

The Decibel and its Usage

Consider a two-stage amplifier system, as shown in Fig. 1. Each amplifier provides an increase of the *signal* power. This effect is referred to as the power gain, A_p , of the amplifier. This means that the signal output power from an amplifier is greater than its signal input power. This power gain may be expressed as:

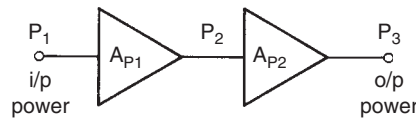


Fig. 1

$$A_p = \frac{P_{out}}{P_{in}} \quad \text{or} \quad \frac{P_o}{P_i}$$

For the system shown, if the input power P_1 is 1 mW, and the gains of the two amplifier stages are 10 times and 100 times respectively, then the final output power, P_3 , may be determined as follows.

$$A_{p1} = \frac{P_2}{P_1}; \quad \text{so } P_2 = A_{p1} \times P_1 \text{ watt}$$

$$\text{therefore, } P_2 = 10 \times 10^{-3} = 10 \text{ mW}$$

$$A_{p2} = \frac{P_3}{P_2}; \quad \text{so } P_3 = A_{p2} \times P_2 \text{ watt}$$

$$\text{therefore, } P_3 = 100 \times 10 \times 10^{-3} = 1 \text{ W}$$

From these results it may be seen that the output power of the system is 1000 times that of the input.

In other words, overall signal power gain,

$$A_p = \frac{P_3}{P_1} = \frac{1}{10^{-3}} = 1000$$

or, overall signal power gain,

$$A_p = A_{p1}A_{p2}$$

In general, when amplifiers (or other devices) are cascaded in this way, the overall gain (or loss) is given by the product of the individual stage gains (or losses).

Note: The efficiency of any machine or device is defined as the ratio of its output power to its input power. However, this does NOT mean that an amplifier is more than 100% efficient! The reason is that only the *signal* input and output powers are considered when quoting the power gain. No account is taken of the comparatively large amount of power injected from the d.c. power supply, without which the amplifier cannot function. In practice, small signal voltage amplifiers will have an *efficiency* figure of less than 25%. Power amplifiers may have an efficiency in the order of 70%.

It is often more convenient to express power gain ratios in a logarithmic form, known as the Bel (named after Alexander Graham Bell). Thus a power gain expressed in this way is:

$$A_p = \log \frac{P_o}{P_i} \text{ Bel}$$

where P_i = input power; P_o = output power; and logarithms to the base 10 are used.

The Bel is an inconveniently large unit for practical purposes, so the decibel (one tenth of a Bel) is used.

$$\text{Hence, } A_p = 10 \log \frac{P_o}{P_i} \text{ decibel} \quad (1)$$

The unit symbol for the decibel is dB. For the two-stage amplifier system considered, the power gains would be expressed as follows:

$$\begin{aligned} A_{p1} &= 10 \log 10 = 10 \text{ dB} \\ A_{p2} &= 10 \log 100 = 20 \text{ dB} \\ \text{and } A_p &= 10 \log 1000 = 30 \text{ dB} \end{aligned}$$

Note that the overall system gain, A_p , when expressed in dB is simply the sum of the individual stage gains, also expressed in dB.

Worked Example 1

- Q** A communications system, involving transmission lines and amplifiers, is illustrated in Fig. 2. Each section of transmission line attenuates (reduces) the signal power by a factor of 35.5%, and each amplifier has a gain ratio of 5 times. Calculate the overall power gain of the system as (a) a power ratio, and (b) in decibels.

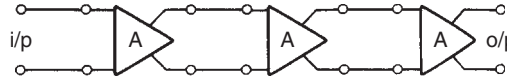


Fig. 2

A

- (a) For each line, $\frac{P_o}{P_i} = 0.355$
 so, total loss = $0.355 \times 0.355 \times 0.355 \times 0.355$
 $= 0.0159$

For each amplifier, $\frac{P_o}{P_i} = 5$
 so, total gain = $5 \times 5 \times 5 = 125$
 therefore, overall gain = 125×0.0159
 $= 2 \text{ times Ans}$

- (b) For each line, attenuation is

$10 \log 0.355 = -4.5 \text{ dB}^*$
 so total attenuation = -4.5×4
 $= -18 \text{ dB}$
 For each amplifier, gain = $10 \log 5$
 $= +7 \text{ dB}$
 so total gain = $7 \times 3 = 21 \text{ dB}$

