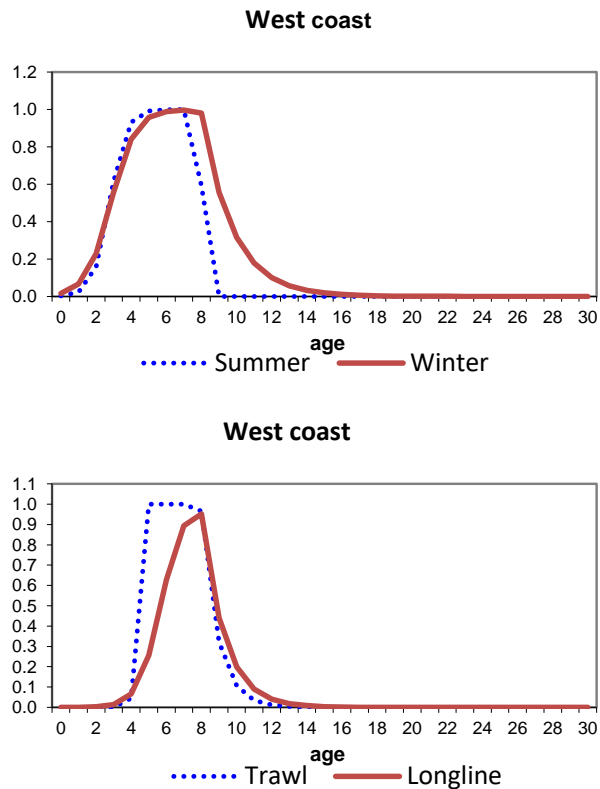


Figure 5b. Estimated selectivity curves for West coast surveys (top plots) and for the trawl and longline fishery (bottom plots) for the **two stock hypothesis alternative model**.

