

Wilcoxon Rank Sum Test: Correction for Tied Data (with more than 30 participants)

In this explanation of how to correct for tied data when calculating the Wilcoxon Rank Sum Test for a large dataset, there are tied scores within one group and across the two groups. Here's a description of the (fabricated) study.

Example study. One of Professor Oliva's colleagues had the opportunity to conduct research similar to his; however, her classes were larger than Professor Oliva's. The independent variable for her study was *type of feedback*, and the dependent variable was *level of motivation*. The research question was the same as Professor Oliva's: "Is there a statistically significant difference in the posttest level of motivation of students who received feedback designed to promote autonomy and motivation versus those who received the usual type of feedback?" Her non-experimental class had 17 students and her experimental class had 14. Because she was replicating Professor Oliva's study, she followed the same procedures and required her students to complete the motivation questionnaire at the beginning and end of the term.

The posttest motivation scores are presented in Table 1. There are tied values for the scores of 43, 45, 46, 47, 48, 49, 50, 51, and 52.

Table 1

Posttest motivation scores for non-experimental (NE) and experimental (E) groups

Student	Non-experimental group	Student	Experimental group
NE ₁	40	E ₁	39
NE ₂	42	E ₂	41
NE ₃	43	E ₃	43
NE ₄	44	E ₄	45
NE ₅	45	E ₅	46
NE ₆	46	E ₆	48
NE ₇	46	E ₇	49
NE ₈	47	E ₈	49
NE ₉	47	E ₉	50
NE ₁₀	48	E ₁₀	50
NE ₁₁	49	E ₁₁	50
NE ₁₂	50	E ₁₂	52
NE ₁₃	50	E ₁₃	52
NE ₁₄	51	E ₁₄	54
NE ₁₅	51		
NE ₁₆	52		
NE ₁₇	54		

The steps in statistical logic, with the correction factor for tied scores, follow.

Step 1: State the formal research hypotheses.

Null hypothesis: There is no significant difference between the rankings of the posttest motivation scores for the non-experimental and experimental groups.

Alternative hypothesis 1: The rankings of the posttest motivation scores for the experimental group are systematically higher than those of the non-experimental group.

Alternative hypothesis 2: The rankings of the posttest motivation scores for the experimental group are systematically lower than those of the non-experimental group.

Step 2: Set alpha. Alpha is set at .05.

Step 3: Select the appropriate statistic for analysis of the data. The Wilcoxon rank sum test is appropriate for analysis of the data because the nominal independent variable has two levels and the posttest motivation values are rankable. The calculations include correction for numerous ties among the data.

Step 4: Collect the data. The fabricated data are presented in Table 1.

Step 5: Verify that the assumptions for the statistic are met. There are two conditions that must be met when using the Wilcoxon rank sum test statistic.

1. The independent variable is *nominal* and has only two levels; the two levels are represented by different participants.
2. The dependent variable yields rankable data.

In addition, when there are ties within the data, a correction procedure must be followed. *R* corrects for tied data (Verzani, 2005, p. 245), but when calculations are done with a calculator, the correction factor explained in Step 6 must be carried out.

Step 6: Calculate the observed value of the statistic. The calculation steps, with the correction for tied values, follow. The value of W is calculated first, as when there are no tied scores. The correction for ties takes place in the conversion of W_{observed} to a z -score.

The values for W_{NE} and W_E are calculated first. Combine the non-experimental group and the experimental group and rank the scores. When there are ties, the mean ranking for the ties is used for each of the tied scores, as shown in Table 2 and explained in Chapter Seven.

