CHAPTER 10 MEMBRANE THEORY FOR THIN-WALLED CIRCULAR CYLINDERS AND SPHERES

EXERCISE 44, Page 234

1. A thin-walled circular cylinder of internal diameter 10 m is subjected to a maximum internal pressure of 50 bar. Determine its wall thickness, if η_c = 0.8 and η_L = 0.4.

Assume that the maximum permissible stress is 300 MPa. (Note that 1 bar = 10^5 Pa.)

$$\sigma_H = \frac{PR}{\eta_L t}$$

from which, $$t = \frac{PR}{\eta_L \sigma_H} = \frac{50\times10^5\times\left(\frac{10}{2}\right)}{0.4\times300\times10^6}$$

i.e. **wall thickness, *t* = 0.208 m = 208 mm**

$$\sigma_L = \frac{PR}{2\eta_c t}$$

from which, $$t = \frac{PR}{2\eta_c \sigma_L} = \frac{50\times10^5\times5}{2\times0.8\times300\times10^6}$$

i.e. **wall thickness, *t* = 0.052 m = 52 mm**

and **design wall thickness = 208 mm** – the larger of the two design thicknesses

2. Assuming that a submarine pressure hull of external diameter 12 m, and wall thickness 5 cm, can be designed using internal pressure theory, and neglecting failure due to buckling, determine its permissible diving depth for a safety factor of 2.

Assume that the density of sea water = 1020kg/m^3, g = 9.81 m/s^2, η_L = 0.48, η_c = 0.5 and the yield stress of the material of construction is 400 MPa.

$$\sigma_H = \frac{PR}{\eta_L t}$$

from which, $$P = \frac{\eta_L \sigma_H t}{R} = \frac{0.48 \times 400 \times 10^6 \times 5 \times 10^{-2}}{12/2}$$

i.e. **P = 1.6 MPa**

$$P = \rho g h$$

from which, $$h = \frac{P}{\rho g}$$

$$\sigma_L = \frac{PR}{2\eta_c t}$$

from which, $$P = \frac{2\eta_c \sigma_L t}{R} = \frac{2 \times 0.5 \times 400 \times 10^6 \times 5 \times 10^{-2}}{12/2}$$

i.e. **P = 3.333 MPa**

Hence, design pressure = 1.6 MPa

Therefore, **diving depth**, $h = \frac{P}{\rho g} = \frac{1.6 \times 10^6}{1020 \times 9.81}$ = **159.9 m**

3. Design the wall thickness for an aircraft fuselage of internal diameter 6 m, subjected to an internal pressure of 0.5 bar. Assume that η_L = 0.48, η_c = 0.5 and σ_{yp} = 200 MPa. (Note that 1 bar = 10^5 Pa.)

$$\sigma_H = \frac{PR}{\eta_L t}$$

from which, $$t = \frac{PR}{\eta_L \sigma_H} = \frac{0.5 \times 10^5 \times \left(\frac{6}{2}\right)}{0.48 \times 200 \times 10^6}$$

i.e. **wall thickness, t = 1.56×10^{-3} m = 1.56 mm**

$$\sigma_L = \frac{PR}{2\eta_c t}$$

from which, $$t = \frac{PR}{2\eta_c \sigma_L} = \frac{0.5\times10^5 \times 3}{2\times0.5\times200\times10^6}$$

i.e. **wall thickness, $t = 7.5\times10^{-4}$ m = 0.75 mm**

Hence, **the design wall thickness = 1.56 mm** – the larger of the two design thicknesses

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EXERCISE 45, Page 239

1. A boiler, which may be assumed to be composed of a thin-walled cylindrical shell body of internal diameter 4 m, is blocked off by two thin-walled hemispherical dome ends. Neglecting the effects of discontinuity at the intersection between the dome and cylinder, determine suitable thicknesses for the cylindrical shell body and the hemispherical dome ends.

The following may be assumed:

maximum permissible stress = 100 MN/m^2, design pressure = 1 MPa,

longitudinal joint efficiency = 75% and circumferential joint efficiency = 50%

For the cylinder

$$\sigma_H = \frac{pr}{\eta_L t} \quad 1$$

from which, $$\boldsymbol{t} = \frac{pr}{\eta_L \sigma_H} = \frac{1\times10^6 \times 2}{0.75\times100\times10^6} 2 = 0.027 \text{ m} = \textbf{2.67 cm}$$

For the dome

$$\boldsymbol{t} = \frac{pr}{2\times\eta_L \sigma_H} = \frac{1\times10^6 \times 2}{2\times0.5\times100\times10^6} 34 = 0.02 \text{ m} = \textbf{2 cm}$$