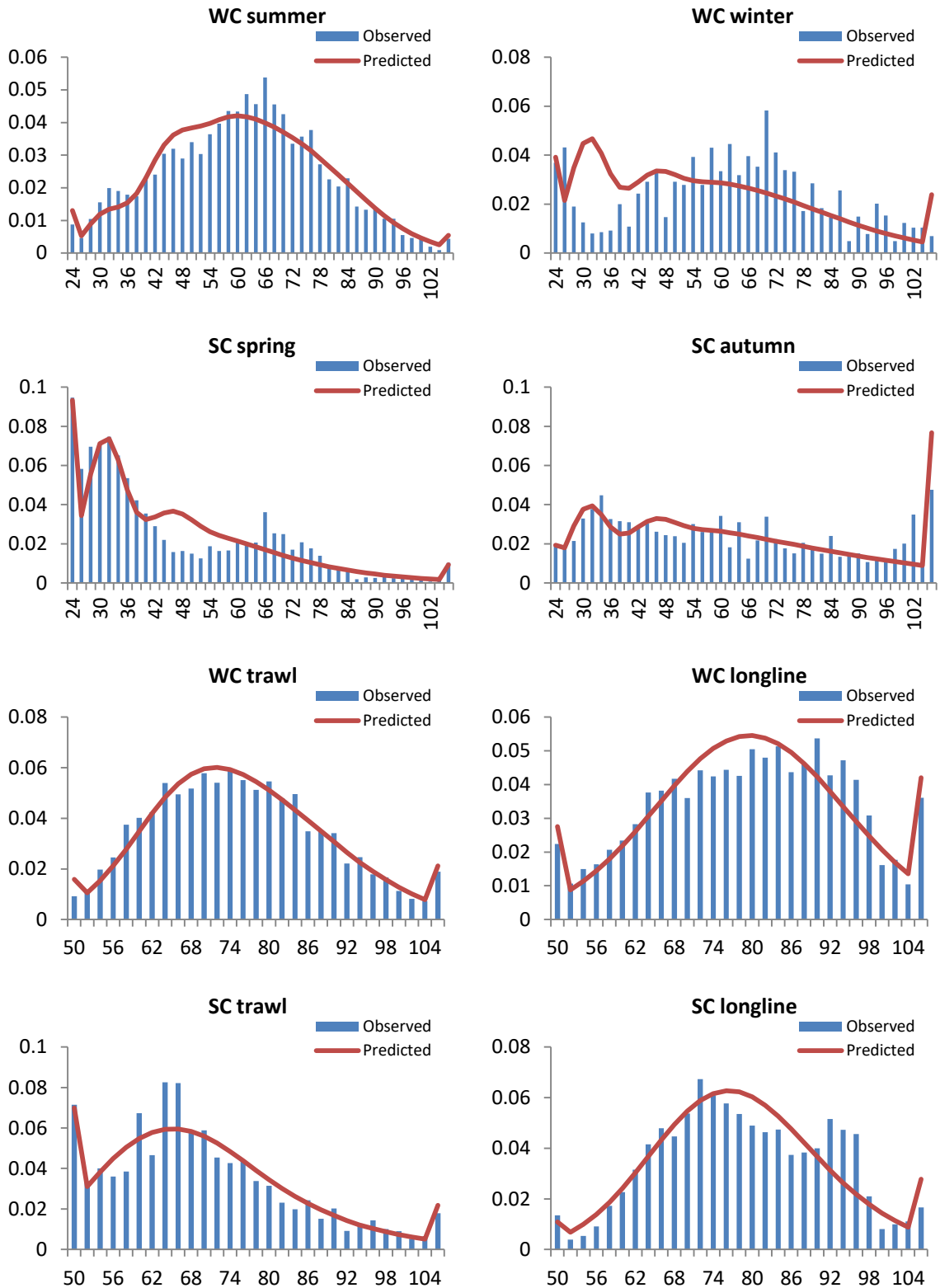


Figure 6a. Observed and **one stock hypothesis base case model** assessment predictions for the average catch-at-length proportions in the West and South coast research surveys (top four plots) and in the trawl and longline fishery (bottom four plots). Note that lengths below 24 and above 106 cm are combined into minus- and plus-groups for the West and South coast surveys and lengths below 50 and above 106 cm for the West and South coast commercial fishery.