Table 2
Ranking of total posttest motivation scores for non-experimental and experimental groups

Non-experimental (NE) or experimental (NE)	Total motivation score	Ranking
E ₁	39	1
NE ₁	40	2
E ₂	41	3

NE ₂	42	4
E ₃	43	5.5
NE ₃	43	5.5
NE ₄	44	7
E ₄	45	8.5
NE ₅	45	8.5
E ₅	46	11
NE ₆	46	11
NE ₇	46	11
NE ₈	47	13.5
NE ₉	47	13.5
E ₆	48	15.5
NE ₁₀	48	15.5
E ₇	49	18
E ₈	49	18
NE ₁₁	49	18
E ₉	50	22
E ₁₀	50	22
E ₁₁	50	22
NE ₁₂	50	22
NE ₁₃	50	22
NE ₁₄	51	24.5
NE ₁₅	51	24.5
E ₁₂	52	28
E ₁₃	52	28
NE ₁₆	52	28
E ₁₄	54	30
NE ₁₇	54	30

Then separate the two groups and find the sum of the ranks for each group, as shown in Table 3.

Table 3
Sum of ranks for non-experimental and experimental groups

Student	Non-experimental group	Student	Experimental group
NE ₁	2	E ₁	1
NE ₂	4	E ₂	3
NE ₃	5.5	E ₃	5.5
NE ₄	7	E ₄	8.5
NE ₅	8.5	E ₅	11
NE ₆	11	E ₆	15.5
NE ₇	11	E ₇	18
NE ₈	13.5	E ₈	18
NE ₉	13.5	E ₉	22

NE₁₀	15.5	E₁₀	22
NE₁₁	18	E₁₁	22
NE₁₂	22	E₁₂	28
NE₁₃	22	E₁₃	28
NE₁₄	24.5	E₁₄	30
NE₁₅	24.5		
NE₁₆	28		
NE₁₇	30		
Sum of ranks for non-experimental group	$\sum R_{NE} = 260.5$	Sum of ranks for experimental group	$\sum R_E = 232.5$

Next determine the value of W_{NE} using this formula:

$$W_{NE} = n_{NE}n_E + \frac{n_{NE}(n_{NE} + 1)}{2} - \sum R_{NE} =$$

$$(17)(14) + \frac{17(17 + 1)}{2} - 260.5 =$$

$$238 + \frac{(17)(18)}{2} - 260.5 =$$

$$238 + \frac{306}{2} - 260.5 =$$

$$238 + 153 - 260.5 =$$

$$391 - 260.5 = \mathbf{130.5}$$

Then calculate W_E using this formula:

$$W_{NE} = n_{NE}n_E + \frac{n_E(n_E + 1)}{2} - \sum R_E =$$

$$(17)(14) + \frac{14(14 + 1)}{2} - 232.5 =$$

$$238 + \frac{14(15)}{2} - 232.5 =$$

$$238 + \frac{210}{2} - 232.5 =$$

$$238 + 105 - 232.5 =$$

$$343 - 232.5 = \mathbf{110.5}$$

The smaller of the two values is the observed value of the Wilcoxon Rank Sum statistic; $W_{\text{observed}} = 110.5$. W_{observed} is converted to a z -score for interpretation when there are more than 30 participants. It is in the conversion of W_{observed} to a z -score that the adjustment for tied scores is made.

The formula for converting W_{observed} to a z -score is:

$$z = \frac{W_{\text{observed}} - \bar{X}_W}{s_W}$$

The formula for \bar{X}_W is:

$$\bar{X}_W = \frac{n_{NE}n_E}{2}$$

For this study, the value of \bar{X}_W is: $\frac{(17)(14)}{2} = \frac{238}{2} = 119$.

The usual formula for s_W is:

$$s_W = \sqrt{\frac{n_{NE}n_E(n_{NE} + n_E + 1)}{12}}$$

However, the formula used when there are tied scores is:

$$s_{W\text{-correction}} = \sqrt{\left(\frac{n_{NE}n_E}{N(N-1)}\right)\left(\frac{N^3 - N}{12} - \sum T\right)}$$

The new value in the $s_{W\text{-correction}}$ formula, $\sum T$, is determined following these steps:

(1) Identify the scores for which there are ties and the number of tied scores for each.

2 scores of 43
 2 scores of 45
 3 scores of 46
 2 scores of 47
 2 scores of 48
 3 scores of 49
 5 scores of 50

2 scores of 51
3 scores of 52
2 scores of 54

(2) Then cube the number of tied scores for each value, subtract the number of tied scores, and divide by 12, as shown in Table 4. The total of these values is $\sum T$.