2. If the vessel of Problem 1 is just filled with water, determine the additional water that is required to be pumped in, to raise the pressure by 1 MPa. The following may be assumed to apply:

Length of cylindrical portion of vessel = 6 m, $E = 2\times10^{11}\,\text{N/m}^2$, $\nu = 0.3$ and $K = 2\times10^9\,\text{N/m}^2$

For the cylinder

$$\boldsymbol{\sigma_H} = \frac{1\times10^6 \times 2}{0.0267} 5 = \textbf{74.91 MN/m}^\mathbf{2}$$

$\boldsymbol{\sigma_L}$ = **37.455 MPa**

$$\varepsilon_H = \frac{1}{E} 6(\sigma_H - \upsilon\sigma_L) = 3.184\times10^{-4}$$

$$w = 6.367\times10^{-4}\,\text{m}$$

$$\boldsymbol{\delta V_1 = 2\pi RwL = 0.048 m^3}$$

$$\sigma_L = 37.455 \text{ MN/m}^2$$

$$\varepsilon_L = \frac{1}{E} 7(\sigma_L - \upsilon\sigma_H) = 7.491 \times 10^{-5}$$

$$u = 4.495 \times 10^{-4}$$

$$\boldsymbol{\delta V_2 = \pi R^2 u = 0.0056 m^3}$$

For the sphere

$$\sigma = 37.455 \text{ MN/m}^2$$

$$\varepsilon = \frac{1}{E} 8(\sigma - \upsilon\sigma) = 1.311 \times 10^{-4}$$

$$w = 2.622 \times 10^{-4}$$

$$\boldsymbol{\delta V_3 = 4\pi R^2 w = 0.0132 m^3}$$

For the water

$$\text{Volume} = \pi R^2 L = \frac{4}{3} 9\, \pi R^3$$

$$= 75.4 + 33.5$$

i.e. **Volume = 108.91 m^3**

$$\delta V_4 = \frac{pV}{K} 10 = 0.055$$

$$\delta V = \delta V_1 + \delta V_2 + \delta V_3 + \delta V_4$$

i.e. **δV = 0.122 m^3**

3. A copper pipe of internal diameter 1.25 cm and wall thickness 0.16 cm is to transport water from a tank that is situated 30 m above it. Determine the maximum stress in the pipe, given the following: density of water = 1000 kg/m3, g = 9.81 m/s^2.

$$p = \rho g h = 1000 \times 9.81 \times 30 = 0.294 \text{ MPa}$$

Hence, **maximum stress,** $\sigma_H = \dfrac{pr}{t} = \dfrac{0.294\times10^6 \times \dfrac{0.0125}{2}}{0.16\times10^{-2}}$ = **1.148 MN/m^2**

4. What would be the change in diameter of the pipe of Problem 3 due to the applied head of water? Assume that for copper: $E = 1\times10^{11}\,\text{N/m}^2$, v = 0.33 and $K = 2\times10^9\,\text{N/m}^2$.

$$\frac{w}{r} = \varepsilon_H = \frac{1}{E}(\sigma_H - v\sigma_L)$$

Now $r = 0.625\times10^{-2}$

$$w = 11\frac{r}{E}(1.15 - 0.33 \times 1.15/2)\times10^6$$

i.e. $w = 6\times10^{-8}$

$\delta = 12\times10^{-8}$

i.e. $\boldsymbol{\delta} = 1.2\times10^{-7}$

= **0.12 micrometres**

5. A thin-walled spherical pressure vessel of 1 m internal diameter is fed by a pipe of internal diameter 3 cm and wall thickness 0.16 cm. Assuming that the material of construction of the spherical pressure vessel has a yield stress of 0.7 of that of the pipe, determine the wall thickness of the spherical shell.

$$\sigma = \frac{pr}{t} = \frac{0.294\times10^6 \times \dfrac{0.0125}{2}}{0.16\times10^{-2}}\,12$$

Therefore, $\dfrac{\sigma}{p} = 9.37513$

For the spherical shell

$$\frac{\sigma}{p} = 0.7 \times 9.375 = 6.5615$$

Now, $\dfrac{\sigma}{p} = \dfrac{r}{2t}$

Hence, $6.5625 = \dfrac{0.5}{2t}14$

from which, **wall thickness, *t*** = 0.0381 m = **3.81 cm**

6. A spherical pressure vessel of internal diameter 2 m is constructed by bolting together two hemispherical domes with flanges. Assuming that the number of bolts used to join the two hemispheres together is 12, determine the wall thickness of the dome and the diameter of the bolts, given the following: Maximum applied pressure = 0.7 MPa,