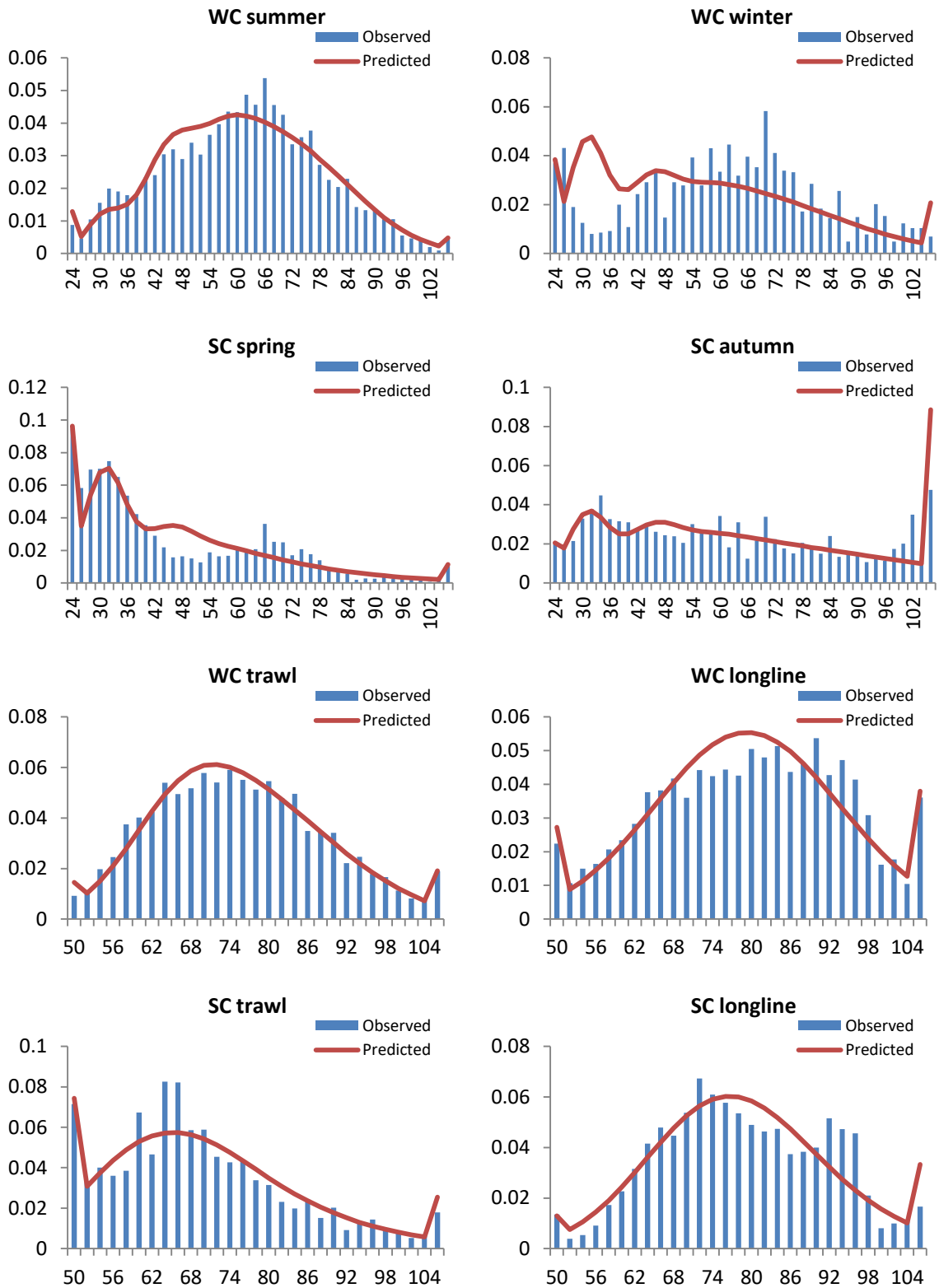


Figure 6b. Observed and **two stock hypothesis base case model** assessment predictions for the average catch-at-length proportions in the West and South coast research surveys (top four plots) and in the trawl and longline fishery (bottom four plots). Note that lengths below 24 and above 106 cm are combined into minus- and plus-groups for the West and South coast surveys and lengths below 50 and above 106 cm for the West and South coast commercial fishery.

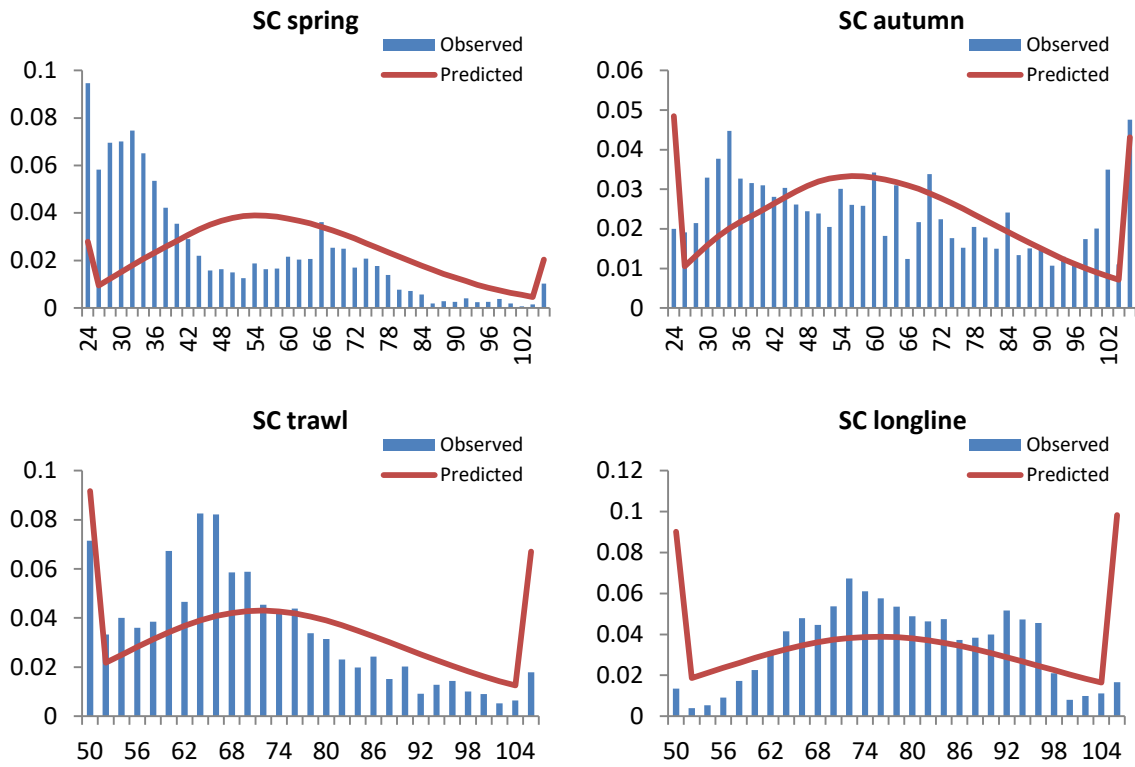


Figure 7. Observed and **two stock hypothesis alternative model** (fixed selectivity functions from West coast two stock model) assessment predictions for the average catch-at-length proportions in the South coast research surveys (top four plots) and in the trawl and longline fishery (bottom four plots). Note that lengths below 24 and above 106 cm are combined into minus- and plus-groups for the South coast surveys and lengths below 50 and above 106 cm for the South coast commercial fishery.

