A Network-based Frequency Analysis of Inclusive Wealth to Track Sustainable Development in World Countries

Nasir Ahmad [a b], Sybil Derrible [a c], and Shunsuke Managi [d e]

[a] Complex and Sustainable Urban Networks (CSUN) Laboratory, 2095 Engineering Research Facility, 842 W. Taylor Street (M/C 246), University of Illinois at Chicago, Chicago, IL 60607-7023.

[b] sahmad38@uic.edu

[c] derrible@uic.edu

[d] managi.s@gmail.com

[e] Departments of Urban and Environmental Engineering, School of Engineering, Kyushu University

Abstract

Using human (HC), natural (NC), and produced (PC) capital from Inclusive Wealth as representatives of the triple bottom line of sustainability and utilizing elements of network science, we introduce a Network-based Frequency Analysis (NFA) method to track sustainable development in world countries from 1990 to 2014. The method compares every country with every other and links them when values are close. The country with the most links becomes the main trend, and the performance of every other country is assessed based on its 'orbital' distance from the main trend. Orbital speeds are then calculated to evaluate country-specific dynamic trends. Overall, we find an optimistic trend for HC only, indicating positive impacts of global initiatives aiming towards socio-economic development in developing countries like the Millennium Development Goals and 'Agenda 21'. However, we also find that the relative performance of most countries has not changed significantly in this period, regardless of their gradual development. Specifically, we measure a decrease in produced and natural capital for most countries, despite an increase in GDP, suggesting unsustainable development. Furthermore, we develop a technique to cluster countries and project the results to 2050, and we find a significant decrease in NC for nearly all countries, suggesting an alarming depletion of natural resources worldwide.

Keywords

sustainable development, inclusive wealth, network science.

1. Introduction

The world has changed dramatically since the end of the 20th century, and the pace of change shows no sign of slowing down. The global societal aspiration for development seems to systematically lead to the consumption of more resources, which not only puts additional pressure on the world's natural resources but it also jeopardizes the environment and plays a key role in altering the climate. According to several studies (Meadows et al., 1972; Rockström et al., 2009; Wackernagel et al., 2002), the depletion of resources, climate change, and the degradation of the environment bear clear signs of unsustainable development. Moreover, studies have also shown that standards of living can be on the rise even as stresses on the environment are increasing (DeFries, 2014; Johnson, 2000; Raudsepp-Hearne et al., 2010). Therefore, tracking progress (or lack of it) in sustainable development is critical (Bettencourt and Kaur, 2011; Clark and Dickson, 2003; Kates, 2001; Levin and Clark, 2010; Parris and Kates, 2003). The Gross Domestic Product (GDP), Human Development Index (HDI), and ecological footprint have long been used to track human development (Rees, 1992; UNDP, 2016; Wackernagel and Rees, 1996), but many have shown that these indices can fail to capture whether a development is sustainable or not (Dasgupta et al., 2015; Polasky et al., 2015). By contrast, the Inclusive Wealth (IW) index, also called Comprehensive Wealth or Genuine Wealth, was designed purposefully to track sustainable development (Arrow et al., 2004; UNU-IHDP and UNEP, 2014; World Bank, 2010). IW is defined as the sum of three capitals (defined in monetary terms) representing the triple bottom line of sustainability: human (i.e., society), produced (i.e., economy), and natural capital (i.e., environment); see supplementary information for details on what variables are included in the three capitals.

The main goal of this study is to capture and evaluate the main sustainable development trends and trajectories of world countries relative to every other using IW data from 1990 to 2014 (UNU-IHDP and UNEP, 2014; Urban Institute and UNEP, 2017). Traditionally, the arithmetic mean or median are used to capture the general trend, but both can be easily biased by the presence of extreme values in a dataset. For this study, we prefer to adopt a Network-based Frequency Analysis (NFA) approach (Derrible and Ahmad, 2015), which is not affected by extreme values and which is able to capture general trends in a more robust way. Moreover, NFA allows us to measure the evolution of an entity relative to the evolution of other entities. More specifically, the objectives of this study are to: a) create a method to estimate the main human, natural, and produced capital trends in the world using concepts from network science; b) assess the relative performance of every country from the general trends captured; c) cluster countries based on their performance in all the three capitals; and d) project current trends up to 2050 to depict the potential future performance of every country. For this work, we use data from the 2014 and 2017 Inclusive Wealth Report (UNU-IHDP and UNEP, 2014; Urban Institute and UNEP, 2017). We selected human, natural, and produced capital for all the countries for our analysis.

One of the main contributions of this work is the deployment of the NFA method, whose results can offer a significant contribution to current knowledge and complement traditional statistical approaches. Briefly, among 140 analyzed countries, 110 showed an increasing trend for human capital, whereas only 4 countries for natural capital and 6 countries for produced capital showed an increasing trend from 2000 to 2014. Moreover, based on the combined performance of human, natural, and produced capital, only 3 countries show an optimistic trend, whereas 18 countries show a decreasing trend, and the 119 remaining countries have not changed significantly from 2000 to 2014.

