Spectroscopy via Lattice QCD

Norfolk, VA [Home to ODU]

http://hadspec.org

AMBER @ CERN 2020

JLab, VA
Exotics in QCD

to access and understand the nature of a state like the \( \pi_1 \) requires us to face some of the major theoretical challenges in QCD.
Exotics in QCD

What are the necessary features needed to be able to claim **accuracy**?

*broad resonance*
Exotics in QCD

πη

π(1)

I^G(J^{PC}) = 1^-(1+) - 1

broad resonance

strongly coupled

What are the necessary features needed to be able to claim **accuracy**?

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Exotics in QCD

What are the necessary features needed to be able to claim **accuracy**?

- **broad resonance**
- **strongly coupled**
- **non-zero spin systems**
- **three-particle system**

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Exotics in QCD

What are the necessary features needed to be able to claim **accuracy**?

- **broad resonance**
- **strongly coupled**
- **non-zero spin systems**
- **three-particle system**
- **structure of resonant states**

π₁, \(I^G(J^{PC})=1^{-}(1^{++})\)

γ*

πη

COMPASS

to access and understand the nature of a state like the \(π₁\) requires us to face some of the major theoretical challenges in QCD.
**GOAL:**
Get insights to the governing patterns and rules of QCD from emergent phenomena

Observables to test our understanding:
• Production and decay
• Exotic states
  • ...

Possible outcomes:
• Source of masses
• Role of glue
  • Structure of excited states
  • ...

|\(\langle n\rangle_{\text{QCD}} = c_0 + c_1 + c_2 + c_3 + c_4 + \cdots\) |

... perhaps there is a hierarchy [e.g. \(c_0 > c_1 > c_2 > c_3 > c_4\)]
QCD spectroscopy

Amplitude analysis

Experiments

QCD

Sheet I

Resonance poles

Transition form factors

Identification of states, production/decay mechanisms

Partial wave amplitudes

Electroweak amplitudes

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pentaquarks
molecules
hybrids

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QCD spectroscopy

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lattice QCD

finite-volume spectrum

finite-volume matrix elements

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QCD spectroscopy
QCD spectroscopy

structure and nature of states
elastic form factors

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electric form factors

lattice QCD
finite-volume spectrum
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QCD

models & EFTs
pentaquarks, molecules, hybrids

Experiments
resonance poles
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QCD spectroscopy
Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- Quark masses: $m_q \rightarrow m_q^{\text{phys}}$
- Lattice spacing
- Finite volume
Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -it_E$
- Monte Carlo sampling
- Quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
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- Finite volume

![Graph showing scattering observables from lattice QCD](image)

**BMW Collaboration (2015)**
Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \to -i t_E$
- Monte Carlo sampling
- Quark masses: $m_q \to m_q^{\text{phys}}$
- Lattice spacing
- Finite volume

\[ D_\mu = \left( \begin{array}{c} \vdots \\ \end{array} \right) \] \( (L/a)^3 \times (T/a) \)
Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: $t_M \rightarrow -i t_E$
- Monte Carlo sampling
- Quark masses: $m_q \rightarrow m_q^{\text{phys.}}$
- Lattice spacing
- Finite volume

No free states, no asymptotic states, no scattering.

Partial wave amplitudes and spectrum with no-trivial relation.
Scattering observables from lattice QCD

- Wick rotation [Euclidean spacetime]: \( t_M \rightarrow -i t_E \)
- Monte Carlo sampling
- quark masses: \( m_q \rightarrow m_q^{\text{phys}} \)
- lattice spacing
- finite volume

Scattering processes and resonances from lattice QCD
Published in Rev.Mod.Phys. 90 (2018) no.2, 025001
JLAB-THY-17-2495, ADP-17-28-T1034
DOI: 10.1103/RevModPhys.90.025001

References | BibTeX | LaTeX(US) | LaTeX(EU) | Harvmac | EndNote
ADS Abstract Service
Detailed record - Cited by 102 records 100+
Interesting but “old-school spectroscopy”

Evaluate: \( C^{2pt.}_{ab}(t, P) \equiv \langle 0 | O_b(t, P) O_a^+(0, P) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t} \)

...using a large number [10-30] of local ops, \( O_b \sim \bar{q} \Gamma_b q \)
Interesting but "old-school spectroscopy"

Evaluate: \( C_{ab}^{2pt.}(t, P) \equiv \langle 0 | \mathcal{O}_b(t, P) \mathcal{O}_a^\dagger(0, P) | 0 \rangle = \sum_n Z_{b,n} Z_{a,n}^* e^{-E_n t} \)

