CHAPTER 12 THEORIES OF ELASTIC FAILURE

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1. A submarine pressure hull, which may be assumed to be a long thin-walled circular cylinder, of external diameter 10 m and wall thickness 5 cm, is constructed from high tensile steel. Assuming that buckling does not occur, determine the maximum permissible diving depths that the submarine can achieve, without suffering elastic failure, based on the five major theories of yield and given the following: $\sigma_{yp} = -\sigma_{ypc} = 400 \text{ MN/m}^2$ $\quad v = 0.3$

Density of water = 1020 kg/m^3 $\quad g = 9.81 \text{ m/s}^2$

$\sigma_{ypc} = \sigma_{yp}$

Pressure, p = ρgh = 1020 × 9.81 × h = 10006 h = 0.01 h MPa

$\sigma_1 = -p$

$\sigma_2 = -\dfrac{pR}{2t} = -50p1$

$\sigma_3 = -\dfrac{pR}{t} = -100p2$

Maximum principal stress theory

$-100\,p = -400$ or $p = 4 = 0.01\,h$

i.e. **h** = 4/0.01 = **400 m**

Maximum principal strain

$\sigma_3 - \upsilon(\sigma_1 + \sigma_2) = \sigma_{ypc}$

$-100\,p - \upsilon(-p - 50\,p) = -400$

i.e. $-100\,p + 0.3(51)p = -400$

or $-84.7\,p = -400$

i.e. p = 400/84.7 = 4.723 = 0.01 h

from which, **h = 472.3 m**

Total strain energy theory

$$p^2 + 2500\, p^2 + 10000\, p^2 - 0.6\,(50 + 100 + 5000)\, p^2 = 400^2$$

or $$9411\, p^2 = 400^2$$

from which, $$p = \sqrt{\frac{400^2}{9411}} = 4.123 = 0.01\, h$$

i.e. $$\mathbf{h = 412.3\ m}$$

Maximum shear stress theory

$$\sigma_1 - \sigma_3 = \sigma_{yp}$$

i.e. $$-p + 100\, p = 400$$

or $$p = 4.04 = 0.01\, h$$

i.e. $$\mathbf{h = 404\ m}$$

Shear strain energy theory

$$(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 = 2\sigma_{yp}^{\ 2}$$

i.e. $$(49p)^2 + (99p)^2 + (50p)^2 = 2 \times 400^2$$

i.e. $$14702\, p^2 = 2 \times 400^2$$

and $$p = \sqrt{\frac{2 \times 400^2}{14702}} = 4.665 = 0.01\, h$$

i.e. $$\mathbf{h = 466.5\ m}$$

2. A circular-section torsion specimen, of diameter 2 cm, yields under a pure torque of 0.25 kN m. What is the shear stress due to yield? What is the yield stress according to (a) Tresca, (b) Hencky-von Mises? What is the ratio τ_{yp} / σ_{yp} according to these two theories?

Polar second moment of area, $J = \dfrac{\pi \times \left(2 \times 10^{-2}\right)^4}{32} = 1.571 \times 10^{-8}\, \mathrm{m}^4$

Yield stress, $\tau = \dfrac{T\,r}{J} = \dfrac{0.25 \times 10^3 \times 1 \times 10^{-2}}{1.571 \times 10^{-8}}$ = **159.1 MPa**

σ_1 = 159.1 MPa, and σ_2 = – 159.1 MPa

(a) **Tresca**

$\boldsymbol{\sigma_{yp}}$ = 159.1 × 2 = **318.3 MPa**

(b) **Hencky-von Mises**

$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_{yp}^2$

$159.1^2 + 159.1^2 + 159.1^2 = \sigma_{yp}^2$

i.e. $75938.43 = \sigma_{yp}^2$

and $\boldsymbol{\sigma_{yp}}$ **= 275.6 MPa**

(c) **Tresca ratio** = $\dfrac{159.1}{318.3}$ = **0.50**3

Hencky-von Mises ratio = $\dfrac{159.1}{275.6}$ 4= **0.577**

3. A shaft of diameter 0.1 m is found to yield under a torque of 30 kN m. Determine the pure bending moment that will cause a similar shaft, with no torque applied to it, to yield, assuming that the Tresca theory applies. What would be the bending moment to cause yield if the Hencky-von Mises theory applied?

Polar second moment of area, J = $\dfrac{\pi\times(0.1)^4}{32} = 9.817\times10^{-6}\,\mathrm{m}^4$

Now $\dfrac{\tau}{r} = \dfrac{T}{J}$ 5

from which, $\tau = \dfrac{T\,r}{J} = \dfrac{30\times10^3\times0.05}{9.817\times10^{-6}}$ = 152.8 MPa

Hence, σ_1 = 152.8 MPa, and σ_2 = – 152.8 MPa

and $\sigma_1 - \sigma_2 = \sigma_{yp}$

i.e. σ_{yp} = 305.6 MPa

Due to M:

$$I = 0.5 \times 9.817 \times 10^{-6} = 4.909 \times 10^{-6}\,\text{m}^4$$

Now, $$\frac{\sigma_1}{y} = \frac{M}{I}$$

and $$\sigma_2 = 0$$

Tresca

$$\sigma_1 - \sigma_2 = 305.6 \text{ MPa}$$

i.e. $$\sigma_1 = 305.6 \text{ MPa}$$

Hence, **pure bending moment**, $M = \dfrac{\sigma_1 I}{y} = \dfrac{305.6 \times 10^6 \times 4.909 \times 10^{-6}}{0.05}$ **= 30 kN m**

Hencky-von Mises

$$\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2 = \sigma_{yp}^2$$

$$\sigma_1 = \sigma_{yp}$$

Hence, **bending moment, *M* = 30 kN m**