2. Methodology

With the profusion of data and availability of virtually limitless computing power, new algorithmic solutions have been developed and applied since the early 2000s (Ahmad et al., 2017, 2016). In this study, we adopt a Network-based Frequency Analysis (NFA) approach that is able to capture global trends in a dataset and measure the relative evolution of an entity (Ahmad and Derrible, 2015) . Although not rooted in the general field of Network Science (Newman, 2010), the method essentially compares to forming a network. From a Machine Learning perspective, the foundation of the methodology is closest to kernel density estimation (KDE) (Parzen, 1962; Rosenblatt, 1956), where the critical challenge is to select the optimum bandwidth. The NFA method works as follows. First, we convert each dataset to a network by comparing the performance of all countries with every other. Formally, we connect two countries together when their values are within a certain range, ζ, of each other. A network is analytically represented by an adjacency matrix, A_{ij}, where the cells take a value of 1 if nodes i and j are connected and 0 otherwise. For this study, adjacency matrices are defined as:

$$A_{ij} = \begin{cases} 1 \, if \; (x_i - \zeta) \leq x_j \leq (x_i + \zeta) \\ 0 \; otherwise \end{cases} \tag{1}$$

where, x_i and x_j represent the value (e.g., HC) of countries i and j.

To form the network, the selection of an optimal ζ is critical. To find the optimal ζ, first, we create a network using a ζ as 1% of the median of the data and we then gradually increase it. We rapidly observe the evolution of a giant cluster, which includes a significant portion of the nodes of that network. Moreover, we also observe that the giant cluster increases gradually with the increment of ζ, but it then stabilizes after a certain value of ζ. The value of ζ for which the size of the giant cluster remains stable is selected as the optimal ζ. Detailed information about network formation can be found in Derrible and Ahmad (2015) and is recalled in the supplementary information, including details on how the optimal value of ζ is determined. After forming the network, the country with the highest degree (i.e., number of connections) in each dataset becomes the most representative country, thus capturing the main trend. We refer to it as the Network-based (N) mode since it captures the statistical mode of a frequency distribution.

The N mode is then used as a benchmark to evaluate each country's performance relative to the general trend. For this, we use the concept of geodesic distance in network science (Newman, 2010) . In other words, we measure how far a node is from the N mode. We then refer to this distance as the "orbital position" of a country. Because this distance is related to the cutoff ζ_t calculated (for more details, see the methodology section in the supplementary information), we can calculate the orbital position as:

$$O_{i,t} = \frac{x_{i,t} - M_t}{\zeta_t} + C \tag{2}$$

where, $O_{i,t}$ is the orbital position for country i in year t, $x_{i,t}$ is the corresponding value for country i in year t, M_t is the N mode in year t, and ζ_t is the cutoff in year t. The orbital position of the N mode is set at 100 (i.e., C = 100). Therefore, orbital positions with values less than the N mode will be less than 100, and conversely, orbital positions with values greater than the N mode will be greater than 100. Moreover, the change in orbital positions from one year to the next is computed by taking the difference between orbital positions in two consecutive years and is defined as “orbital distance.” Equation 2 can be used to track the “orbital speed,” that is the speed at which a country’s orbital positions are changing.

$$O'_i = \frac{\sum_{2}^{T} O_{i,t} - O_{i,t-1}}{T_i} \qquad (3)$$

where, O'_i is the orbital speed for country i and T_i is the number of available data points within the timeline of 1 (i.e. 1990) to T (i.e. 2014) for country i.

The evolution of the orbital positions of all the countries is visualized by plotting them in polar format and designated as an “orbital diagram.” If a country went through significant changes from 1990 to 2014, considerable changes are observed in the orbital diagrams. Conversely, if a country did not go through significant changes from 1990 to 2014, no or few changes are observed in the orbital diagrams, regardless of their gradual increment in all the capitals (i.e., human, natural, and produced capital) during the analyzed period.

Moreover, as IW is the summation of HC, NC, and PC, a high capital value (e.g., HC) can compensate for the losses in other capital values (e.g., NC). Therefore, to further supplement IW, we can compute the combined performance of all three capitals. While constructing the networks, we systematically observe the presence of a giant cluster (i.e., sub-network containing most countries) for each dataset, in which all countries (i.e., nodes) are directly or indirectly connected to one another (for more details, see the methodology section in the supplementary information). Therefore, for each capital and each year, a country can be either part of the giant cluster or not, thus yielding two distinct groups for each dataset. As we have three datasets (i.e., human, natural and produced) for each year, we have $2^3 = 8$ possible combinations per year, and therefore we can group all countries into 8 different clusters. To find the combination for each country, we assign a value of ‘2’ if a country is in the giant cluster and a value of ‘1’ if it is not.

Subsequently, using orbital speeds and assuming that on average future trends will follow current ones, we can project the potential future orbital position of a country:

$$O_{i,t} = O_{i,t_0} + O'_i \times (t - t_0) \qquad (4)$$

where $O_{i,t}$ is the orbital position for the country i in the year t and O'_i is the orbital speed for the country i. The year 2014 is used as t_0. Then, we find the maximum orbital position of the country within the giant cluster for HC, NC, and PC from 1990 to 2014 and use those values as cutoffs to decide whether a country will be in the giant cluster or not in the future.

3. Results and Discussion

Figure 1 presents three ways of representing the main trends in Human Capital (HC), Natural Capital (NC), and Produced Capital (PC) of all the countries in the world. The two standard representations are the arithmetic mean and median. To these two, we add the N mode that captures the main trend from the Network-based Frequency Analysis (NFA); essentially, the N mode arguably better represents the general trend since it is not skewed by asymmetrical distributions unlike the mean and the median.

