Chapter 9. Quantitative

1. (a)

(b) Setting $MB = MC$ and solving for $HC^*$ yields

$MB = MC$

$800 - 25HC^* = 400$

$HC^* = 400/25 = 16$

(c)
The new solution is
\[ MB_0 = MC \]
\[ 800 - 20HC^* = 400 \]
\[ HC^* = 400/20 = 20 \]
The result is an increase in the optimal level of human capital investment.

2. (a) The present value of the income stream from not going to school is \( 200 + 250/1.1 = 427.27 \). The present value of the income stream from going to school is \( -100 + 575/1.1 = 422.73 \). Thus, she will not choose to go to school in period 1.
(b) The internal rate of return is the discount rate such that the benefits of going to school equal the costs of going to school. The total cost of going to school is the sum of the direct costs and the foregone earnings in period 1. The benefit from schooling is the higher wage in period 2. Thus, the internal rate of return, \( r^* \), is given by
\[
\frac{325}{1 + r^*} = 200 + 100
\]
\[ r^* = 0.083 \]
As a result, only for discount rates less than \( r^* \) will she find it optimal to go to school in period 1.

3. (a) For Billy, the present value of the additional income stream from taking the course is \( 2000(1/1.05 + 1/1.052 + 1/1.053 + 1/1.054 + 1/1.055 + 1/1.056 + 1/1.057 + 1/1.058 + 1/1.059 + 1/1.0510) = 15,443.47 \), which exceeds the cost. Thus, Billy should enroll in the course. For Bob, the present value of the additional income stream from taking the course is
(b) For the course to be beneficial, the benefits must be greater than the costs. Thus, 
\[ \Delta y \left( \frac{1}{1.05} + \frac{1}{1.05^2} + \cdots + \frac{1}{1.05^T} \right) > 10,000, \]
where \( \Delta y \) is the amount of the raise ($2000/yr for Billy, $1000/yr for Bob) and \( T \) is the years until retirement. For Bob, if he works 14 more years, the discounted PV of the benefits from the course total $9,898.64; for 15 years, the benefits total $10,379.66. Thus, Bob must plan on working until age 65 if taking the course is to be optimal. For Billy, if he works 5 more years, the discounted PV of the benefits from the course total $8,658.95; for 6 years, the benefits total $10,151.38. Thus, Billy must plan on working until age 56 if taking the course is to be optimal.

4. (a) Since \( \text{wb} = ab \), \( \text{wb} = 25 \). Likewise, \( \text{wf} = 5 \).
(b) Estimated return to a college diploma is the average difference in wages between those with and without a diploma. This is \( 25 - 5 = $20/hr. \)
(c) For those who chose to become bankers, the average difference in wages with and without a college diploma is \( 25 - 3 = $22/hr. \) Thus, the ‘usual’ calculation of the return to a college education (that obtained in (b)) is biased down for bankers. For those who opted to become farmers, the return to a college diploma would have been \( 20 - 5 = $15/hr. \)
Thus, the ‘usual’ calculation of the return to a college education (that obtained in (b)) is biased up for farmers. This illustrates the idea behind selection bias.

5. (a) Firms will assume white workers with more than \( e^* \) years of schooling are high ability, with productivity equal to 10. Thus, these workers will receive a wage of 10. Firms will assume white workers with less than \( e^* \) years of schooling are low ability, with productivity equal to 5. Thus, these workers will receive a wage of 5. Since there is no benefit for high ability workers to go to school more than \( e^* \) years, high ability white workers will go to school exactly \( e^* \) years, whereas low ability white workers will not go to school at all (or, at least not continue past that required by compulsory attendance laws) since the costs exceed the benefits.
(b) Since the ‘signaling’ ability of education is not assumed to hold for minorities (as differences in schooling tend to reflect household wealth and not differences in the cost of schooling related to ability), all minority workers are treated the same and paid the average productivity. Thus, all minority workers — both high and low ability — are paid the average wage, \( (5 + 10)/2 = 7.50. \)
Since all minority workers are paid the identical wage independent of schooling, both high and low ability minorities will not go to school at all (or, at least not continue past that required by compulsory attendance laws) since there is no benefit to continued schooling.