Chapter 12. Quantitative

1. Assume the occupational distribution of males and females is as follows:

<table>
<thead>
<tr>
<th>Occupation</th>
<th>Male</th>
<th>Female</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>60%</td>
<td>5%</td>
</tr>
<tr>
<td>B</td>
<td>20</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>10</td>
<td>40</td>
</tr>
<tr>
<td>D</td>
<td>10</td>
<td>50</td>
</tr>
</tbody>
</table>

Calculate the index of dissimilarity (also known as the Duncan Index). Explain its interpretation.

2. One measure of wage discrimination, say between men and women, is the portion of the gender wage gap that is explained by differences in the payoffs to productive characteristics. After analyzing wage data (using regression analysis), you obtain the following expressions for male and female wages:

\[ \omega_f = 2 + 0.2EDUC + 0.2EXP + 0.5TEN \]
\[ \omega_m = 3 + 0.4EDUC + 0.3EXP + 0.5TEN \]

where \( \omega \) is the hourly wage, EDUC is years of education, EXP is total years of experience, and TEN is tenure (years worked) with one’s current employer. Furthermore, the average female wage (\( \omega_f \)) is $10/hour, the average male wage (\( \omega_m \)) is $18/hour, the average EDUC for males is 15 years, the average EXP for males is 20 years, and the average TEN for males is six years.

(a) What is the gender wage gap between males and females?

(b) What is the hypothetical wage of a female with characteristics the same as the “average” male?

(c) What fraction of the gender wage gap is explained by differences in the productive characteristics of men and women? What fraction of the gender wage gap is explained by differences in the payoffs to productive characteristics?

3. Suppose that the firm’s production function is given by

\[ q = 20 \sqrt{L_w + L_b} \]

where \( L_w \) and \( L_b \) are the number of white and black workers, respectively. The \( MP_L \) is given by

\[ MP_L = \frac{10}{\sqrt{L_w + L_b}} \]

Suppose that the market wage is $5/hour for black workers and $10/hour for white workers and the price of output is $10.

(a) How many workers would a non-discriminating firm hire? What is the firm’s profit level?
(b) Consider a discriminating firm that devalues the contributions of black workers at a rate of \( d = 8 \) dollar/hour. How many of each type of workers does this firm hire and what is its profit level?

4. Suppose a college admissions board assigns each applicant a score based on the applicant’s race and high school GPA. Because minorities are more likely to attend inner city public schools with an assumed higher degree of variability in terms of quality, the scores for white and black applicants, respectively, are generated as follows:

\[
\begin{align*}
S_f &= GPA_w \\
S_b &= 0.5 + 0.5GPA_b
\end{align*}
\]

Admission is granted to all students with a score over 2.

(a) Plot the admission board’s score for white and minority applicants against their high school GPA on the same graph.

(b) What is the minimum GPA required for white and minority students to gain admission? Is this an example of statistical discrimination?

5. Assume the marginal revenue product (MRP) for minority workers is given by

\[ MRP = 40 - 5N_m, \]

where \( N_m \) is the number of minority workers. The market wage for minorities is 5 dollar/hr. Discriminating firms devalue the contributions of minorities at a rate of 5 dollar/hr (i.e., \( d = 5 \)).

(a) How many minority workers do non-discriminating firms hire? What is their profit level?

(b) How many minority workers do discriminating firms hire? What is their profit level?

6. Suppose that the market demand for female workers is given by:

\[
\begin{align*}
\omega_f &= \omega_m \\
\omega_m &= \begin{cases} 
1 & \text{if } N_f \leq 100 \\
1.2 - 0.002N_f & \text{if } N_f \geq 100
\end{cases}
\]

where \( N_f \) is number of female applicants.

(a) How many females can be employed in this market prior to the presence of some discriminating firms being harmful to women?

(b) If non-discriminating firms or entrepreneurs buy out some discriminating firms such that now

\[
\begin{align*}
\omega_f &= \omega_m \\
\omega_m &= \begin{cases} 
1 & \text{if } N_f \leq 200 \\
1.2 - 0.001N_f & \text{if } N_f \geq 200
\end{cases}
\]

how does \( \frac{\omega_f}{\omega_m} \) change if \( N_f = 150 \) before the takeovers and does not change?

7. As in the previous question, suppose that the market demand for female workers is given by

\[
\begin{align*}
\omega_f &= \omega_m \\
\omega_m &= \begin{cases} 
1 & \text{if } N_f \leq 200
\end{cases}
\]
and the aggregate labor supply of females is given by \( N^*_f = 500 \frac{\omega_f}{\omega_m} \).

(a) Graph the labor supply and demand curves.

(b) Calculate the equilibrium level of employment for females and wage ratio.