

FORMULAE/REVISION HINTS FOR SECTION B

FURTHER NUMBER AND ALGEBRA

Factor theorem

If $x = a$ is a root of the equation $f(x) = 0$,

then $(x - a)$ is a factor of $f(x)$

Remainder theorem

If $(ax^2 + bx + c)$ is divided by $(x - p)$, the remainder will be: $ap^2 + bp + c$

or if $(ax^3 + bx^2 + cx + d)$ is divided by $(x - p)$, the remainder will be: $ap^3 + bp^2 + cp + d$

Arithmetic progression

If $a =$ first term and $d =$ common difference, then the arithmetic progression is:

$$a, a + d, a + 2d, \dots$$

The n th term is $a + (n - 1)d$

Sum of n terms, $S_n = \frac{n}{2} [2a + (n - 1)d]$

Geometric progression

If $a =$ first term and $r =$ common ratio, then the geometric progression is:

$$a, ar, ar^2, \dots$$

The n th term is: ar^{n-1}

Sum of n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$ or $\frac{a(r^n-1)}{(r-1)}$

If $-1 < r < 1$, $S_\infty = \frac{a}{(1-r)}$

Partial fractions

Provided that the numerator $f(x)$ is of less degree than the relevant denominator, the following identities are typical examples of the form of partial fractions used:

$$\frac{f(x)}{(x+a)(x+b)(x+c)} \equiv \frac{A}{(x+a)} + \frac{B}{(x+b)} + \frac{C}{(x+c)}$$

$$\frac{f(x)}{(x+a)^3(x+b)} \equiv \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+a)^3} + \frac{D}{(x+b)}$$

$$\frac{f(x)}{(ax^2+bx+c)(x+d)} \equiv \frac{Ax+B}{(ax^2+bx+c)} + \frac{C}{(x+d)}$$

Binomial series

$$(a+b)^n = a^n + na^{n-1}b + \frac{n(n-1)}{2!}a^{n-2}b^2 + \frac{n(n-1)(n-2)}{3!}a^{n-3}b^3 + \dots$$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \dots$$

Maclaurin's series

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!}f''(0) + \frac{x^3}{3!}f'''(0) + \dots$$

Hyperbolic functions

$$\sinh x = \frac{e^x - e^{-x}}{2} \quad \operatorname{cosech} x = \frac{1}{\sinh x} = \frac{2}{e^x - e^{-x}}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \quad \operatorname{sech} x = \frac{1}{\cosh x} = \frac{2}{e^x + e^{-x}}$$

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}} \quad \operatorname{coth} x = \frac{1}{\tanh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$\cosh^2 x - \sinh^2 x = 1 \quad 1 - \tanh^2 x = \operatorname{sech}^2 x \quad \operatorname{coth}^2 x - 1 = \operatorname{cosech}^2 x$$

Newton–Raphson iterative method

If r_1 is the approximate value for a real root of the equation $f(x) = 0$, then a closer approximation to the root, r_2 , is given by:

$$r_2 = r_1 - \frac{f(r_1)}{f'(r_1)}$$

Boolean algebra

Laws and rules of Boolean algebra

Commutative laws: $A + B = B + A$

$$A \cdot B = B \cdot A$$

Associative laws: $A + B + C = (A + B) + C$

$$A \cdot B \cdot C = (A \cdot B) \cdot C$$

Distributive laws: $A \cdot (B + C) = A \cdot B + A \cdot C$

$$A + (B \cdot C) = (A + B) \cdot (A + C)$$

Sum rules: $A + \bar{A} = 1$

$$A + 1 = 1$$

$$A + 0 = A$$

$$A + A = A$$

Product rules: $A \cdot \bar{A} = 0$

$$A \cdot 0 = 0$$

$$A \cdot 1 = A$$

$$A \cdot A = A$$

Absorption rules: $A + A \cdot B = A$

$$A \cdot (A + B) = A$$

$$A + \bar{A} \cdot B = A + B$$

De Morgan's Laws

$$\overline{A + B} = \bar{A} \cdot \bar{B}$$

$$\overline{A \cdot B} = \bar{A} + \bar{B}$$
