

# FORMULAE/REVISION HINTS FOR SECTION M

## LAPLACE TRANSFORMS

### Laplace transforms

Function $f(t)$	Laplace transforms $\mathcal{L}\{f(t)\} = \int_0^{\infty} e^{-st} f(t) dt$
1	$\frac{1}{s}$
$k$	$\frac{k}{s}$
$e^{at}$	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2 + a^2}$
$\cos at$	$\frac{s}{s^2 + a^2}$
$t$	$\frac{1}{s^2}$
$t^n$ ( $n =$ positive integer)	$\frac{n!}{s^{n+1}}$
$\cosh at$	$\frac{s}{s^2 - a^2}$
$\sinh at$	$\frac{a}{s^2 - a^2}$
$e^{-at} t^n$	$\frac{n!}{(s+a)^{n+1}}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2 + \omega^2}$
$e^{-at} \cos \omega t$	$\frac{s+a}{(s+a)^2 + \omega^2}$
$e^{-at} \cosh \omega t$	$\frac{s+a}{(s+a)^2 - \omega^2}$
$e^{-at} \sinh \omega t$	$\frac{\omega}{(s+a)^2 - \omega^2}$

## The Laplace transforms of derivatives

*First derivative*      $\mathcal{L}\left\{\frac{dy}{dx}\right\} = s\{y\} - y(0)$      where  $y(0)$  is the value of  $y$  at  $x = 0$

*Second derivative*      $\mathcal{L}\left\{\frac{d^2y}{dx^2}\right\} = s^2\{y\} - s y'(0) - y''(0)$      where  $y'(0)$  is the value of  $\frac{dy}{dx}$  at  $x = 0$

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## Heaviside unit step function

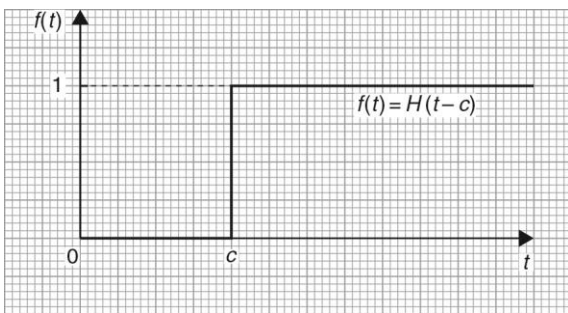


Figure M.1

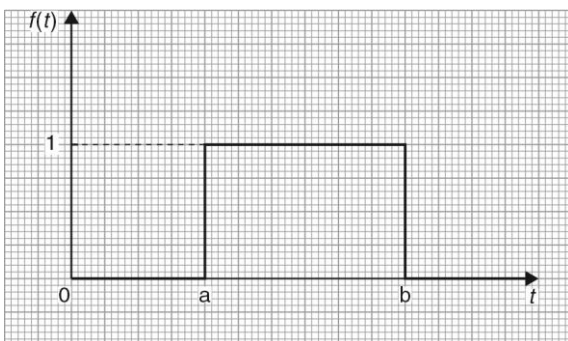


Figure M.2

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### Laplace transform of $H(t - c)$

$$\mathcal{L}\{H(t - c)\} = \frac{e^{-cs}}{s} \quad \text{and} \quad \mathcal{L}\{H(t)\} = \frac{1}{s}$$

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### Laplace transform of $H(t - c)f(t - c)$

$$\mathcal{L}\{H(t - c)f(t - c)\} = e^{-cs} F(s) \quad \text{where } F(s) = \mathcal{L}\{f(t)\}$$

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### Inverse Laplace transform of Heaviside functions

$$\text{if } F(s) = \mathcal{L}\{f(t)\}, \text{ then } e^{-cs} F(s) = \mathcal{L}\{H(t - c)f(t - c)\}$$

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