

Main formulae for Part 2 Electrical principles and technology

A.c. theory

$$T = \frac{1}{f} \text{ or } f = \frac{1}{T} \quad I = \sqrt{\left(\frac{i_1^2 + i_2^2 + i_3^2 + \dots + i_n^2}{n}\right)}$$

For a sine wave: $I_{AV} = \frac{2}{\pi} I_m$ or $0.637 I_m$

$$I = \frac{1}{\sqrt{2}} I_m \text{ or } 0.707 I_m$$

Form factor = $\frac{\text{r.m.s.}}{\text{average}}$ Peak factor = $\frac{\text{maximum}}{\text{r.m.s.}}$

General sinusoidal voltage: $v = V_m \sin(\omega t \pm \phi)$

Single-phase circuits

$$X_L = 2\pi fL \quad X_C = \frac{1}{2\pi fC}$$

$$Z = \frac{V}{I} = \sqrt{(R^2 + X^2)}$$

Series resonance: $f_r = \frac{1}{2\pi\sqrt{LC}}$

$$Q = \frac{V_L}{V} \text{ or}$$

$$\frac{V_C}{V} = \frac{2\pi f_r L}{R} = \frac{1}{2\pi f_r C R} = \frac{1}{R} \frac{L}{C}$$

$$Q = \frac{f_r}{f_2 - f_1} \text{ or}$$

$$(f_2 - f_1) = \frac{f_r}{Q}$$

Parallel resonance (LR-C circuit):

$$f_r = \frac{1}{2\pi} \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$I_r = \frac{VRC}{L} \quad R_D = \frac{L}{CR}$$

$$Q = \frac{2\pi f_r L}{R} = \frac{I_C}{I_r}$$

$$P = VI \cos \phi \text{ or}$$

$$I^2 R \quad S = VI$$

$$Q = VI \sin \phi \quad \text{power factor} = \cos \phi = \frac{R}{Z}$$

D.c. transients

C-R circuit $\tau = CR$

Charging: $v_C = V(1 - e^{-(t/CR)})$
 $v_r = Ve^{-(t/CR)}$

$$i = Ie^{-(t/CR)}$$

Discharging: $v_C = v_R = Ve^{-(t/CR)}$
 $i = Ie^{-(t/CR)}$

L-R circuit $\tau = \frac{L}{R}$

Current growth: $v_L = Ve^{-(Rt/L)}$

$$v_R = V(1 - e^{-(Rt/L)})$$

$$i = I(1 - e^{-(Rt/L)})$$

Current decay: $v_L = v_R = Ve^{-(Rt/L)}$
 $i = Ie^{-(Rt/L)}$

Operational amplifiers

$$\text{CMRR} = 20 \log_{10} \frac{\text{differential voltage gain}}{\text{common mode gain}} \text{ dB}$$

Inverter: $A = \frac{V_o}{V_i} = -\frac{R_f}{R_i}$

Non-inverter: $A = \frac{V_o}{V_i} = 1 + \frac{R_f}{R_i}$

Summing: $V_o = -R_f \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$

Integrator: $V_o = -\frac{1}{CR} \int V_i dt$

Differential If $V_1 > V_2$: $V_o = (V_1 - V_2) - \frac{R_f}{R_1}$

If $V_2 > V_1$:

$$V_o = (V_2 - V_1) \frac{R_3}{R_2 + R_3} + \frac{R_f}{R_1}$$

Three-phase systems

Star: $I_L = I_p \quad V_L = \sqrt{3} V_p$

Delta: $V_L = V_p \quad I_L = \sqrt{3} I_p$

$$P = \sqrt{3} V_L I_L \cos \phi \text{ or } P = 3 I_p^2 R_p$$

Two-wattmeter method $P = P_1 + P_2$

$$\tan \phi = \sqrt{3} \frac{(P_1 - P_2)}{(P_1 + P_2)}$$

Transformers

$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{I_2}{I_1} \quad I_0 = \sqrt{(I_M^2 + I_C^2)}$$

$$I_M = I_0 \sin \phi_0 \quad I_C = I_0 \cos \phi_0$$

$$E = 4.44 f \Phi_m N \quad \text{Regulation} = \left(\frac{E_2 - E_1}{E_2} \right) \times 100\%$$

$$\text{Equivalent circuit: } R_e = R_1 + R_2 \left(\frac{V_1}{V_2} \right)^2$$

$$X_e = X_1 + X_2 \left(\frac{V_1}{V_2} \right)^2$$

$$Z_e = \sqrt{(R_e^2 + X_e^2)}$$

$$\text{Efficiency, } \eta = 1 - \frac{\text{losses}}{\text{input power}}$$

$$\text{Output power} = V_2 I_2 \cos \phi_2$$

$$\text{Total loss} = \text{copper loss} + \text{iron loss}$$

$$\text{Input power} = \text{output power} + \text{losses}$$

$$\text{Resistance matching: } R_1 = \left(\frac{N_1}{N_2} \right)^2 R_L$$

D.c. machines

$$\text{General e.m.f. } E = \frac{2p\Phi nZ}{c} \propto \Phi \omega$$

$$(c = 2 \text{ for wave winding, } c = 2p \text{ for lap winding})$$

$$\text{Generator: } E = V + I_a R_a$$

$$\text{Efficiency, } \eta = \left(\frac{VI}{VI + I_a^2 R_a + I_f V + C} \right) \times 100\%$$

$$\text{Motor: } E = V - I_a R_a$$

$$\text{Efficiency, } \eta = \left(\frac{VI - I_a^2 R_a - I_f V - C}{VI} \right) \times 100\%$$

$$\text{Torque} = \frac{EI_a}{2\pi n} = \frac{p\Phi Z I_a}{\pi c} \propto I_a \Phi$$

Three-phase induction motors

$$n_s = \frac{f}{p} \quad s = \left(\frac{n_s - n_r}{n_s} \right) \times 100$$

$$f_r = s f \quad X_r = s X_2$$

$$I_r = \frac{E_r}{Z_r} = \frac{s \left(\frac{N_2}{N_1} \right) E_1}{\sqrt{[R_2^2 + (s X_2)^2]}} \quad s = \frac{I_r^2 R_2}{P_2}$$

$$\text{Efficiency, } \eta = \frac{P_m}{P_l}$$

$$= \frac{\text{input} - \text{stator loss} - \text{rotor copper loss} - \text{friction and windage loss}}{\text{input power}}$$

$$\text{Torque, } T = \left(\frac{m(N_2/N_1)^2}{2\pi n_s} \right) \left(\frac{s E_1^2 R_2}{R_2^2 + (s X_2)^2} \right) \propto \frac{s E_1^2 R_2}{R_2^2 + (s X_2)^2}$$