

Exercises for Chapter 10

Exercise 10.1 Consider the following symmetric matrices:

$$\mathbf{A} = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 3 & -2 \\ 1 & -2 & 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 3 & 2 & 1 \\ 2 & 12 & 4 \\ 1 & 4 & 2 \end{pmatrix}.$$

Confirm that both \mathbf{A} and \mathbf{B} are positive definite.

Exercise 10.2 Consider the following matrices:

$$\mathbf{A} = \begin{pmatrix} -3 & 2 & 5 \\ 2 & -6 & 1 \\ 5 & 1 & -13 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 2 & 1 \\ 2 & -1 & -1 \\ 1 & -1 & -7 \end{pmatrix}.$$

Confirm that both \mathbf{A} and \mathbf{B} are negative definite.

Exercise 10.3 Consider the matrix:

$$\mathbf{A} = \begin{pmatrix} \frac{1}{4} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{9} \end{pmatrix}$$

Show that \mathbf{A} is positive definite for all $\mathbf{x} = (x_1, x_2)$ satisfying $(\frac{1}{2}, \frac{2}{3}) \cdot (x_1, x_2) = 0$.

Exercise 10.4 Consider the matrix:

$$\mathbf{A} = \begin{pmatrix} \frac{1}{4} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{32}{9} \end{pmatrix}.$$

Show \mathbf{A} that is positive definite for all $\mathbf{x} = (x_1, x_2)$ satisfying $(2, \frac{8}{3}) \cdot (x_1, x_2) = 0$.

Exercise 10.5 Consider the matrix:

$$\mathbf{Q}(\alpha, \beta, \gamma) = \begin{pmatrix} \alpha(\alpha-1) & \alpha\beta & \alpha\gamma \\ \alpha\beta & \beta(\beta-1) & \beta\gamma \\ \alpha\gamma & \beta\gamma & \gamma(\gamma-1) \end{pmatrix}.$$

Let $\mathbf{x}'\mathbf{Q}(\alpha, \beta, \gamma)\mathbf{x}$ be a quadratic form. Also, let $\mathbf{b} = (w, r, p)$ be a vector of strictly positive entries. Suppose that $\alpha = \beta = \gamma = \frac{1}{4}$. Show that $\mathbf{x}'\mathbf{Q}(\alpha, \beta, \gamma)\mathbf{x} < 0$ for all \mathbf{x} satisfying $\mathbf{b}'\mathbf{x} = 0$.