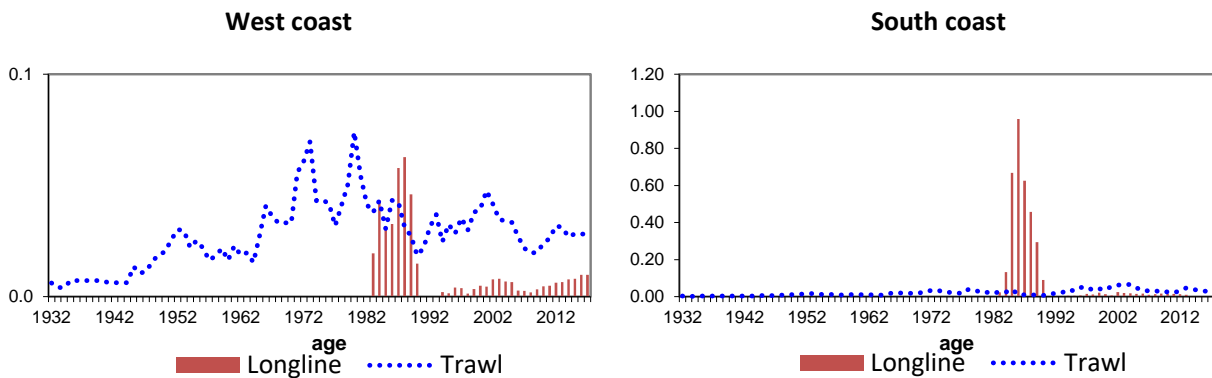


Figure 8. Estimated fishing mortality of kinglip for the **one stock hypothesis base case model** in the West (left) and South (right) coasts.

APPENDIX

THE AGE-STRUCTURED PRODUCTION MODEL FOR KINGKLIP

THE POPULATION DYNAMICS

The kingklip population dynamics are represented by:

$$\begin{aligned}
 N_{y+1,a} &= R_{y+1} && \text{if } a = 0 \\
 N_{y+1,a} &= (N_{y,a-1} e^{-\frac{M_{a-1}}{2}} - C_{y,a-1}) e^{-\frac{M_{a-1}}{2}} && \text{if } 1 \leq a < x \\
 N_{y+1,a} &= (N_{y,a} e^{-\frac{M_a}{2}} - C_{y,a}) e^{-\frac{M_a}{2}} + (N_{y,a-1} e^{-\frac{M_{a-1}}{2}} - C_{y,a-1}) e^{-\frac{M_{a-1}}{2}} && \text{if } a = m
 \end{aligned} \tag{A.1}$$

where:

- $N_{y,a}$ is the number of kingklip of age a at the start of year y ,
- $C_{y,a}$ is the number of kingklip of age a taken by the fishery (both longline and trawl) in year y ,
- R_y is the number of 0 year olds at the start of year y ,
- M_a is the instantaneous rate of natural mortality of kingklip of age a (assumed to be age invariant with a value of 0.2 yr^{-1}), and
- m is the largest age considered (i.e. the “plus group”, taken to be 30 years).

Note that in the interests of simplicity this approximates the fishery as a pulse fishery at the start of the year. Given that kingklip is relatively long-lived with low natural mortality, such an approximation would seem adequate.

For a two-coast (West and South) and a two-gear (or “fleet”, trawl and longline) fishery, the total predicted number of fish of age a caught in year y is given by:

$$C_{y,a} = \sum_{c=1}^2 \sum_{f=1}^2 C_{y,a}^{c,f}, \tag{A.2}$$

where:

$$C_{y,a}^{c,f} = N_{y,a} e^{-\frac{M_a}{2}} S_a^{\text{com},c,f} F_y^{c,f} \tag{A.3}$$

and:

- $F_y^{c,f}$ is the proportion of the resource above age a harvested in coast c , in year y by fleet f , and
- $S_a^{\text{com},c,f}$ is the commercial fishing selectivity at age a for fleet f and coast c .

The mass-at-age is given by the combination of a von Bertalanffy growth equation $\ell(a)$ defined by constants ℓ_∞ , κ and t_0 and a relationship relating length to mass. Note that ℓ refers to standard length.

$$\ell(a) = \ell_\infty [1 - e^{-\kappa(a-t_0)}] \tag{A.4}$$

$$w_a = c[\ell(a)]^d \tag{A.5}$$

where:

- w_a is the mass of a fish at age a , and
- c, d are the mass-length relationship parameters.

The coast and fleet-specific total catch by mass in year y is given by:

$$C_y^{c,f} = \sum_{a=0}^m w_{a+0.5} C_{y,a}^{c,f} = \sum_{a=0}^m w_{a+0.5} S_a^{com,c,f} F_y^{c,f} N_{y,a} e^{-\frac{M_a}{2}} \quad (A.6)$$

which can be re-written as:

$$F_y^{c,f} = \frac{C_y^{c,f}}{\sum_{a=0}^m w_{a+0.5} S_{y,a}^{com,c,f} N_{y,a} e^{-\frac{M_a}{2}}} = \frac{C_y^{c,f}}{B_{y,mid}^{exp,c,f}}, \quad (A.7)$$

where

$B_{y,mid}^{exp,c,f}$ is the mid-year coast and fleet-specific exploitable component of the biomass, and

$w_{a+0.5}$ is the mid-year mass of fish of age a assumed to be the same for each coast.