...using a large number [10-30] of local ops, \( \mathcal{O}_b \sim \bar{q} \Gamma_b q \)
Isoscalar spectra: S-wave dominant

Spectrum including a large basis: \( \{ \pi\pi, K\bar{K}, \eta\eta, \ell\bar{\ell}, s\bar{s} \} \)

\( m_\pi = 391 \text{ MeV} \)
Isoscalar spectra: $S$-wave dominant

Spectrum including a large basis: \( \{ \pi\pi, K\overline{K}, \eta\eta, \ell\ell, s\overline{s} \} \)

$2m_\pi \sim 782 \text{ MeV}$

$B \sim 37 \text{ MeV}$

$2 - 3 \text{ fm}$
Isoscalar spectra: S-wave dominant

Spectrum including a large basis: \(\{\pi\pi, K\bar{K}, \eta\eta, \ell\bar{\ell}, s\bar{s}\}\)

\(m_\pi = 391\) MeV

No simple story…
lattice spectroscopy

lattice QCD

finite-volume spectrum

PW amplitudes

analytic continuation

resonance poles

identification of
- states [masses & widths],
- production/decay mechanisms
analytic continuation
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Im[s]
Re[s]

Im[s]
Re[s]

Sheet II

no continuum of states:
- no cuts
- no sheet structure
- no resonances

no continuum of states:
- no cuts
- no sheet structure
- no resonances

$s_R = (E_R - \frac{i}{2} \Gamma_R)^2$
two-particle spectrum satisfies: \( \det[F^{-1}(P, L) + \mathcal{M}(P)] = 0 \)

- Lüscher (1986, 1991)
- Rummukainen & Gottlieb (1995)
- Kim, Sachrajda, & Sharpe/Christ, Kim & Yamazaki (2005)
- Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
- RB (2014)
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identification of states [masses & widths], production/decay mechanisms

**two-particle spectrum satisfies:** \( \text{det}[F^{-1}(P, L) + \mathcal{M}(P)] = 0 \)

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Feng, Li, & Liu (2004); Hansen & Sharpe / RB & Davoudi (2012)
RB (2014)

**single channel:** “easy”
\(\pi\pi\) Spectrum - \((l=1\ \text{channel})\)

\[ \det[F^{-1}(P, L) + \mathcal{M}(P)] = 0 \]

\[\mathcal{M} \propto \frac{1}{\cot \delta_1 - i}\]

\[m_\pi \sim 240\ \text{MeV}\]

Wilson, RB, Dudek, Edwards & Thomas (2015)
\( \pi\pi \) Spectrum - \((l=1\) channel\)

\[ [000] T_{1^-} \]

\[ \text{det}[F^{-1}(P, L) + M(P)] = 0 \]

\( \mathcal{M} \propto \frac{1}{\cot \delta_1 - i} \)

\( m_\pi \sim 240 \text{ MeV} \)

Wilson, RB, Dudek, Edwards & Thomas (2015)
ππ scattering - (l=1 channel)

$D\pi \pi = 391\,\text{MeV}$

$m_{\pi} = 236\,\text{MeV}$

Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
The $\rho$ vs $m_\pi$

$m_\rho = \text{Re}(E_\rho)/\text{MeV}$

$m_\pi = 536$ MeV
$m_\pi = 700$ MeV
$m_\pi = 391$ MeV

$m_\pi = 236$ MeV

$m_\pi = 140$ MeV, Lattice QCD + $\chi$PT
$m_\pi = 140$ MeV, Roy Equation

Lin et al. (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
The $\rho$ vs $m_\pi$

$\Gamma_\rho = 2 \cdot \text{Im}(E_\rho)/\text{MeV}$

$m_\rho = \text{Re}(E_\rho)/\text{MeV}$

$m_\pi = 536 \text{ MeV}$

$m_\pi = 700 \text{ MeV}$

$m_\pi = 391 \text{ MeV}$

$m_\pi = 236 \text{ MeV}$

$m_\pi = 140 \text{ MeV}, \text{ Lattice QCD + $U_\chi PT$}$

$m_\pi = 140 \text{ MeV}, \text{ Roy Equation}$

Lin et al. (2009)
Dudek, Edwards, Guo & Thomas (2013)
Dudek, Edwards & Thomas (2012)
Wilson, RB, Dudek, Edwards & Thomas (2015)
$\pi\pi$ scattering - (l=0 channel)

\[ M_0 = \frac{16\pi E_{cm}}{p \cot \delta_0 - ip} \]
The $\sigma$ vs $m_\pi$

![Diagram showing the relationship between $\sigma$, $m_\pi$, and $E_\sigma$ in MeV.](image)
\( \pi K \) scattering - (\( l=1/2 \) channel)

\[ M \sim \frac{1}{p \cot \delta - ip} \]

