


DIMER MONOMER


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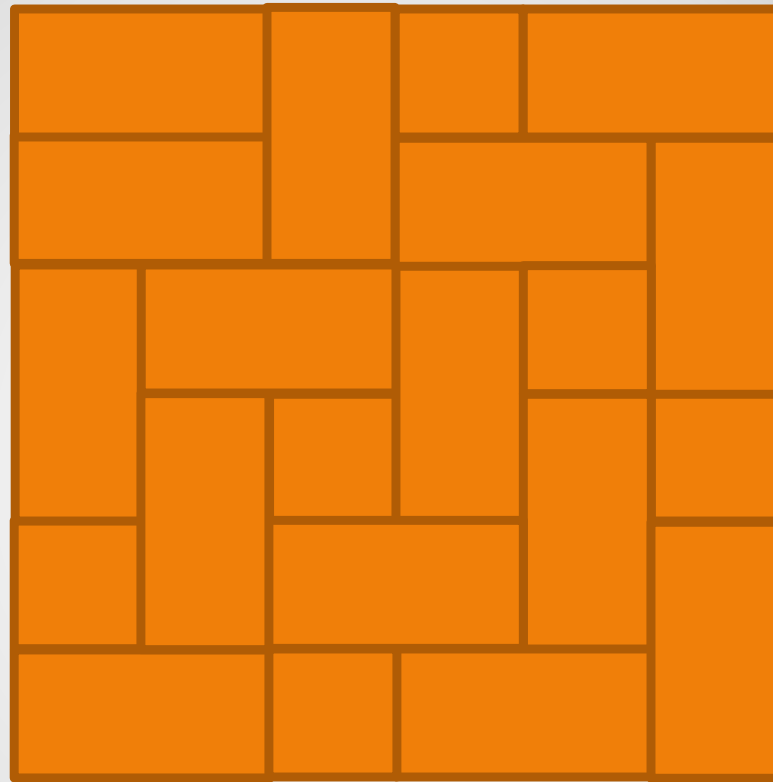
Partially supported by Wyoming EPSCoR

- Planar monomer-dimer tilings

- Monomer – 1 by 1 tile 

- Dimer – 2 by 1 tile or 1 by 2 

Monomer-Dimer Tilings



Monomer-Dimer Tilings

- What properties does a “random” monomer-dimer tiling of a region have?
 - Expected number of Dimers?
 - Is a given cell more likely to be a monomer?
- How can we generate a uniform, random monomer-dimer tiling of a region?

Fundamental Questions:

- **Methods**
 - Computer-based simulations
 - Kastelyn's Theorem
 - Probabilistic Models (matrices)
 - Analytical Models
- Sage Math (Python) was used throughout the year to aid us

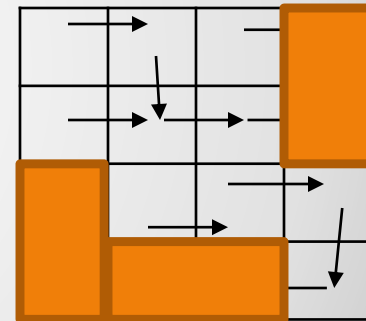
Monomer-Dimer Tilings

- N by N grids w/ 2 monomers, rest dimers
- 4 by 4
 - 552 possible tilings
 - Monomer's probable locations: corners, sides, center
- 6 by 6
 - 363,536 possible tilings
 - Probable Locations:
- 8 by 8
 - 1,760,337,760 tilings

1	6	2	2	6	1
6	5	4	4	5	6
2	4	3	3	4	2
2	4	3	3	4	2
6	5	4	4	5	6
1	6	2	2	6	1

Beginning Work

- Explored various regions: N by N , 2 by N , 1 by N
- Began as all monomers, “paired” to become dimers.
- Pair until “frozen”



Generating tilings

- Computer simulations
- Thousands of Iterations
- 4 by 4, 6 by 6, 8 by 8:
 - Average # of Steps: $\approx 3.5, 4.3, 4.8,$ respectively
 - SD(# of Steps): 0.85, regardless of N
 - Average # of Dimers/Area: ≈ 0.4578

Simulations

Using Probabilities

- Use theory of Markov chains and one-step probabilities
- 2 by 3
 - Expected # of dimers = $371/130 \approx 2.853846$
 - Expected # of steps = 2.39230760230769
- 1 by N

- Recursive Formulas:
 - $F_N = 1 + E_N + F_{N-3} + (F_0 + E_{N-2}) + (F_k + E_{N-k-2})$
 - $E_N = (F_{N-k-2} + E_k) + (F_0 + E_{N-2})$
- $E_N = E(\text{Dimers})$
- $E_{20000} / 20000 = 0.43344653343386$
- Bounded ... Must Converge

1 by N : Mathematical Discoveries

N	E_N	F_N	E_N/N
1	0.000000000000000000	1.000000000000000000	0.000000000000000000
2	1.000000000000000000	2.000000000000000000	0.500000000000000000
3	1.000000000000000000	2.000000000000000000	0.3333333333333330000
4	1.750000000000000000	2.718750000000000000	0.437500000000000000
5	2.000000000000000000	3.000000000000000000	0.400000000000000000
6	2.531250000000000000	3.500976562500000000	0.421875000000000000
7	2.921875000000000000	3.908935546875000000	0.4174107142857140000
8	3.372070312500000000	4.350563049316400000	0.4215087890625000000
9	3.800048828125000000	4.781750679016110000	0.4222276475694440000
10	4.235084533691400000	5.215733960270880000	0.4235084533691410000
11	4.668152809143060000	5.649077356792980000	0.4243775281039150000
12	5.101685479283330000	6.082547590580360000	0.4251404566069440000
13	5.535122551955280000	6.515996303797010000	0.4257786578427140000
14	5.968576318562550000	6.949448180993270000	0.4263268798973250000
15	6.402027546804900000	7.382899662521050000	0.4268018364536600000
16	6.835479100543120000	7.816351186864410000	0.4272174437839450000
17	7.268930618098660000	8.249802707266080000	0.4275841540058030000
9999			0.4334415451746800000
10999			0.4334424521226300000
15000			0.4334448710133280000
20000			0.4334465334338600000

Table of Values

- Still hoping to find a uniform way to generate a random tiling
- Finding a closed formula for $E(\text{dimers})$
 - What is 0.0433447?
- Applications in diatomic nuclear bonding and ice-formations

Future Potential Research