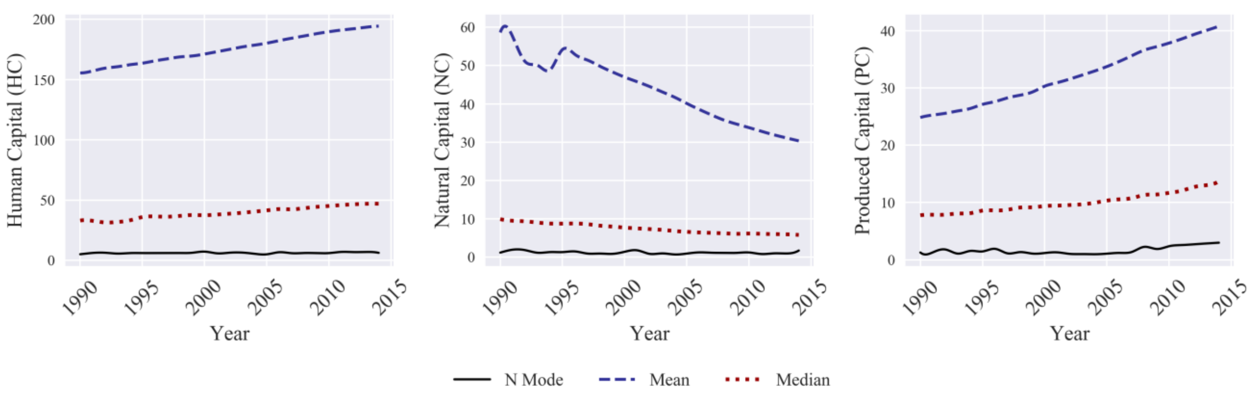

Figure 1. Evolution of the Network-based (N) mode, mean, and median for per-capita human capital, natural capital, and produced capital from 1990 to 2014. (All indicators are reported in constant 2005 U.S. dollars in thousands.)

An increasing global trend is detected in both the mean and the median values for HC and PC, while NC is decreasing (Figure 1). In contrast, the N mode stayed relatively stable for HC and NC, and it has been showing signs of increase for PC since 2009. Moreover, we find on average that 78% of HC, 83% of NC, and 72% of PC are below the corresponding mean, suggesting a large right-tail in the frequency distribution of the three capitals. Furthermore, we observe that among the 140 analyzed countries, 139 countries for HC, 13 countries for NC, and

120 countries for PC have higher capitals in the year 2014 than in the year 1990. Overall, when summing the three variables to compute the IW, only 90 countries have higher values in 2014 compared to 1990, mainly because 127 countries see a significant decrease in NC.

Figure 2 shows the orbital diagrams and orbital speeds for the United States (USA), France, Brazil, and China for human, natural, and produced capital from 1990 to 2014. Orbital diagrams and orbital speeds for all countries are provided in the supplementary section.

