

FORMULAE/REVISION HINTS FOR SECTION N

FOURIER SERIES

Fourier series

If $f(x)$ is a **periodic function of period 2π** , then its Fourier series is given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$$

where, for the range $-\pi$ to $+\pi$.

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx \quad (n = 1, 2, 3, \dots)$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx dx \quad (n = 1, 2, 3, \dots)$$

Even functions

If $f(x)$ is **even** in the range $-\pi < x < \pi$, then $f(-x) = f(x)$ and the Fourier series has **no sine terms**, i.e.

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos nx$$

where $a_0 = \frac{1}{\pi} \int_0^{\pi} f(x) dx$

$$a_n = \frac{2}{\pi} \int_0^{\pi} f(x) \cos nx dx$$

Odd functions

If $f(x)$ is **odd** in the range $-\pi < x < \pi$, then $f(-x) = -f(x)$ and the Fourier series has **no constant term** and **no cosine terms**, i.e.

$$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$$

where

$$b_n = \frac{2}{\pi} \int_0^{\pi} f(x) \sin nx \, dx$$

If $f(x)$ is a **periodic function of period L** , then its Fourier series is given by:

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left\{ a_n \cos\left(\frac{2\pi nx}{L}\right) + b_n \sin\left(\frac{2\pi nx}{L}\right) \right\}$$

where for the range $-\frac{L}{2}$ to $+\frac{L}{2}$:

$$a_0 = \frac{1}{L} \int_{-L/2}^{L/2} f(x) \, dx$$

$$a_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \cos\left(\frac{2\pi nx}{L}\right) \, dx \quad (n = 1, 2, 3, \dots)$$

$$b_n = \frac{2}{L} \int_{-L/2}^{L/2} f(x) \sin\left(\frac{2\pi nx}{L}\right) \, dx \quad (n = 1, 2, 3, \dots)$$

For an even function, $a_0 = \frac{2}{L} \int_0^{L/2} f(x) \, dx$

$$a_n = \frac{4}{L} \int_0^{L/2} f(x) \cos\left(\frac{2\pi nx}{L}\right) \, dx$$

and

$$b_n = 0$$

For an odd function, $a_0 = 0$

$$a_n = 0$$

and

$$b_n = \frac{4}{L} \int_0^{L/2} f(x) \sin\left(\frac{2\pi nx}{L}\right) \, dx$$

Complex or exponential Fourier series

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{j\frac{2\pi nx}{L}} \quad \text{where} \quad c_n = \frac{1}{L} \int_{-L/2}^{L/2} f(x) e^{-j\frac{2\pi nx}{L}} dx$$

For even symmetry, $c_n = \frac{2}{L} \int_0^{\frac{L}{2}} f(x) \cos\left(\frac{2\pi nx}{L}\right) dx$

For odd symmetry, $c_n = -j \frac{2}{L} \int_0^{\frac{L}{2}} f(x) \sin\left(\frac{2\pi nx}{L}\right) dx$