Hence, overall gain of the system = $21 - 18 = 3 \text{ dB Ans}$

*Note that a loss or attenuation expressed in dB has a negative value; whereas a gain has a positive value. A further point to note is that if the gains and attenuations of the system had originally been expressed in dB, then the calculation would simply have been as follows:

Overall system gain = $(3 \times 7) - (4 \times 4.5) = 3 \text{ dB Ans}$

The above example illustrates the convenience of using decibel notation, since it involves only simple addition and subtraction to determine the overall gain or attenuation of a system. It has also been shown that a gain of 2 times is equivalent to a power gain of 3 dB (more precisely, 3.01 dB). It is left to the reader to confirm, by using

a calculator, that an attenuation of 2 times (i.e. a gain of 0.5) is equal to -3 dB. This figure of -3 dB will frequently be met when dealing with the frequency response curves for amplifiers and other frequency-dependent circuits, such as series and parallel tuned circuits.

In order to gain a 'feel' for power gains and losses expressed in dB, the following list shows the corresponding power gain ratios.

gains	10 000 times = 40 dB
	1 000 times = 30 dB
	100 times = 20 dB
	10 times = 10 dB
	2 times = 3 dB
	1 times = 0 dB
losses	0.5 times = -3 dB
	0.1 times = -10 dB
	0.01 times = -20 dB
	0.001 times = -30 dB
	0.0001 times = -40 dB

Worked Example 2

Q

(a) Convert the following gain ratios into dB.

(i) 250, (ii) 50, (iii) 0.4

(b) Convert the following gains and losses into ratios.

(i) 25 dB, (ii) 8 dB, (iii) -15 dB

A

(a) (i) $A_p = 10 \log 250 = 24$ dB **Ans**

(ii) $A_p = 10 \log 50 = 17$ dB **Ans**

(iii) $A_p = 10 \log 0.4 = -4$ dB **Ans**

(b) (i) $25 = 10 \log (\text{ratio})$ dB

so $2.5 = \log (\text{ratio})$

therefore, $(\text{ratio}) = \text{antilog } 2.5 = 316$ times **Ans**

(ii) $8 = 10 \log (\text{ratio})$ dB

$0.8 = \log (\text{ratio})$

therefore, $(\text{ratio}) = \text{antilog } 0.8 = 6.3$ times **Ans**

(iii) $-15 = 10 \log (\text{ratio})$ dB

$-1.5 = \log (\text{ratio})$

therefore, $(\text{ratio}) = \text{antilog } (-1.5) = 0.032$ **Ans**

Although the decibel is defined in terms of a power ratio, it may also be used to express both voltage and current ratios, provided that certain conditions are met. These conditions are that the resistance of the load is the same as that of the source, i.e. the conditions for maximum power transfer. Consider such a system, whereby the two resistance values are R ohm, the input voltage is V_1 volt, and the output voltage is V_2 volt. Let the corresponding currents be I_1 and I_2 ampere.

$$P_1 = \frac{V_1^2}{R} \text{ watt and } P_2 = \frac{V_2^2}{R} \text{ watt}$$

$$\text{gain} = 10 \log \left(\frac{V_2^2}{R} \times \frac{R}{V_1^2} \right)$$

$$= 10 \log \left(\frac{V_2}{V_1} \right)^2$$

hence, voltage gain, $A_v = 20 \log \frac{V_2}{V_1}$ dB

Also, using the fact that $P_1 = I_1^2 R$ watt and $P_2 = I_2^2 R$ watt it is left to the reader to verify that:

$$\text{current gain, } A_i = 20 \log \frac{I_2}{I_1} \text{ dB} \quad (2)$$

As shown in *Further Electrical and Electronic Principles*, Chapter 2, a tuned circuit used as a pass-band or stop-band filter has a bandwidth. The same concept also applies to a.c. amplifiers. In the latter case, the frequency response curve for a voltage amplifier would be similar to that shown in Fig. 3. The bandwidth is defined as that range of frequencies over which the voltage gain is greater than, or equal to, $A_{vm}/\sqrt{2}$, where A_{vm} is the mid-frequency gain. A similar response curve for the amplifier current gain could also be plotted.