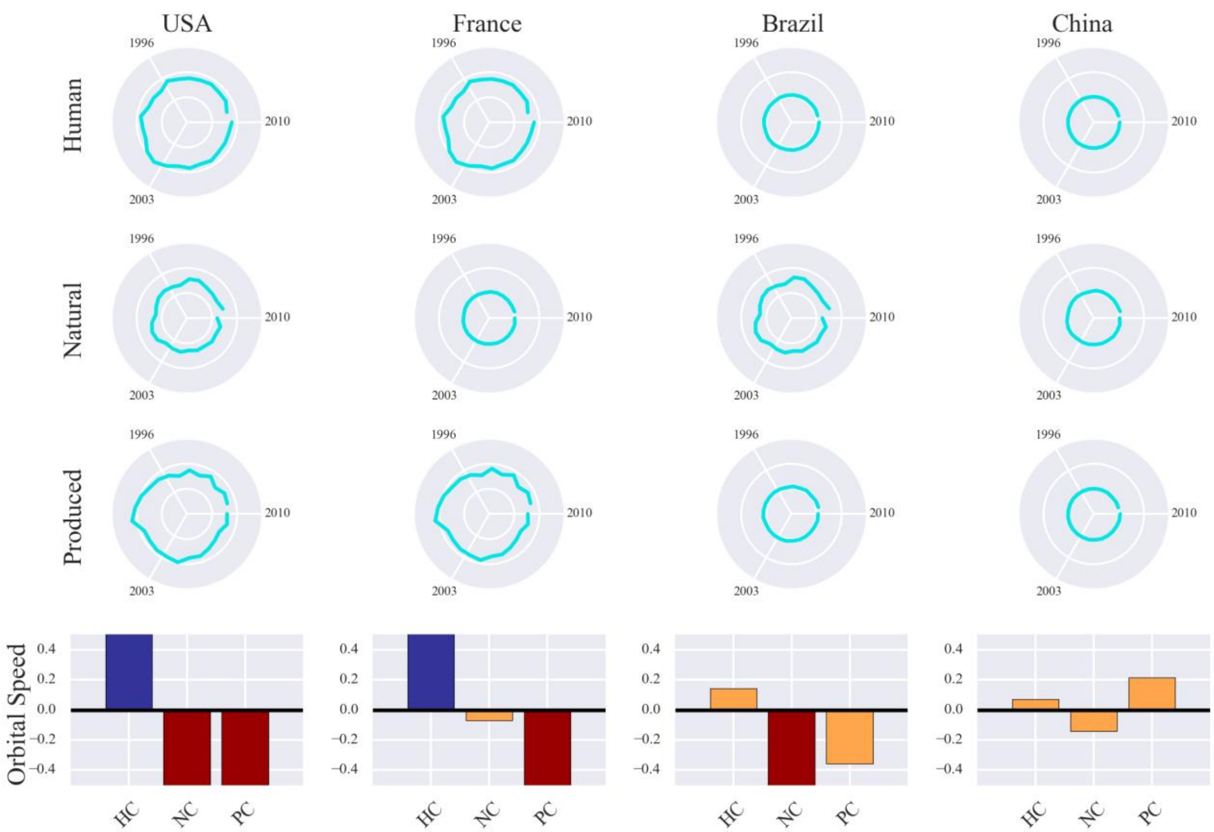

Figure 2. Orbital Diagrams and orbital Speeds of the United States (USA), France, Brazil, and China. The top three rows show the orbital diagrams, where the inner most circles denote the orbital position of the mode at 100 and the outer most circles denote the orbital position of 250. The bottom row shows the orbital speeds since 2000. Orbital speeds between -0.5 and 0.5 are marked with beige, whereas orbital speeds less than 0.5 are marked with red and orbital speeds greater than 0.5 are marked with purple.

Overall, we find mixed results when analyzing each country's performance relative to the N mode. For instance, from Figure 2, we can see that the orbital positions of the USA for all capitals are consistently located away from the N mode (i.e., the inner circle). Additionally, except for HC, we observe a decreasing trend for the two other capitals, indicating that the USA is depleting its natural and produced capitals. Only a few countries have a scenario similar to the USA. In contrast, the orbital positions of all capitals for most countries (i.e., about 70%) are close to the N mode, akin to China that barely changed its orbital positions during the analyzed period. Indeed, despite China's increase in HC and PC, per capita values remain nearly constant and close to the N mode from 1990 to 2014. France's orbital positions are close to the N mode only for NC, with a slightly negative orbital speed; the orbital positions of HC and PC are away from the N mode with a negative orbital speed for PC and a positive orbital speed for HC. Therefore, similar to the USA, France is also depleting its natural and produced capitals, but the depletion rate for the natural capital is significantly lower (i.e., 0.07 geodesic distance per year) than that of the USA (i.e., 0.71 geodesic distance per year). Conversely, in the case of Brazil, except for NC, the orbital positions of the other two capitals remain close to N mode. We also observe a decreasing trend for NC and PC, whereas HC remains marginally positive from 1990 to 2014 for Brazil.

Orbital speed captures whether a country's orbital positions are increasing, decreasing, or remaining constant. We compute the orbital speeds of all countries from 1990 to 2014, and from years 2000 to 2014 to capture more recent dynamics. Figure 3 shows the frequency analysis of all countries based on whether they are increasing, decreasing, or remaining constant.

From Figure 3, we can see that for HC, few countries have decreasing orbital speeds. In

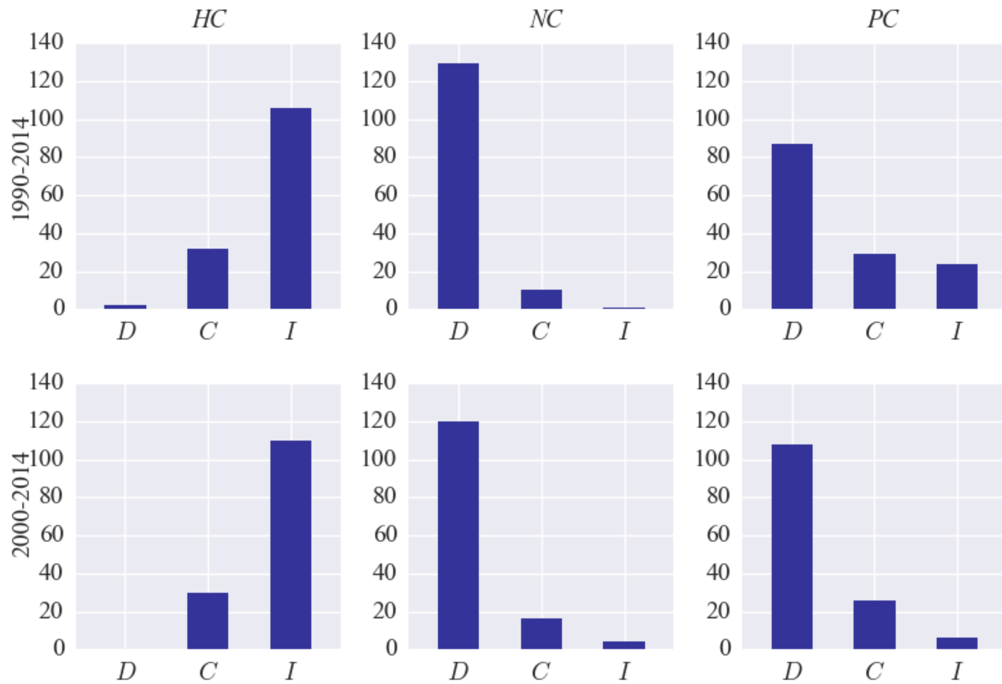

Figure 3 Number of countries with increasing, decreasing or constant orbital speed. The top row shows the orbital speed since 1990 and the bottom row shows the orbital speed since 2000 for human (first column), natural (second column) and produced (third column) capital. D, C, and I stand for Decrease, Constant, and Increase.