Wilson, RB, Dudek, Edwards, & Thomas (2019)
**πK scattering - (l=1/2 channel)**

\[ M \sim \frac{1}{p \cot \delta - ip} \]

\[ -\Gamma = 2 \text{Im}\sqrt{s}/\text{MeV} \]

\[ m = \text{Re}\sqrt{s}/\text{MeV} \]

\[ \delta_1/° \]

\[ E_{\text{cm}}/\text{MeV} \]

Wilson, RB, Dudek, Edwards, & Thomas (2019)
multi-channel systems - the cutting edge!

the necessary formalism for doing coupled-channel scattering of

<table>
<thead>
<tr>
<th>Channel Configurations</th>
<th>Authors and Journal (Year)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi\pi$, KK, $\eta\eta$ [isoscalar]</td>
<td>RB, Dudek, Edwards, Wilson - PRL (2017)</td>
</tr>
<tr>
<td>$\pi\eta$, KK</td>
<td>Dudek, Edwards, Wilson - PRD (2016)</td>
</tr>
<tr>
<td>$D\pi$, $D\eta$, $D_sK$</td>
<td>Moir, Peardon, Ryan, Thomas, Wilson - JHEP (2016)</td>
</tr>
</tbody>
</table>

to date, the Hadron Spectrum collaboration is the only one to have extracted coupled-channel scattering amplitude information from QCD.
Above $2m_K$, there is not a one-to-one correspondence

$$\det \left[ \begin{array}{cc} F_{\pi\pi}^{-1} + M_{\pi\pi,\pi\pi} & M_{\pi\pi,K\bar{K}} \\ M_{\pi\pi,K\bar{K}} & F_{K\bar{K}}^{-1} + M_{K\bar{K},K\bar{K}} \end{array} \right] = 0$$

Feng, Li, & Liu (2004), Hansen & Sharpe / RB & Davoudi (2012)

- infinite volume:  
  - coupled, scattering amplitude 
  - branch cuts

- finite volume:  
  - power-law finite-volume effects

multi-channel systems - the cutting edge!
Above $2m_K$, there is not a one-to-one correspondence

$$\text{det} \begin{bmatrix} F_{\pi\pi}^{-1} + M_{\pi\pi,\pi\pi} & M_{\pi\pi,K\bar{K}} \\ M_{\pi\pi,K\bar{K}} & F_{K\bar{K}}^{-1} + M_{K\bar{K},K\bar{K}} \end{bmatrix} = 0$$

Feng, Li, & Liu (2004),
Hansen & Sharpe / RB & Davoudi (2012)

In general, must constrain $(1/2) [N^2 + N]$ functions of energy

Need that many energy levels at the same energy

Alternatively, parametrize scattering amplitude and do a global fit
coupled-channels analysis

- S-wave above $2m_\pi, 2m_K, \text{ and } 2m_\eta$

- Ansatz $K^{-1}(s) = \begin{pmatrix} a + bs & c + ds & e \\ c + ds & f & g \\ e & g & h \end{pmatrix}$

\begin{align*}
\chi^2/N_{\text{dof}} = \frac{44.0}{57 - 8} = 0.90 \\
57 \text{ energy levels}
\end{align*}
scalar $\pi\pi-KK$