Now, power gain = voltage gain \times current gain

$$\text{or, } A_p = A_v \times A_i$$

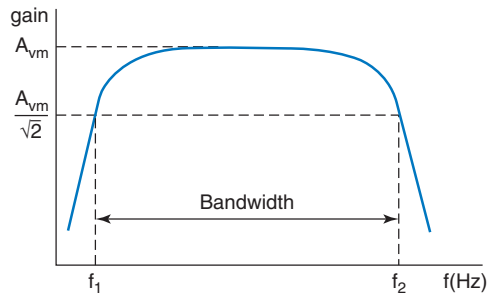


Fig. 3

The cut-off frequencies, f_1 and f_2 , define the bandwidth, and at these frequencies, the current and voltage gains will be:

$$A_v = \frac{A_{vm}}{\sqrt{2}} \text{ and } A_i = \frac{A_{im}}{\sqrt{2}} \text{ respectively}$$

$$\text{thus } A_p = \frac{A_{vm}}{\sqrt{2}} \times \frac{A_{im}}{\sqrt{2}} = \frac{A_{pm}}{2}$$

These points on the response curve are therefore referred to as either the cut-off points, the half-power points, or the -3 dB points.

Worked Example 3

Q An amplifier is fed with a 50 mV, 200 μ A signal. The amplifier has a voltage gain of 75 times and a current gain of 150 times. Determine (a) the voltage, current and power gains, expressed in dB, and (b) the output voltage, current and power. You may assume that input and output resistances are the same value.

A

$$V_1 = 50 \times 10^{-3} \text{ V}; I_1 = 200 \times 10^{-6} \text{ A}; A_v = 75; A_i = 150$$

$$(a) \quad A_v = 20 \log 75 \text{ dB} = 37.5 \text{ dB Ans}$$

$$A_i = 20 \log 150 \text{ dB} = 43.5 \text{ dB Ans}$$

$$A_p = A_v \times A_i = 75 \times 150 = 11\,250 \text{ times}$$

$$\text{so, } A_p = 10 \log 11\,250 \text{ dB} = 40.5 \text{ dB Ans}$$

$$(b) \quad V_2 = V_1 \times A_v \text{ volt} = 50 \times 10^{-3} \times 75 = 3.75 \text{ V Ans}$$

$$I_2 = I_1 \times A_i \text{ amp} = 200 \times 10^{-6} \times 150 = 30 \text{ mA Ans}$$

$$P_2 = V_2 I_2 \text{ watt} = 3.75 \times 0.03 = 112.5 \text{ mW Ans}$$

Worked Example 4

Q An amplifier has a voltage gain of 10 times at a frequency of 150 Hz; 60 times between 2 kHz and 12 kHz; and 15 times at 35 kHz; Determine (a) the voltage gain, expressed in decibel, for each case, and (b) the voltage gain at the limits of its bandwidth.

A

$$(a) \quad \text{at 150 Hz: } A_v = 20 \log 10 = 20 \text{ dB Ans}$$

$$\text{at mid-frequencies: } A_v = 20 \log 60 = 35.6 \text{ dB Ans}$$

$$\text{at 35 kHz: } A_v = 20 \log 15 = 23.5 \text{ dB Ans}$$

(b) The limits of the bandwidth occur when A_v is 3 dB down on the mid-frequency value, or $A_v = A_{vm}/\sqrt{2}$.

$$\text{Therefore, } A_v = A_{vm} - 3 = 35.6 - 3 = 32.6 \text{ dB Ans}$$

$$\text{or, } A_v = \frac{A_{vm}}{\sqrt{2}} = \frac{60}{\sqrt{2}} = 42.426 \text{ times}$$

$$\text{hence, } A_v = 20 \log 42.426 = 32.6 \text{ dB Ans}$$

The decibel is defined in terms of a power *ratio*. Thus, the power output of a device cannot be quoted directly in dB. For example, to say that an amplifier has an output power of 20 dB is completely meaningless. In this case it only makes sense to say that the amplifier has a power *gain* of 20 dB. However, actual values of output power may be expressed in decibel form *provided* a known reference power input is used. In electronic work, this reference power is one milliwatt (1 mW), and powers expressed in this way are designated as dBm.