fact, orbital speeds largely increase for most countries. Considering the N mode has stayed nearly constant, this indicates a global gain of human capital, which is desirable. On the contrary, the scenario for the natural and produced capitals are reversed as 129 and 87 countries since 1990, and 120 and 108 countries since 2000 have decreasing orbital speeds for NC and PC. Moreover, from Figure 1, we found decreasing trends in both the mean and the median values for NC. Looking at trends in orbital speeds, we find decreasing trends for both NC and PC for most countries. This observation notably suggests that despite a global increase in PC, 108 countries failed to keep pace with the global trend since 2000. In contrast, both the traditional and NFA approaches corroborate the progressive trend in HC, likely thanks to major global initiatives that aim to significantly encourage socio-economic development in developing countries like 'Agenda 21' (United Nations Division for Sustainable Development, 1992) and the Millennium Development Goals (UNDP, 2000).

Moreover, for each capital, the emergence of a giant cluster provides a means to separate countries in two groups: those within and those outside the giant cluster. Since we have three different capitals (i.e., human, natural, and produced) with two distinct options (i.e., within the giant cluster or not), we can essentially cluster the countries into $2^3 = 8$ groups. Figure 4 shows the clusters of all countries for the years 1990, 2000, and 2014. For example, in 2014, France was outside the giant cluster for HC, inside the giant cluster for NC, and outside the giant cluster for PC, it is, therefore, part of the cluster with a combination of '121' and group of 6, as we assign a value of '2' if a country is in the giant cluster and a value of '1' if it is not. Among the eight possible clusters (i.e. two distinct groups for HC, NC, and PC, thus 2^3=8), four of them are much more common: 1) countries with high HC, NC, and PC (e.g., the USA); 2) countries with high HC and PC but low NC (e.g., France); 3) countries with low HC and PC but high NC (e.g., Brazil); and 4) countries with low HC, NC, and PC (e.g., China). Overall, about a quarter of the world countries maintained higher values (i.e., values outside the giant cluster) for at least two capitals from 1990 to 2014. These countries mostly include developed countries including the USA, Australia, and European countries. Among the remaining 75% of world countries, we find that most countries' capitals for all three capitals remain within the giant cluster. Moreover, we find that the number of countries per group did not change significantly from 1990 to 2014. Table 1 shows the number of countries per group for the years 1990, 2000, and 2014. A heatmap tracking the cluster of every country every year is available in the supplementary information (Figure S6 and S7).