FISHING SELECTIVITY

When no catch-at-length data is used in the model fitting procedure, the coast and fleet-specific commercial fishing selectivity, $S_a^{c,f}$, is assumed to be of the same form as in Punt and Japp (1994). In this instance, the fishing selectivities are assumed to be the same for the West and the South coasts. For the longline this is given by:

$$S_a^{com,c,L} = \left\{ 1 + \exp \left[- (a - a_{50}^L) / \delta^L \right] \right\}^{-1} \quad (A.8)$$

and for the trawl fishery by:

$$S_a^{com,c,T} = \begin{cases} \left\{ 1 + \exp \left[- (a - a_{50}^T) / \delta^T \right] \right\}^{-1} & \text{if } a \leq a_{50}^T \\ \frac{\exp \left[-\gamma (a - a_{50}^T) \right]}{\left\{ 1 + \exp \left[- (a - a_{50}^T) / \delta^T \right] \right\}^{-1}} & \text{if } a > a_{50}^T \end{cases} \quad (A.9)$$

where:

$a_{50}^{L/T}$ is the age-at-50% selectivity for the longline/trawl fishery,

$\delta^{L/T}$ is the parameter which determines the width of the age-specific selectivity function for the longline/trawl fishery, and

γ is (approximately) the negative of the slope of the selectivity function for large ages.

For the model that includes both commercial and survey catch-at-length data, the coast and

fleet-specific commercial selectivity, $S_a^{com,c,f}$, and the coast and survey-specific selectivity, $S_a^{surv,c,s}$, are assumed to be described by a logistic curve, modified by a decreasing selectivity for fish older than age $a_{ch}^{surv/com,c}$. The commercial selectivity is given by:

$$S_a^{com,c,f} = \begin{cases} \left[1 + e^{-\left(a - a_{50}^{com,c,f}\right) / \delta^{com,c,f}} \right]^{-1} & \text{for } a \leq a_{ch}^{com,c} \\ \left[1 + e^{-\left(a - a_{50}^{com,c,f}\right) / \delta^{com,c,f}} \right]^{-1} e^{-\omega^{com,c,f} \left(a - a_{ch}^{com,c}\right)} & \text{for } a > a_{ch}^{com,c} \end{cases} \quad (\text{A.10})$$

where:

- $a_{50}^{com,c,f}$ is the age-at-50% selectivity (in years) for the commercial fishery f and coast c ,
- $\delta^{com,c,f}$ relates to the steepness of the ascending section of the commercial selectivity curve (in years⁻¹) for fishery f and coast c , and
- $\omega^{com,c,f}$ specifies the steepness of the descending section of the commercial selectivity curve for fish older than age $a_{ch}^{com,c}$ for fishery f and coast c (for all the results reported in this paper, $a_{ch}^{com,c}$ for the commercial selectivities is fixed at 2 yrs for the South coast and at 7.968 for the West coast).

The survey selectivity is of the same form as the commercial selectivity given by equation (A.10), however the age at which the selectivity function decreases for older fish is fixed at $a_{ch}^{surv,WC} = 7.968$ and $a_{ch}^{surv,SC} = 5$ for the West and South coasts respectively. These values as well as the value for $a_{ch}^{com,c}$ were chosen from the best attempt at estimating them. However, convergence problems were encountered and therefore the values were fixed in further analyses.

STOCK-RECRUITMENT RELATIONSHIP

The spawning biomass in year y is given by:

$$B_y^{sp} = \sum_{a=1}^m w_a f_a N_{y,a} = \sum_{a=a_m}^m w_a N_{y,a} \quad (\text{A.11})$$

where:

- f_a is the proportion of fish of age a that are mature (assumed to be knife-edge at age a_m).

The number of recruits at the start of year y is assumed to relate to the spawning biomass at the start of year y , B_y^{sp} , by a Beverton-Holt stock-recruitment relationship (assuming deterministic recruitment):

$$R(B_y^{sp}) = \frac{B_y^{sp}}{\alpha + \beta B_y^{sp}}. \quad (\text{A.12})$$

The values of the parameters α and β can be calculated given the unexploited equilibrium (pristine) spawning biomass K^{sp} and the steepness of the curve h , using equations (A.13)–(A.14) below. If the pristine recruitment is $R_0 = R(K^{sp})$, then steepness is the recruitment (as a fraction of R_0) that results when spawning biomass is 20% of its pristine level, i.e.:

$$hR_0 = R(0.2K^{sp}) \quad (\text{A.13})$$

from which it can be shown that:

$$h = \frac{0.2(\alpha + \beta K^{sp})}{\alpha + \beta 0.2K^{sp}} \quad (\text{A.14})$$

and hence:

$$\begin{aligned} \alpha &= K^{sp}(1-h)/(4hR_0) \\ \beta &= (5h-1)/(4hR_0) \end{aligned} \quad (\text{A.15})$$

The model estimate of the mid-year exploitable component of the biomass for each fleet is given by:

$$B_{y,mid}^{exp,c,f} = \sum_{a=0}^m w_{a+0.5} S_a^{com,c,f} N_{y,a} e^{-M/2}, \quad (\text{A.16})$$

where:

$w_{a+0.5}$ is the mid-year weight of kingklip of age a .