Table 4
Calculation of $\sum T$

2 scores of 43	$2^3 - 2/12 =$	$8 - 2/12 = 6/12 = .5$
2 scores of 45	$2^3 - 2/12$	$8 - 2/12 = 6/12 = .5$
3 scores of 46	$3^3 - 3/12$	$27 - 3/12 = 24/12 = 2$
2 scores of 47	$2^3 - 2/12$	$8 - 2/12 = 6/12 = .5$
2 scores of 48	$2^3 - 2/12$	$8 - 2/12 = 6/12 = .5$
3 scores of 49	$3^3 - 3/12$	$27 - 3/12 = 24/12 = 2$
5 scores of 50	$5^3 - 5/12$	$125 - 5/12 = 120/12 = 10$
2 scores of 51	$2^3 - 2/12$	$8 - 2/12 = 6/12 = .5$
3 scores of 52	$3^3 - 3/12$	$27 - 3/12 = 24/12 = 2$
2 scores of 54	$2^3 - 2/12$	$8 - 2/12 = 6/12 = .5$
$\sum T = 19$		

With the value of $\sum T$, $s_{W-corrected}$ can be calculated. N is the total number of participants in the study.

$$s_{W-corrected} = \sqrt{\left(\frac{n_{NE}n_E}{N(N-1)}\right)\left(\frac{N^3 - N}{12} - \sum T\right)} =$$

$$\sqrt{\left(\frac{(14)(17)}{31(31-1)}\right)\left(\frac{31^3 - 31}{12} - 19\right)} =$$

$$\begin{aligned}
&= \sqrt{\left(\frac{238}{31(30)}\right)\left(\frac{29791-31}{12}-19\right)} = \\
&\sqrt{\left(\frac{238}{930}\right)\left(\frac{29760}{12}-19\right)} = \\
&\sqrt{(.25591)(2480-19)} = \\
&\sqrt{(.25591)(2461)} = \\
&\sqrt{629.79451} = 25.09571
\end{aligned}$$

Now W can be converted to a z -score for interpretation; the value of W_{observed} is 110.5; the value of \bar{X}_W is 119; $s_{W\text{-corrected}}$ is 25.09571.

$$z = \frac{W_{\text{observed}} - \bar{X}_W}{s_W} = \frac{110.5 - 119}{25.09571} = \frac{-8.5}{25.09571} = -0.33870$$

Steps 7 and 8: Using the critical value of z , interpret z_{observed} . A z -score equal to or exceeding -1.96 or $+1.96$ is statistically significant at $p = .05$. The observed value of z is -0.33870 , so the null hypothesis must be accepted.

Null hypothesis: There is no significant difference between the rankings of the posttest motivation scores for the non-experimental and experimental groups.

Step 9: Interpret the findings as a probability statement. Alpha is used in making the probability statement.

There's 95% certainty of no statistically significant difference between the rankings of the posttest level of motivation for the non-experimental and experimental groups.

Step 10: Interpret the meaningfulness of the findings. Meaningfulness is interpreted with reference to the research question: "Is there a statistically significant difference in the posttest outcomes of students who received the feedback designed to promote autonomy and motivation versus those who received the usual type of feedback?" Additionally effect size can be calculated, though it will be quite small. The formula for effect size is:

$$\text{effect size} = \frac{|z|}{\sqrt{N}}$$

The symbol $|z|$ is the absolute value of z_{observed} , so effect size is:

$$\text{effect size} = \frac{.33870}{\sqrt{31}} = \frac{.33870}{5.56776} = .06$$

The interpretation of the meaningfulness of the findings would look something like this:

For these 31 students, I conclude that the type of feedback does not have an impact on students' level of motivation; there is no statistically significant difference between the motivation scores of learners receiving the feedback designed to promote autonomy and motivation and learners who received the usual type of feedback ($z = -0.34257, p = .05$). The effect size (.06) is very small.

References

Verzani, J. (2005). *Using R for introductory statistics*. Boca Raton, FL: Chapman & Hall/CRC Press.