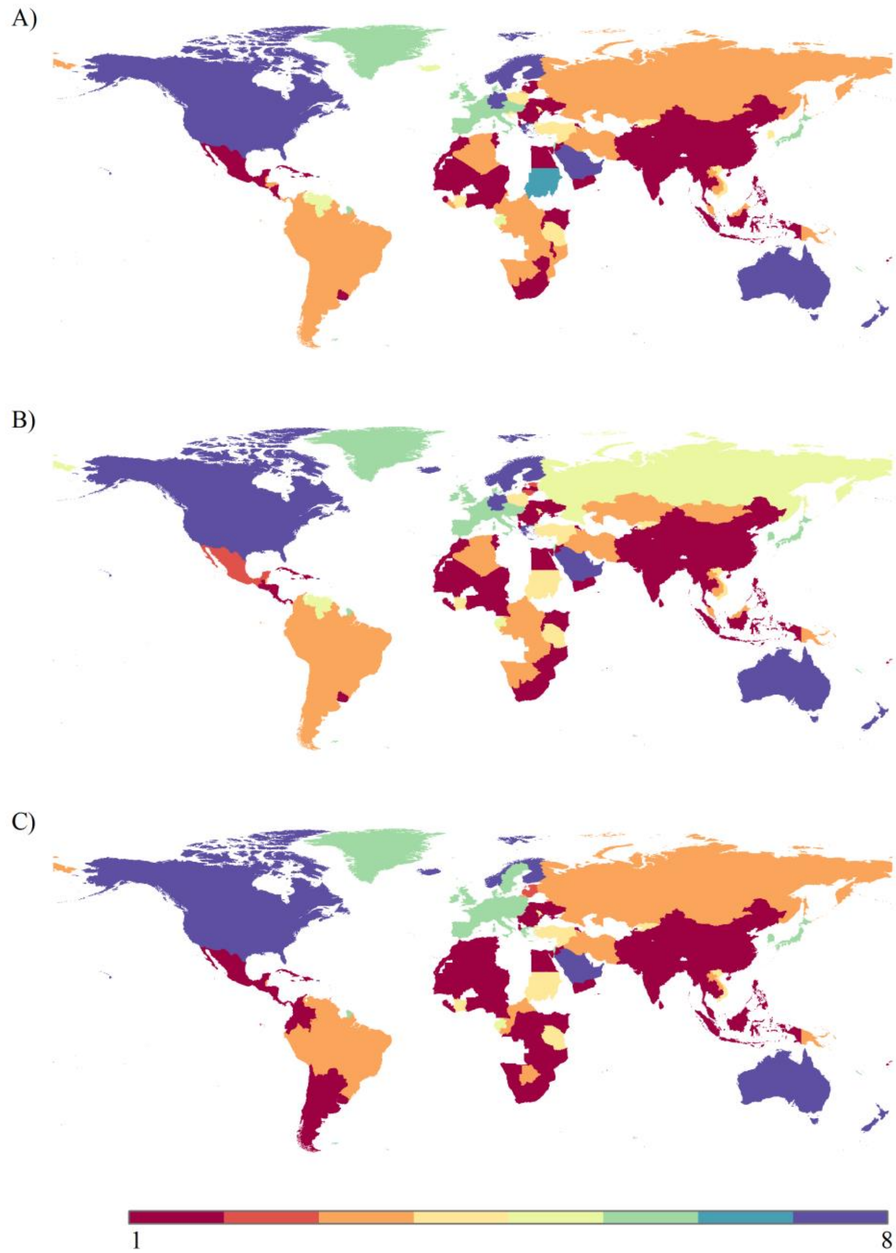

Figure 4 Combinations of Human, natural and produced capital for all the countries for the years: A) 1990, B) 2000, and C) 2014. Combinations 222, 221, 212, 122, 211, 121, 112, and 111 denote groups 1, 2, 3, 4, 5, 6, 7 and 8 respectively (see color bar and table S1 in supplementary section for details about the combinations).

Table 1 Number of Countries per Group

Group Description	**Group**	**1990**	**2000**	**2014**
High human, natural and produced capital	8 (111)	11	14	11
High human and natural but scarce produced capital	7 (112)	1	0	0
High human and produced but scarce natural capital	6 (121)	20	22	27
Scarce human but high natural and produced capital	5 (211)	6	4	1
High human but scarce natural and produced capital	4 (122)	10	9	8
Scarce human and produced capital but high natural capital	3 (212)	31	25	16
Scarce human and natural capital but high produced capital	2 (221)	2	6	6
Scarce human, natural and produced capital	1 (222)	59	60	71

Then, using equation 4, we project orbital positions for all the countries for the years 2020, 2030, 2040, and 2050. Subsequently, we use the maximum orbital positions of the country within the giant cluster for HC, NC, and PC from 1990 to 2014 (i.e., 119 for NC and 118 for both HC and PC) to decide whether a country will be in the giant cluster in 2020, 2030, 2040, and 2050. For example, the orbital position for Russia's NC was 158 in the year 2014, but it will decrease to 117 in the year 2050, which is below the cutoff of NC (i.e., 118). Thus, although Russia was outside the giant cluster in 2014, it is projected to become part of the giant cluster by the year 2050 if it follows the current decreasing trend for NC. Moreover, we also find the projected cluster for each country in the future. As we found an increasing trend for HC and decreasing trends for NC and PC for most countries, most of them are projected to move or try to move away from the giant cluster for HC while simultaneously trying to come closer or enter the giant cluster for NC and PC by the year 2050. As a consequence, by the year 2050, we expect two predominant cluster combinations: 1) 77 countries within the giant cluster for HC, NC, and PC; and 2) 59 countries outside the giant cluster for HC but within the giant cluster for NC and PC. Figure 5 shows the projected clusters for all the countries for the years 2020, 2030, 2040, and 2050. Moreover, we also find that countries with low HC and PC but with high NC (e.g., Brazil, Iran, Congo) will be affected more significantly and fail to keep their high NC, thus quickly joining the cluster with low HC, NC, and PC.