The model estimate of the start of the year exploitable component of the biomass for each coast and each fleet is given by:

$$B_{y,st}^{exp,c,f} = \sum_{a=0}^m w_a S_a^{com,c,f} N_{y,a} \quad (\text{A.17})$$

The model estimate of the survey biomass at the start of the year for each coast is given by:

$$B_{y,st}^{surv,c} = \sum_{a=0}^m w_a S_a^{surv,c,s} N_{y,a} \quad (\text{A.18})$$

where:

$S_a^{surv,c,s}$ is the survey selectivity at age a for survey s and coast c ,

and the model estimate of the survey biomass at mid-year is given by:

$$B_{y,mid}^{surv,c} = \sum_{a=0}^m w_{a+0.5} S_a^{surv,c,s} N_{y,a} e^{-M/2}. \quad (\text{A.19})$$

PAST STOCK TRAJECTORY AND FUTURE PROJECTIONS

Given a value for the pre-exploitation equilibrium spawning biomass (K^{sp}) of kingklip, and the assumption that the initial age structure is at pre-exploitation equilibrium, it follows that:

$$K^{sp} = R_0 \left(\sum_{a=1}^{m-1} w_a f_a e^{-Ma} + \frac{w_m f_m e^{-Mm}}{1 - e^{-M}} \right) \quad (\text{A.20})$$

which can be solved for R_0 .

The resource is assumed at its unexploited equilibrium when catches start in 1932 for the West and South coasts. The initial numbers at each age a for the trajectory calculations, corresponding to the deterministic equilibrium, are given by:

$$N_{1932,a} = R_0 n_a, \quad (\text{A.21})$$

where:

$$n_a = \begin{cases} 1 & \text{if } a = 0 \\ \exp\left(-\sum_{a=0}^{a-1} M_a\right) & \text{if } 1 \leq a < m \\ \exp\left(-\sum_{a=0}^{m-1} M_a\right) / (1 - \exp(-M_m)) & \text{if } a = m \end{cases} \quad (\text{A.22})$$

Numbers-at-age for subsequent years are then computed by means of equations (A1.1)-(A1.3) and (A1.6)-(A1.11) under the series of annual catches given.

THE LIKELIHOOD FUNCTION

The age-structured production model (ASPM) is fitted to the coast and fleet-specific CPUE data and to the coast and season-specific (i.e. summer or winter) survey abundance series.

CPUE abundance data

The likelihood is calculated assuming that the observed abundance indices are lognormally distributed about their expected values:

$$I_y^{c,i} = \hat{I}_y^{c,i} e^{\varepsilon_y^{c,i}} \quad \text{or} \quad \varepsilon_y^{c,i} = \ln(I_y^{c,i}) - \ln(\hat{I}_y^{c,i}) \quad (\text{A.23})$$

where:

$I_y^{c,i}$ is the abundance index of type i for year y and coast c , where for example, $i = CPUE_{trawl}$, when dealing with the CPUE index for the trawl fishery,

$\hat{I}_y^{c,i} = \hat{q}_i^c \hat{B}_{y,t}^{\text{exp},c,f}$ is the corresponding model estimated value, where $B_{y,t}^{\text{exp},c,f}$ is the model value for exploitable resource biomass for coast c ($c = WC$ for the West coast or $c = SC$ for the South coast), for fishery f ($f = T$ for trawl or $f = L$ for longline fishery) and time of the year t ($t = mid$ for mid-year or $t = st$ for the start of year),

\hat{q}_i^c is a constant of proportionality for abundance index i and coast c , and

$\varepsilon_y^{c,i}$ is normally distributed with mean zero and standard deviation $\sigma_y^{c,i}$.

The contribution to the negative of the log-likelihood function (after removal of constants) by the trawl and longline CPUE abundance data is given by:

$$-\ln L_{CPUE} = \sum_c \sum_i \sum_y \left[\ln \sigma_y^{c,i} + \left(\varepsilon_y^{c,i} \right)^2 / 2 \left(\sigma_y^{c,i} \right)^2 \right], \quad (\text{A.24})$$

where the model estimate of exploitable resource biomass is given by equation (A.16) and homoscedasticity of residuals is assumed, so that $\sigma_y^{c,i} = \sigma^{c,i}$ and this is estimated by its maximum likelihood value:

$$\hat{\sigma}^{c,i} = \sqrt{\frac{1}{n_{c,i}} \sum_y \left(\ln I_y^{c,i} - \ln \hat{q}_i^c \hat{B}_{y,mid}^{\text{exp},c,f} \right)^2},$$

where:

$n_{c,i}$ is the number of data points for the abundance series i and coast c ,

and the maximum likelihood estimate of q_i^c is given by:

$$\ln \hat{q}_i^c = \frac{1}{n_{c,i}} \sum_y \left\{ \ln I_y^{c,i} - \ln \hat{B}_{y,mid}^{\text{exp},c,T} \right\}.$$

Survey abundance data

Survey abundance indices are treated in a similar way to the CPUE series, where abundance index of type i now represent those from the research surveys, for example $i = \text{Survey}_{\text{summer}}$ when dealing with the abundance index from the summer survey, and so on. The West coast winter survey and the South coast autumn survey were taken to correspond to the middle of the year, while the West coast summer survey and the South coast spring survey were taken to correspond to the beginning of the year. Some survey abundance indices were obtained using either the old or the new gear on the *Africana*, and sometimes the *Africana* or the *Nansen* was used for the survey. To differentiate between gear type and between vessels, the term $\varepsilon_y^{c,i}$ in Equation (A.23) is modified to:

$$\varepsilon_y^{c,i} = \ln(I_y^{c,i}) - \ln(\hat{q}_i^c \hat{B}_{y,t}^{\text{exp},c,T})$$

where:

\hat{q}_i^c is a constant of proportionality for abundance index i , where

$$\hat{q}_i^c = \begin{cases} \hat{q}_i^{\text{old},c} & \text{if index } i \text{ in year } y \text{ is from } \textit{Africana} \text{ with old gear} \\ \hat{q}_i^{\text{old},c} \hat{q}_i^{\text{d},c} & \text{if index } i \text{ in year } y \text{ is from } \textit{Africana} \text{ with new gear} \\ \hat{q}_i^{\text{old},c} \hat{q}_i^{\text{d}2,c} & \text{if index } i \text{ in year } y \text{ is from } \textit{Nansen} \end{cases}$$

When survey catch-at-length data are not used in the model fitting procedure, the surveys are assumed to have the same selectivity as the trawl fishery, and the time of year when the survey took place is taken into account. Thus, for the summer survey, the corresponding model estimated

exploitable resource biomass is given by equation (A.17) and for the winter survey, $B_{y,t}^{\text{exp},c,f}$ is given by equation (A.16), with the corresponding selectivity function being that for the trawl fishery and the specific coast being considered.

When survey catch-at-length data are used in the model fitting procedure, survey specific selectivity functions can be estimated and thus the survey model estimated biomass $B_{y,t}^{\text{surv},c}$ given by equation (A.18) or equation (A.19) is used in this case.

The contribution of the survey abundance data to the negative of the log-likelihood function (after removal of constants) is then given by:

$$-\ln L_{\text{survey}} = \sum_c \sum_i \sum_y \left[\ln \sigma_y^{c,i} + (\varepsilon_y^{c,i})^2 / 2(\sigma_y^{c,i})^2 \right] \quad (\text{A.25})$$

where:

$\sigma_y^{c,i}$ are given by $\ln(1 + (CV_y^{c,i})^2)$, where the $CV_y^{c,i}$ are the coefficients of variation of the resource abundance estimate for coast c and index i for year y . These CVs are input and are given in Table 2.

The catchability coefficient $q_i^{\text{old},c}$ for the survey abundance index i and coast c is estimated by its maximum likelihood value and is given by:

$$\ln \hat{q}_i^{\text{old},c} = \frac{\sum_y X_y^{c,i} (1/(\sigma_y^{c,i})^2)}{\sum_y 1/(\sigma_y^{c,i})^2}, \quad (\text{A.26})$$

where:

$$X_y^{c,i} = \begin{cases} \ln I_y^{c,i} - \ln \hat{B}_{y,t}^{\text{exp},c,T} & \text{if index } i \text{ in year } y \text{ is from } \textit{Africana} \text{ with old gear} \\ \ln I_y^{c,i} - \ln(\hat{q}_i^{d,c} \hat{B}_{y,t}^{\text{exp},c,T}) & \text{if index } i \text{ in year } y \text{ is from } \textit{Africana} \text{ with new gear} \\ \ln I_y^{c,i} - \ln(\hat{q}_i^{d2,c} \hat{B}_{y,t}^{\text{exp},c,T}) & \text{if index } i \text{ in year } y \text{ is from } \textit{Nansen} \end{cases}$$

and $q_i^{d,c}$ and $q_i^{d2,c}$ are estimated in the non-linear search.

In the case when survey catch-at-length data are included in the model fit, $\hat{B}_{y,t}^{\text{exp},c,T}$ in equation (A.26) is replaced by $\hat{B}_{y,t}^{\text{surv},c}$.

