

Section Chapter 1: Introduction to Income

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1 Income comparisons

The first dimension of development that we consider in the course is Income and Income Growth. Income is what gives a household the monetary capacity to consume, invest, or save. Thus, income *growth* has to be a key instrument to achieve higher wellbeing. Income growth may reduce poverty directly through income generation by the poor or make it more politically feasible to do so through taxation and redistribution.

Comparisons over time Are countries becoming better off over time? To compare levels of well-being, we need to adjust GDPpc for changes in price levels. We do this by deflating GDPpc in each year using a price deflator, typically the consumer price index (CPI) or GDP deflator. In this course we focus on adjustments using a fixed base year.¹ Assuming CPI in the base year is equal to 100:

$$\text{RealGDPpc} = \frac{\text{CPI}_{base}}{\text{CPI}_{curr}} (\text{NomGDPpc}) = 100 \times \frac{\text{NomGDPpc}}{\text{CPI}_{curr}} \quad (1)$$

Notice that this formula is general. In principle, we can use *a single CPI series* to calculate real GDP in constant 2005 dollars or constant 2003 dollars if that's what we want.

Exercise using GDP data from Brazil:

Variable	Formula	2000	2005	2010
GDPpc current reales	Data	6,759	11,535	19,312
Growth past 5 years- nominal	$\dot{Y}_{Nom} = \frac{\text{NomGDPpc}_T - \text{NomGDPpc}_{T-5}}{\text{NomGDPpc}_{T-5}}$	n/a	71%	67%
CPI (2000 base year)	Data	100	159	224
Real GDPpc (2000 reales)	$100 \times \frac{\text{NomGDPpc}}{\text{CPI}}$	6,759	7,269	8,617
Growth past 5 years- real	$\dot{Y}_{Real} = \frac{\text{RealGDPpc}_T - \text{RealGDPpc}_{T-5}}{\text{RealGDPpc}_{T-5}}$	n/a	8%	19%
CPI (2005 base year)	Data	66	100	126
Real GDPpc (2005 reales)	$100 \times \frac{\text{NomGDPpc}}{\text{CPI}}$	10,240	11,535	15,326
Growth past 5 years- real	$\dot{Y}_{Real} = \frac{\text{RealGDPpc}_T - \text{RealGDPpc}_{T-5}}{\text{RealGDPpc}_{T-5}}$	n/a	13%	33%

Study questions:

¹Different methods are sometimes used to calculate price indices and perform GDP adjustments. For example, the World Bank data you will use in your problem sets use CPIs that are typically *Laspeyeres indices*, but you may also see *Paasche indices*. For a given price index, we might use different methods to adjust GDP. In addition to fixed base year adjustments, you may also hear about *chain-weighted* GDP adjustments. The details can be important but are outside the scope of this class.

- Why are the estimated growth rates different using year 2000 vs. year 2005 CPI values? Which are more correct?

Accounting for compounded growth rates Recall from your math classes that relatively small percentage changes, such as interest or growth rates, have a large effect over time due to compounding. This is especially important to keep in mind when thinking about income growth rates for countries.

If income Y grows at a constant growth rate g for T periods, we would have

$$Y_T = Y_0 (1 + g)^T \quad (2)$$

Given values for Y_0, Y_1, T , we can solve for g , the compound average growth rate over the period:

$$g = \left(\frac{Y_T}{Y_0} \right)^{\frac{1}{T}} - 1 \quad (3)$$

Using the Brazil data to obtain values for Y_{2000} , Y_{2005} , and Y_{2010} , we have:

Variable	Formula	2000	2005	2010
Real GDPpc (2005 reales)	(calculated above)	10,240	11,535	15,326
Annual growth past 5 years - Real	$\left(\frac{\text{RealGDPpc}_T}{\text{RealGDPpc}_{T-5}} \right)^{\frac{1}{5}} - 1$	n/a	2.4%	5.8%

One way to put these growth rates into perspective is to think about how long it would take for growth rates to double or to reach a given income threshold. In order to solve for the exponent T , as usual we want to take logarithms. Taking logs of both sides of equation (2):

$$\ln Y_T = \ln Y_0 + T \ln(1 + g) \approx \ln Y_0 + Tg \quad (4)$$

where the first equality is by definition, but we simplify by exploiting that for small g , $\ln(1 + g) \approx g$.² Re-arranging, we have

$$T = \frac{\ln(Y_T/Y_0)}{g} \quad (5)$$

To calculate the number of years required for income to double, we are interested in the value of $\frac{\ln 2}{g} \approx \frac{70}{g \times 100}$. Calculating the number of years needed to reach an arbitrary threshold Y_T is analogous.

Study questions:

- How long will it take for Brazil to double per capita GDP if it maintains the same average annual growth rate as between 2005 and 2010?

Comparisons across countries National data is sometimes given in local currency units (LCU). To compare incomes across countries, we need to convert to a common unit. We have two options:

²Purely as an aside, some of you may be curious where this approximation comes from. Notice that a Taylor series for the quantity $\ln(1 + x)$ evaluated at $x = 0$ tells us that

$$\ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots \quad \text{for } |x| \leq 1$$

If x is small, then all the higher order terms are close to zero, and we can write that $\ln(1 + x) \approx x$.

1. *Official exchange rate:* The exchange rate is expressed as $e = \frac{LCU}{\$1}$, where LCU is units of local currency. Therefore, we can compare GDPpc across countries in dollar terms by calculating:

$$GDP_{pc}^{\$} = \frac{1}{e} \times GDP_{pc}^{LCU} \quad (6)$$

2. *Purchasing Power Parity (PPP) adjusted exchange rate:* One dollar does not buy the same in Berkeley as in Nairobi, so we need to adjust for the relative purchasing power of currencies. PPPe is calculated for each country based on the number of LCU required to buy the same amount of goods as \$1 buys in the US.

$$PPP_{adjusted}GDP_{pc}^{\$} = \frac{1}{PPP_e} \times GDP_{pc}^{LCU} \quad (7)$$

	2010GDPpc LCU	e	PPPe	GDPpc\$	PPPGDPpc\$
Azerbaijan	1,637	0.802	0.474	2,041	3,453
Cote d'Ivoire	219,323	495.3	305.8	442	717
Haiti	9,599	39.80	23.99	241	400
Sweden	55,7178	7.21	9.03	77,278	61,703
United States	46,702	1	1	46,702	46,702

Study questions:

- For which countries is $PPPe$ greater than e ? Why?