Considering such an amplifier, it could be said to have a power output of 20 dBm. This would mean that its actual output power was 0.1 W. This figure is obtained thus:

$$\begin{aligned} 10 \log \frac{P_2}{P_1} &= 20 \text{ dBm} \\ \log \frac{P_2}{P_1} &= 2 \\ \frac{P_2}{P_1} &= \text{antilog } 2 = 100 \end{aligned}$$

and since $P_1 = 1 \text{ mW}$, then $P_2 = 100 \times 10^{-3} = 0.1 \text{ W}$ or 100 mW.

Supplementary Worked Example 1

Q An attenuator circuit has input and output voltages of 8.5 V and 2.4 V respectively. Determine the attenuation, expressed in decibels.

A

$$\begin{aligned} V_i &= 8.5 \text{ V}; V_o = 2.4 \text{ V} \\ \text{attenuation} &= 20 \log \frac{V_o}{V_i} \text{ decibel} \\ &= 20 \log \frac{2.4}{8.5} = 20 \log 0.2824 \\ \text{so, attenuation} &= -11 \text{ dB} \text{ **Ans**} \end{aligned}$$

Supplementary Worked Example 2

Q An amplifier has a power gain of 40 dB. Express this gain as a power ratio.

A

$$\begin{aligned} 40 &= 10 \log (\text{ratio}) \text{ decibel} \\ 4 &= \log (\text{ratio}) \\ \text{antilog } 4 &= \text{ratio} \\ \text{so, ratio} &= 10^4 \text{ **Ans**} \end{aligned}$$

Supplementary Worked Example 3

- Q** For the attenuator pad shown between terminals AB and CD in Fig. 4, calculate (a) the output (V_O) and input (V_i) voltages, (b) the voltage attenuation in decibels, and (c) the output power.

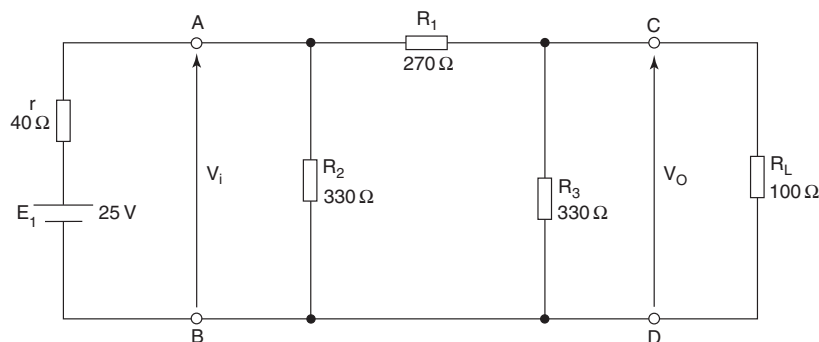


Fig. 4

A

- (a) Using Thévenin's theorem and looking in at terminals CD the circuit will appear as shown in Fig. 5.

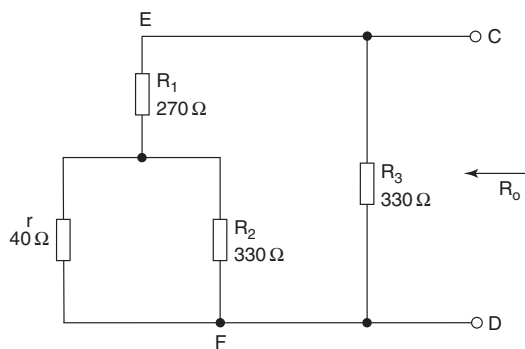


Fig. 5

$$R_{EF} = R_1 + \frac{rR_2}{r + R_2} \text{ ohm}$$

$$= 270 + \frac{40 \times 330}{40 + 330}$$

$$R_{EF} = 305.68 \Omega$$

$$R_O = \frac{R_{EF}R_3}{R_{EF} + R_3} \text{ ohm} = \frac{305.68 \times 330}{305.68 + 330}$$