EXTENSION TO INCORPORATE CATCH-AT-LENGTH INFORMATION

The model above provides estimates of the commercial catch-at-age ($C_{y,a}^{c,f}$) by number made by the each fleet in the fishery each year from equation (A.3). These in turn can be converted into proportions of the catch of age a :

$$p_{y,a}^{c,f} = C_{y,a}^{c,f} / \sum_a C_{y,a}^{c,f}. \quad (\text{A.27})$$

Using the von Bertalanffy growth equation (A.4), these proportions-at-age can be converted to proportions-at-length – here under the assumption that the distribution of length-at-age remains constant over time:

$$p_{y,\ell}^{c,f} = \sum_a p_{y,a}^{c,f} A_{a,\ell}^{c,f} \quad (\text{A.28})$$

where $A_{a,\ell}^{c,f}$ is the proportion of fish of age a that fall in length group ℓ for fleet f and coast c . Note that therefore:

$$\sum_{\ell} A_{a,\ell}^{c,f} = 1 \quad \text{for all ages } a. \quad (\text{A.29})$$

The A matrix has been calculated here under the assumption that length-at-age is normally distributed about a mean given by the von Bertalanffy equation, i.e.:

$$\ell(a) \sim N^* \left[\ell_{\infty} \left\{ 1 - e^{-\kappa(a-t_0)} \right\}; \theta(a)^2 \right] \quad (\text{A.30})$$

where:

N^* is a normal distribution truncated at ± 3 standard deviations (to avoid negative values), and

$\theta(a)$ is the standard deviation of length-at-age a , which is modelled here to be proportional to the expected length at age a , i.e.:

$$\theta(a) = \nu \ell_{\infty} \left\{ 1 - e^{-\kappa(a-t_0)} \right\} \quad (\text{A.31})$$

with ν a parameter estimated in the model fitting process.

Note that since the model of the population's dynamics is based upon a one-year time step, the value of ν and hence the $\theta(a)$'s estimated will reflect not only the real variability of length-at-age, but also the "spread" that arises from the fact that fish in the same annual cohort are not all spawned at exactly the same time, and that catching takes place throughout the year so that there are differences in the age (in terms of fractions of a year) of fish allocated to the same cohort.

Similarly, the proportion of fish of age a for the surveys in the beginning of year y and coast c are given by:

$$p_{y,a}^{surv,c,s} = C_{y,a}^{surv,c,s} / \sum_{a'} C_{y,a'}^{surv,c,s} = S_a^{surv,c,s} N_{y,a} / \sum_{a'} S_{a'}^{surv,c,s} N_{y,a'} \quad (\text{A.32})$$

and for those at mid year by:

$$p_{y,a}^{surv,c,s} = C_{y,a}^{surv,c,s} / \sum_{a'} C_{y,a'}^{surv,c,s} = S_a^{surv,c,s} N_{y,a} e^{-M_a/2} / \sum_{a'} S_{a'}^{surv,c,s} N_{y,a'} e^{-M_{a'}/2} \quad (\text{A.33})$$

These survey proportions-at-age can be converted to proportions-at-length using the same procedure as for the commercial proportions-at-age.

The observed commercial and survey proportions-at-length are assumed to follow a multinomial distribution, and so the contribution of the survey catch-at-length data to the negative log-likelihood function is given by:

$$-\ln L_{len}^{surv} = -N \sum_{c,s,y,\ell} \left\{ p_{y,\ell}^{surv,c,s} \ln \left(\hat{p}_{y,\ell}^{surv,c,s} \right) \right\} + K \quad (\text{A.34})$$

where:

$p_{y,\ell}^{surv,c,s}$ is the observed proportion by number of the catch for coast c in year y in length group ℓ for survey s ,

$\hat{p}_{y,\ell}^{surv,c,s}$ is the model predicted proportion by number of the catch for coast c in year y in length group ℓ for survey s ,

N is the effective sample size, assumed to be 25 in this application, and

K is a constant used to keep the value of $-\ln L$ to a manageable number of digits. It measures the value that would be attained if the data fit would be exactly the model's expectation, that is:

$$K = N \sum_{c,s,y,\ell} \left\{ p_{y,\ell}^{surv,c,s} \ln \left(p_{y,\ell}^{surv,c,s} \right) \right\}$$

and the contribution of the survey catch-at-length data to the negative log-likelihood function is given by:

$$-\ln L_{len}^{com} = -N \sum_{c,f,y,\ell} \left\{ p_{y,\ell}^{c,f} \ln \left(\hat{p}_{y,\ell}^{c,f} \right) \right\} + K \quad (A.35)$$

In the practical application of equations (A.34) and (A.35), length observations were grouped by 2 cm intervals, with minus- and plus-groups specified below 24 and above 86 cm for the West coast surveys, lengths below 24 and above 106 cm for the South coast surveys and lengths below 50 and above 106 cm for the West and South coast commercial fishery.