Near poles: $\mathcal{M} \sim \frac{g^2}{s_0 - s}$

$\sigma$: Sheet I pole

$\left| \frac{g_{KK}}{g_{\pi\pi}} \right|^2 = 1.4(3)$

$f_0(980)$: Sheet II pole

20 parametrizations
scalar $\pi\pi$-$KK$

Near poles: $\mathcal{M} \sim \frac{g^2}{s_0 - s}$
tensor $\pi\pi - KK$

$J^P = 2^+$

$m_\pi = 391$ MeV

$\rho_{ij} |t_{ij}|^2$

$E_{cm} / \text{MeV}$

$\Gamma_R$

$\pi\pi \rightarrow \pi\pi$

$K\bar{K} \rightarrow K\bar{K}$

$\pi\pi \rightarrow K\bar{K}$

$f_2^b$

$m_R$

$92%$

$8%$

$85%$

$12%$
$\pi\omega - \pi\phi$

and the $b_1$

$J^P = 1^+$

$m_\pi \sim 391$ MeV

Woss et al.
Scalar nonet @ $m_\pi \sim 400$ MeV
lattice spectroscopy

finite-volume spectrum

PW amplitudes

analytic continuation

resonance poles

identification of
states [masses & widths],
production/decay mechanisms

let's take a look at the future…!
lattice spectroscopy

**lattice QCD**

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**inside the box**

Blanton
Sharpe
Hansen

**outside the box**

Romero-López
Szczepaniak

three-particle spectrum satisfies:
\[ \text{det} \left[ F_3^{-1} + \mathcal{K}_{df,3} \right] = 0 \]

**Hansen & Sharpe (2014, 2015)**
**RB, Hansen & Sharpe (2017, 2018)**
**RB, Hansen, Sharpe, & Szczepaniak (2019)**
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three-particle spectrum satisfies:

\[ \det \left[ F_3^{-1} + \mathcal{K}_{df,3} \right] = 0 \]

\[ \Delta = \frac{E^2 - 9m^2}{9m^2} \]

Hörz and Hanlon (2019)
Romero-López, Blanton, & Sharpe (2019)

\[ 3\pi^+ @ m_\pi \sim 200 \text{ MeV} \]
lattice spectroscopy

lattice QCD

finite-volume spectrum

0/1-to-2 FV matrix elements

PW amplitudes

electroweak amplitudes

analytic continuation

resonance poles

transition form factors

identification of states [masses & widths], production/decay mechanisms

Lellouch & Lüscher (2000)
Kim, Sachrajda, & Sharpe
Christ, Kim & Yamazaki (2005)
...
Hansen & Sharpe (2012)
RB, Hansen Walker-Loud (2014)
RB & Hansen (2015)

\[ \left| \langle 2 | \mathcal{J} | 1 \rangle_L \right| = \sqrt{A \mathcal{R} A} \]

Lellouch-Lüscher matrix:

\[ \mathcal{R}(E_n, \mathbf{P}) \equiv \lim_{E \to E_n} \left[ \frac{(E - E_n)}{F^{-1}(P, L) + \mathcal{M}(P)} \right] \]
**lattice spectroscopy**

### lattice QCD

- **finite-volume spectrum**
- **PW amplitudes**
- **electroweak amplitudes**
- 0/1-to-2 FV matrix elements

### Identification of states [masses & widths], production/decay mechanisms

### analytic continuation

### Resonance poles

### Transition form factors

### πγ*-to-ππ

- **FV matrix elements**
- **πγ*-to-ππ transition form factors**

**Example Data**

- $E_{\pi\gamma}/m_\pi = 2.193(12)$
- $E_{\pi\gamma}/m_\pi = 2.193(17)$
- $E_{\pi\gamma}/m_\pi = 2.193(15)$

**Graphs**

- $Q^2 = 0$
- $Q^2 = 0.803 \text{ GeV}^2$
lattice spectroscopy

lattice QCD

finite-volume spectrum

0/1-to-2 FV matrix elements

PW amplitudes

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identification of states [masses & widths],
production/decay mechanisms

electroweak amplitudes

πγ*-to-ππ

Leskovec et al. (2018)
lattice spectroscopy

lattice QCD

- finite-volume spectrum
- 0/1-to-2 FV matrix elements
- 2-to-2 FV matrix elements

PW amplitudes
- electroweak amplitudes

analytic continuation

resonance poles
- transition form factors
- elastic form factors

structure and nature of states

identification of states [masses & widths], production/decay mechanisms

Jackura & Ortega (2020?)

Baroni, RB, Hansen, Ortega (2018)

RB, Hansen, & Jackura (2019, 2020)

RB & Hansen (2015)
lattice spectroscopy

**lattice QCD**

finite-volume spectrum

0/1-to-2 FV matrix elements

2-to-2 FV matrix elements

0/1-to-0/1 with two currents

**PW amplitudes**

**resonance poles**

analytic continuation

identification of states [masses & widths], production/decay mechanisms

**transition form factors**

**elastic form factors**

**Christ, Feng, Martinelli, & Sachrajda (2017)**

**Baroni, RB, Davoudi, Hansen, Schindler (to appear)**
What are the necessary features needed to be able to claim **accuracy**?

- **broad resonance**
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