Permissible stress in spherical shell = 50 MPa

Permissible stress in bolts = 200 MPa

$$\sigma = \frac{pr}{t}$$

from which, $\mathbf{t} = \dfrac{pr}{t} = \dfrac{0.7 \times 1}{2 \times 50} = 7 \times 10^{3}\,\text{m} =$ **0.70 m**

Load on bolts = $\pi r^2 p = 2.2 \times 10^{6}$ MN

$$12 \times \sigma_b \times \frac{\pi d^2}{4} = 2.2 \times 10^{6}$$

i.e. $d^2 = \dfrac{2.2 \times 10^{6} \times 4}{12 \times 200 \times 10^{6} \times \pi}15$

i.e. **wall thickness of the dome, *d*** = 0.0342 m = **3.42 cm**

7. A thin-walled circular cylinder, blocked off by inextensible end places, contains a liquid under zero gauge pressure. Show that the additional liquid that is required to be pumped into the vessel, to raise its internal gauge pressure by *P*, is the same under the following two conditions:

(a) when axial movement of the cylinder is completely free, and

(b) when the vessel is totally restrained from axial movement.

It may be assumed that Poisson's ratio (v) for the cylinder material is 0.25.

(a) $$\sigma_H = \frac{pR}{t}$$

$$\sigma_L = \frac{pR}{2t}$$

$$\varepsilon_H = \frac{w}{R} = \frac{1}{E}(\sigma_H - v\sigma_L)$$

$$= \frac{pR}{tE}(1 - v/2)\ 16$$

i.e. $$w = \frac{pR^2}{tE}(1 - 0.125)$$

and $$\boldsymbol{w = 0.875\left(\frac{pR^2}{tE}\right)}$$

$$\frac{u}{L} = \frac{pR}{tE}(1/2 - v)$$

i.e. $$u = \frac{pRL}{tE}(1/2 - 0.25)$$

i.e. $$\boldsymbol{u = \frac{pRL}{tE} \times 0.25}$$

$$\delta V_1 = \frac{2\pi rl \times 0.875 pR^2}{tE}\ 17$$

i.e. $$\boldsymbol{\delta V_1 = 5.498}$$

$$\delta V_2 = \pi R^2 u$$

$$= \frac{\pi R^2 \times pRL \times 0.25}{tE}\ 18$$

i.e. $$\delta V_2 = \frac{0.785 \times pR^3 L}{tE} \qquad 19$$

$$\delta V_3 = \frac{pV}{K} 20$$

i.e. $$\boldsymbol{\delta V = \frac{6.283\, pR^3 L}{t\, E} + \frac{pV}{K}} \qquad 21$$

(b)

$\sigma_H = \frac{p\,R}{t}$ $\sigma_L = 0$ u = 0

$$w = \frac{R}{E}(\sigma_H)\ 22$$

$$\boldsymbol{w = \frac{pr^2}{t\, E}}$$

$\delta V_1 = 2\pi R \times w \times L = \mathbf{6.283\, pR^3}$

$\delta V_2 = 0$

$\delta V_3 = \frac{pV}{K}$

i.e. $$\boldsymbol{\delta V = \frac{6.283\, pR^3 L}{t\, E} + \frac{pV}{K}}$$

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EXERCISE 46, Page 242

1. A circular cylinder is to be blocked off by hemispherical ends and the whole is subjected to an internal pressure. Given that the internal diameter of the cylinder and hemisphere are 5 m, and the internal pressure is 2.5 bar, and if the joint efficiency is 100%, determine suitable thicknesses for the cylinder and hemispherical ends, assuming that the cylinder ends deflect the same as the hemispherical ends. Assume that the yield stress = 200 MPa and $v = 0.3$. (Note that 1 bar = 10^5 Pa.)

$$\sigma = \frac{PR}{\eta t_c} \quad \text{from which,} \quad t_c = \frac{PR}{\eta\sigma}$$

i.e. $$t_c = \frac{PR}{\eta\sigma} = \frac{2.5\times10^5 \times \left(\frac{5}{2}\right)}{1\times200\times10^6} = 3.125\times10^{-3}\,\text{m}$$

i.e. **cylinder thickness, t_{cyl} = 3.125 mm**

From the formula, $t_s = 0.412 t_c = 0.412 \times 3.125$ (from equation (10.25) in the textbook)

i.e. **hemispherical end thickness = 1.288 mm**
