Work Problems Chapter 6

Suppose that in the population of children in the United States, the average number of minutes per day spent exercising is 45 with a standard deviation of 10. I select a random sample of 16 children and find that they spend an average of 50 minutes exercising with a standard deviation of 12. Please use this information for answer the following questions.

1. Calculate two standard errors of the mean, one using the population standard deviation and the other using the sample standard deviation. Describe what each of these standard errors tells you.

   Standard error of the mean (population s.d.): 10/√16 → 10/4 = 2.50. This tells us that the average difference between the population mean and the sample means, when the samples are randomly selected and *n* = 16, is 2.50 minutes of exercise per day.

   Standard error of the mean (sample s.d.): 12/√16 → 12/4 = 3.00. This tells us that the average difference between the population mean and the sample means, when the samples are randomly selected and *n* = 16, is 3.00 minutes of exercise per day.

2. Using the standard error of the mean that you calculated using the population standard deviation, determine the probability of getting a difference between the sample mean and the population mean that is this large *by chance*. (**Note:** This is a *z* score problem.)

   ***z* = (50 - 45)/2.50 → 5/2.50 = 2.00**
   From Appendix A: *p* = 1 - .9938 = .0062.

   a. Describe your results. What does this probability tell you?

   This means that 0.61% of random samples (*n* = 16) would be expected to have a mean of 50 minutes of exercise per day or less. This is a very small probability.

3. Using the standard error of the mean that you calculated using the sample standard deviation, develop an estimate of the probability of getting a difference between the sample mean and the population mean that is this large *by chance*. (**Note:** This is a *t* value problem.)

   ***t* = (50 - 45)/3.00 → 5/3.00 = 1.67. With a sample size of 16, the degrees of freedom is 16 – 1 = 15. Looking at the values in Appendix B, with 15 degrees of freedom, we can see that our calculated *t* value of 1.67 is between the first two columns of *t* values (i.e., between 1.341 and 1.753. Looking up at the top of these two columns (in the "alpha level for 2-tailed test") we can see that between 10% and 20% of randomly selected samples of this size would be expected to produce a *t* value of this size or larger, by chance, either in the positive or negative direction (i.e., 2**

tailed). If we are only interested in the positive *t* values, we would look at the 1-tailed probabilities and conclude that between 5% and 10% of randomly selected samples would be expected to produce a *positive t* value of this size or larger, by chance.

   a. Describe your results. What does this probability tell you?

These probabilities, both 1-tailed and 2-tailed, tell us that the probability of randomly selecting a sample with a mean this different from the population mean is fairly small (i.e., less than 20%), but not tiny.

4. Compare the two standard errors that you calculated in Questions 2 and 3.

   a. Why are they different?

   Because the sample standard deviation was larger than the population standard deviation, the standard error of the mean using the sample standard deviation was larger than the standard error of the mean that used the population standard deviation.

   b. Why are the probabilities that you found in each problem different?

5. Suppose that instead of a sample size of 16, you had a sample size of 36.

   a. How would this affect the size of the standard error of the mean?

   Larger sample sizes produce smaller standard errors of the mean, assuming the standard deviation size remains the same.

   b. Why does it make sense that a larger sample size has this affect on the size of the standard error of the mean?

   In general, the larger the random sample, the better the sample should represent the population. A more accurate representation of the population means less error in the sample.

6. Using this larger sample size of 36, and the standard deviation from the population, calculate a new probability of obtaining a difference between the sample mean and the population mean.

Standard error of the mean (population s.d.): 10/√36 → 10/6 = 1.67

***z* = (50 - 45)/1.67 → 5/1.67 = 2.99**
From Appendix A: *p* = 1 - .9986 = .0014.

a. Compare this probability with the probability that you obtained in Question 2. How do they differ?

The *p* value obtained in Question 2 was .0062. This is a little bit larger than the *p* value of .0014 obtained in Question 6, using the slightly smaller standard error of the mean.

7. Explain why it makes sense that the larger sample size produces the difference in the two probabilities that you obtained in Question 2 and Question 5.

   Larger sample sizes produce smaller standard errors (assuming the standard deviation is the same). Smaller standard errors produce larger *z* scores, and the probability of obtaining larger *z* scores by change is smaller. We saw this when comparing the results for Question 2 with those for Question 6.

8. Explain how you would decide whether to calculate a *z* score or a *t* value to determine the probability of randomly selecting a sample with a given mean by chance from a population?

   If we know the population standard deviation, we use it in the standard error formula and calculate a *z* score. If not, we calculate a *t* value.