$$R_O = 158.7 \Omega$$

To obtain the Thévenin generator emf E_O , consider Fig. 6

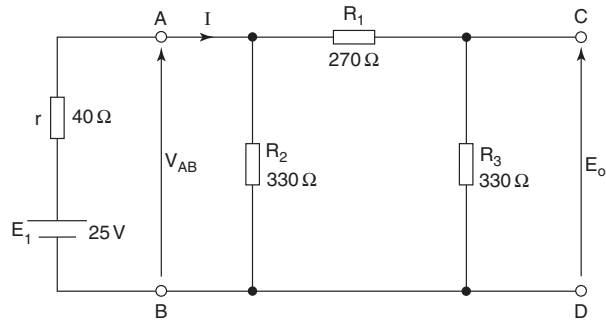


Fig. 6

$$R = r + \frac{R_2(R_1 + R_3)}{R_2 + (R_1 + R_3)} \text{ ohm}$$

Total resistance,

$$= 40 + \frac{330 \times 600}{330 + 600}$$

$$R = 252.9 \Omega$$

$$I = \frac{E_1}{R} \text{ amp} = \frac{25}{252.9} = 0.0989 \text{ A}$$

$$V_{AB} = E_1 - Ir \text{ volt} = 25 - (0.0989 \times 40)$$

$$V_{AB} = 21.05 \text{ V}$$

Resistors R_1 and R_3 act as a potential divider with V_{AB} applied across them, so

$$E_O = \frac{R_3}{R_3 + R_1} \times V_{AB} \text{ volt} = \frac{330}{600} \times 21.05$$

$E_O = 11.575 \text{ V}$, and the Thévenin equivalent circuit is as Fig. 7

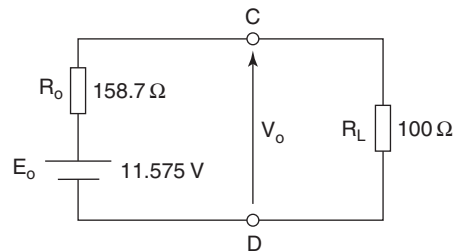


Fig. 7

$$V_o = \frac{R_L}{R_L + R_o} \times E_o \text{ volt} = \frac{100}{258.7} \times 11.575$$

$$V_o = 4.474 \text{ V Ans}$$

Looking in at terminals AB of the original circuit, the resistance 'seen' R_i is illustrated in Fig. 8, and may be obtained as follows:

$$R_{GH} = R_1 + \frac{R_3 R_L}{R_3 + R_L} \text{ ohm}$$

$$= 270 + \frac{330 \times 100}{330 + 100}$$

$$R_{GH} = 346.744 \Omega$$

$$R_i = \frac{R_{GH} R_2}{R_{GH} + R_2} \text{ ohm} = \frac{346.7 \times 330}{346.7 + 330}$$

$$R_i = 169.08 \Omega$$

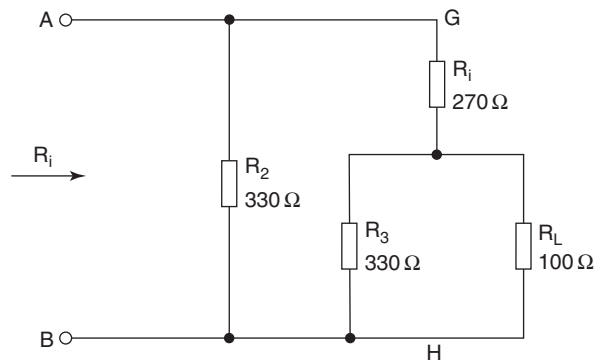


Fig. 8

So the circuit is equivalent to that shown in Fig. 9, from which:

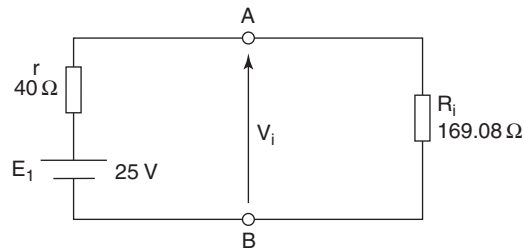


Fig. 9

$$V_i = \frac{R_i}{R_i + r} \times E_1 \text{ volt} = \frac{169.08}{209.08} \times 25$$

$$\text{so, } V_i = 20.217 \text{ V Ans}$$

$$(b) \text{ attenuation} = 20 \log \frac{V_O}{V_i} \text{ decibel}$$

$$= 20 \log \frac{4.474}{20.217}$$

$$\text{attenuation} = -13 \text{ dB Ans}$$

$$(c) P_O = \frac{V_O^2}{R_L} \text{ watt} = \frac{4.474^2}{100}$$

$$P_O = 0.2 \text{ W Ans}$$