

FORMULAE/REVISION HINTS FOR SECTION E

GEOMETRY AND TRIGONOMETRY

Theorem of Pythagoras: $b^2 = a^2 + c^2$

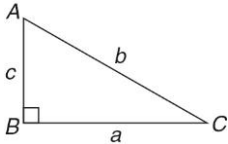


Figure FE1

$$\sin C = \frac{c}{b} \quad \cos C = \frac{a}{b} \quad \tan C = \frac{c}{a} \quad \sec C = \frac{b}{a} \quad \operatorname{cosec} C = \frac{b}{c} \quad \cot C = \frac{a}{c}$$

Trigonometric ratios for angles of any magnitude

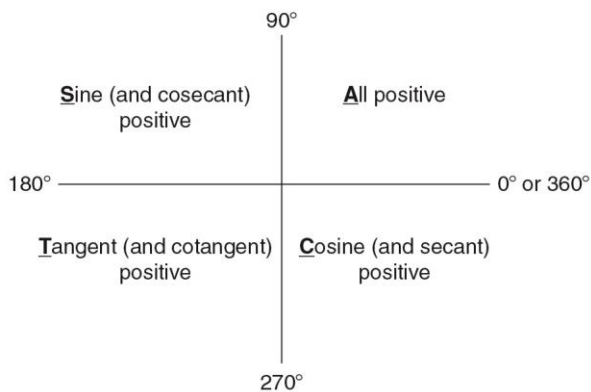


Figure FE2

For a **general sinusoidal function** $y = A \sin(\omega t \pm \alpha)$, then

A = amplitude ω = angular velocity = $2\pi f$ rad/s

$\frac{2\pi}{\omega}$ = periodic time T seconds $\frac{\omega}{2\pi}$ = frequency, f hertz

α = angle of lead or lag (compared with $y = A \sin \omega t$)

$$180^\circ = \pi \text{ rad} \qquad 1 \text{ rad} = \frac{180^\circ}{\pi}$$

Cartesian and polar coordinates

If coordinate $(x, y) = (r, \theta)$ then $r = \sqrt{x^2 + y^2}$ and $\theta = \tan^{-1} \frac{y}{x}$

If coordinate $(r, \theta) = (x, y)$ then $x = r \cos \theta$ and $y = r \sin \theta$

Triangle formulae

With reference to Figure FE3:

Sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$

Area of any triangle (i) $\frac{1}{2} \times \text{base} \times \text{perpendicular height}$

(ii) $\frac{1}{2} ab \sin C$ or $\frac{1}{2} ac \sin B$ or $\frac{1}{2} bc \sin A$

(iii) $\sqrt{[s(s-a)(s-b)(s-c)]}$ where $s = \frac{a+b+c}{2}$

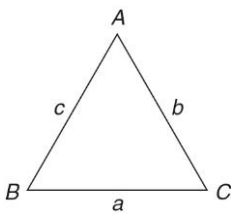


Figure FE3

Identities

$$\sec \theta = \frac{1}{\cos \theta} \qquad \operatorname{cosec} \theta = \frac{1}{\sin \theta} \qquad \cot \theta = \frac{1}{\tan \theta} \qquad \tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cos^2 \theta + \sin^2 \theta = 1 \qquad 1 + \tan^2 \theta = \sec^2 \theta \qquad \cot^2 \theta + 1 = \operatorname{cosec}^2 \theta$$

Compound angle formulae

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

If $R \sin(\omega t + \alpha) = a \sin \omega t + b \cos \omega t$,

$$\text{then } a = R \cos \alpha, \quad b = R \sin \alpha, \quad R = \sqrt{a^2 + b^2} \quad \text{and} \quad \alpha = \tan^{-1} \frac{b}{a}$$

Double angles $\sin 2A = 2 \sin A \cos A$

$$\cos 2A = \cos^2 A - \sin^2 A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

Products of sines and cosines into sums or differences

$$\sin A \cos B = \frac{1}{2} [\sin(A + B) + \sin(A - B)]$$

$$\cos A \sin B = \frac{1}{2} [\sin(A + B) - \sin(A - B)]$$

$$\cos A \cos B = \frac{1}{2} [\cos(A + B) + \cos(A - B)]$$

$$\sin A \sin B = -\frac{1}{2} [\cos(A + B) - \cos(A - B)]$$

Sums or differences of sines and cosines into products

$$\sin x + \sin y = 2 \sin\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2 \cos\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2 \cos\left(\frac{x+y}{2}\right) \cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2 \sin\left(\frac{x+y}{2}\right) \sin\left(\frac{x-y}{2}\right)$$