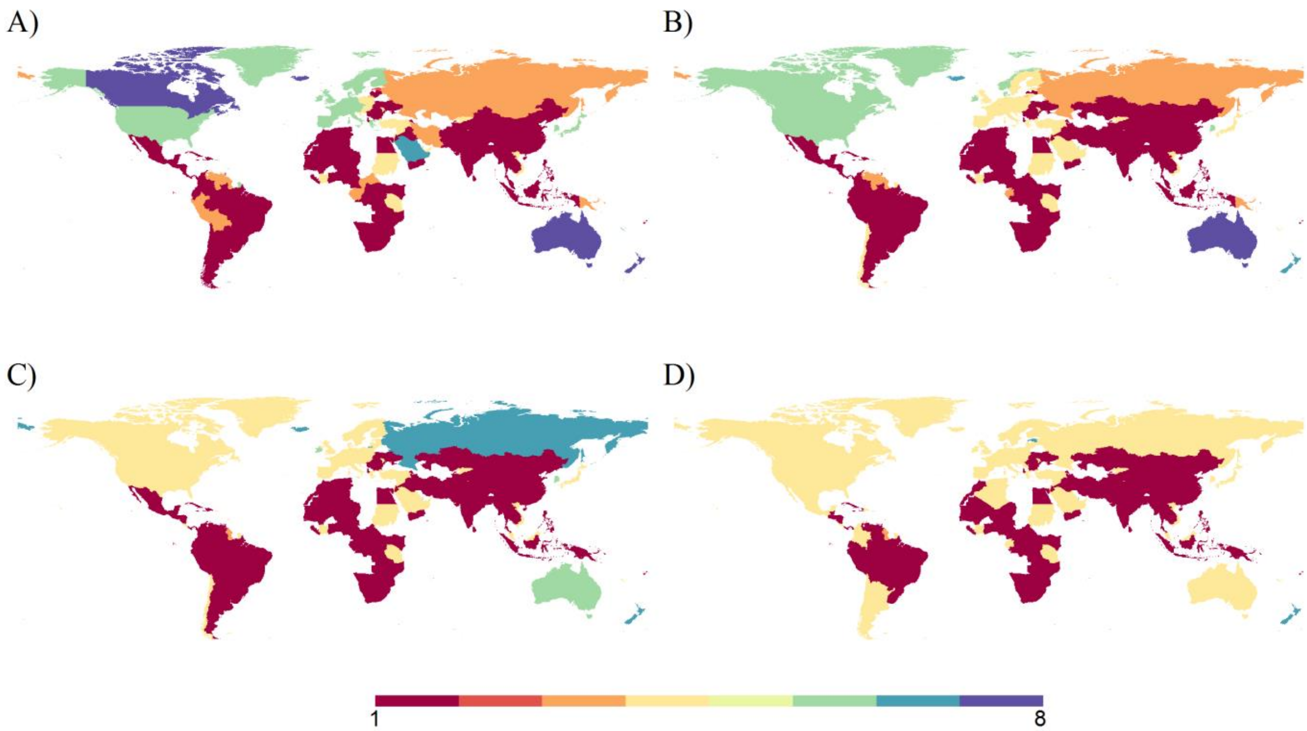

Figure 5 Combinations of human, natural and produced capital for all the countries for the years: A) 2020, B) 2030, C) 2040, and D) 2050. Combinations 222, 221, 212, 122, 211, 121, 112, and 111 denote groups 1, 2, 3, 4, 5, 6, 7 and 8 respectively (see color bar and table S1 in supplementary section for details about the combinations).

4. Conclusion

As the world has entered the Anthropocene (Crutzen, 2002; Steffen et al., 2007), tracking and assessing sustainable development in every country is paramount, if we, as a society, aspire to ensure a safe operating space for humanity (Rockström et al., 2009). For this research, we studied global trends in the evolution of human, natural, and produced capital by adopting a network science approach, which has successfully been applied in other fields (Ahmad and Derrible, 2015; Newman, 2010; Xu et al., 2011). This technique enables us to evaluate the evolution of individual country compared to the main trend by measuring the orbital positions; conventional techniques generally assess evolution in absolute terms only (e.g., GDP growth). To visualize these orbital positions, we generated “orbital diagrams” and we calculated “orbital speeds” to evaluate the evolving patterns (i.e., increasing, decreasing, or stable) of every country. Moreover, based on whether a country is connected to the main trend or not for all three capitals, we classify all countries per year and track their evolution from 1990 to 2014. The method created here can also be applied in a variety of cases; for example to track the performance of individual cities over time. We find an encouraging optimistic trend for human capital, likely thanks to major global initiatives that aim to significantly encourage socio-economic development in developing countries like ‘Agenda 21’ (United Nations Division for Sustainable Development, 1992) and the Millennium Development Goals (UNDP, 2000), which also resulted in a positive trend for IW for 60% of the world countries. Meanwhile, the trends for natural and produced capital are rapidly decreasing and require immediate attention for a more robust sustainable development. Moreover, among 140 analyzed countries, 119 countries remain in the same group in 2014 as they were in 2000, although 90% of these countries’ GDP increased in the same period (Dasgupta et al., 2015; UNU-IHDP and UNEP, 2014), suggesting that the relative performance of most of the countries did not change much in this period, regardless of their gradual development, which is alarming as a decrease in produced and natural capitals along with an increase in GDP also suggests that most of the countries are consuming resources to increase their GDP, thus exhibiting features of unsustainable development. This result is alarming as decrease in produced and natural capital along with increase in GDP also suggests that most of the countries are consuming resources to increase their GDP, thus exhibiting features of unsustainable development. Moreover, by projecting the current trends in the future, we find that the number of countries with low human, natural, and produced capital will rise from 71 in 2014 to 77, and countries with higher values (i.e., values outside the giant cluster) for at least two capitals will decrease from 39 to 3 by 2050, if the current trends hold true. Furthermore, countries with low human and produced capitals but high natural capital (e.g., Brazil, Congo, Iran) will fail to sustain their high natural capital. Greater and more consistent effort should therefore be expanded to sustain natural capital as well as to increase human and produced capital in the years to come rather than focusing solely on the GDP growth.

5. Funding Sources

This research was supported, in part, by the National Science Foundation (NSF) Award CCF-1331800, the NSF CAREER Award 1551731, the NSF CPS Award 1646395, and by the Department of Civil and Materials Engineering at the University of Illinois at Chicago.

6. References

Ahmad, N., Derrible, S., 2015. Evolution of Public Supply Water Withdrawal in the USA: A Network Approach: Water Withdrawal in the USA: A Network Approach. J. Ind. Ecol. 19, 321–330. https://doi.org/10.1111/jiec.12266

Ahmad, N., Derrible, S., Cabezas, H., 2017. Using Fisher information to assess stability in the performance of public transportation systems. R. Soc. Open Sci. 4, 160920. https://doi.org/10.1098/rsos.160920

Ahmad, N., Derrible, S., Eason, T., Cabezas, H., 2016. Using Fisher information to track stability in multivariate systems. R. Soc. Open Sci. 3, 160582. https://doi.org/10.1098/rsos.160582

Arrow, K., Dasgupta, P., Goulder, L., Daily, G., Ehrlich, P., Heal, G., Levin, S., Mäler, K.-G., Schneider, S., Starrett, D., others, 2004. Are we consuming too much? J. Econ. Perspect. 18, 147–172.

Bettencourt, L.M.A., Kaur, J., 2011. Evolution and structure of sustainability science. Proc. Natl. Acad. Sci. 108, 19540–19545. https://doi.org/10.1073/pnas.1102712108

Clark, W.C., Dickson, N.M., 2003. Sustainability science: The emerging research program. Proc. Natl. Acad. Sci. 100, 8059–8061. https://doi.org/10.1073/pnas.1231333100

Crutzen, P.J., 2002. Geology of mankind. Nature 415, 23–23. https://doi.org/10.1038/415023a

Dasgupta, P., Duraiappah, A., Managi, S., Barbier, E., Collins, R., Fraumeni, B., Gundimeda, H., Liu, G., Mumford, K., 2015. How to measure sustainable progress. Science 350, 748–748.

DeFries, R., 2014. The Big Ratchet: How Humanity Thrives in the Face of Natural Crisis. Basic Books.

Derrible, S., Ahmad, N., 2015. Network-Based and Binless Frequency Analyses. PLOS ONE 10, e0142108. https://doi.org/10.1371/journal.pone.0142108

Johnson, D.G., 2000. Population, Food, and Knowledge. Am. Econ. Rev. 90, 1–14. https://doi.org/10.1257/aer.90.1.1

Kates, R.W., 2001. ENVIRONMENT AND DEVELOPMENT: Sustainability Science. Science 292, 641–642. https://doi.org/10.1126/science.1059386

Levin, S., Clark, W., 2010. Toward a science of sustainability: report from toward a science of sustainability conference. Princet. Environ. Inst. Princet.

Meadows, D.H., Meadows, D.L., Randers, J., Behrens, W.W., 1972. The limits to growth. N. Y. 102.

Newman, M., 2010. Networks: An Introduction, 1 edition. ed. Oxford University Press, USA.

Parris, T.M., Kates, R.W., 2003. Characterizing and measuring sustainable development. Annu. Rev. Environ. Resour. 28, 559–586. https://doi.org/10.1146/annurev.energy.28.050302.105551

Parzen, E., 1962. On Estimation of a Probability Density Function and Mode. Ann. Math. Stat. 33, 1065–1076. https://doi.org/10.1214/aoms/1177704472

Polasky, S., Bryant, B., Hawthorne, P., Johnson, J., Keeler, B., Pennington, D., 2015. Inclusive wealth as a metric of sustainable development. Annu. Rev. Environ. Resour. 40, 445–466.

Raudsepp-Hearne, C., Peterson, G.D., Tengö, M., Bennett, E.M., Holland, T., Benessaiah, K., MacDonald, G.K., Pfeifer, L., 2010. Untangling the Environmentalist's Paradox: Why Is

Human Well-being Increasing as Ecosystem Services Degrade? BioScience 60, 576–589. https://doi.org/10.1525/bio.2010.60.8.4
Rees, W.E., 1992. Ecological footprints and appropriated carrying capacity: what urban economics leaves out. Environ. Urban. 4, 121–130.
Rockström, J., Steffen, W., Noone, K., Persson, Å., Chapin, F.S., Lambin, E.F., Lenton, T.M., Scheffer, M., Folke, C., Schellnhuber, H.J., Nykvist, B., de Wit, C.A., Hughes, T., van der Leeuw, S., Rodhe, H., Sörlin, S., Snyder, P.K., Costanza, R., Svedin, U., Falkenmark, M., Karlberg, L., Corell, R.W., Fabry, V.J., Hansen, J., Walker, B., Liverman, D., Richardson, K., Crutzen, P., Foley, J.A., 2009. A safe operating space for humanity. Nature 461, 472–475. https://doi.org/10.1038/461472a
Rosenblatt, M., 1956. Remarks on Some Nonparametric Estimates of a Density Function. Ann. Math. Stat. 27, 832–837. https://doi.org/10.1214/aoms/1177728190
Steffen, W., Crutzen, P.J., McNeill, J.R., 2007. The Anthropocene: Are Humans Now Overwhelming the Great Forces of Nature. AMBIO J. Hum. Environ. 36, 614–621. https://doi.org/10.1579/0044-7447(2007)36[614:TAAHNO]2.0.CO;2
UNDP, 2016. | Human Development Reports [WWW Document]. URL http://hdr.undp.org/en/2016-report (accessed 2.20.18).
UNDP, 2000. Millennium Development Goals [WWW Document]. URL http://www.undp.org/content/undp/en/home/sdgoverview/mdg_goals.html (accessed 1.26.16).
United Nations Division for Sustainable Development, 1992. Agenda 21 [WWW Document]. URL (accessed 10.5.16).
UNU-IHDP, UNEP, 2014. Inclusive Wealth Report 2014. Cambridge University Press.
Urban Institute, UNEP, 2017. Inclusive Wealth Report 2017, Working Draft.
Wackernagel, M., Rees, W., 1996. Our Ecological Footprint. Green Teach. 45, 5–14.
Wackernagel, M., Schulz, N.B., Deumling, D., Linares, A.C., Jenkins, M., Kapos, V., Monfreda, C., Loh, J., Myers, N., Norgaard, R., Randers, J., 2002. Tracking the ecological overshoot of the human economy. Proc. Natl. Acad. Sci. 99, 9266–9271. https://doi.org/10.1073/pnas.142033699
World Bank, 2010. The Changing Wealth of Nations: Measuring Sustainable Development in the New Millennium. The World Bank.
Xu, M., Allenby, B.R., Crittenden, J.C., 2011. Interconnectedness and Resilience of the U.S. Economy. Adv. Complex Syst. 14, 649–672. https://doi.org/10.1142/S0219525911003335