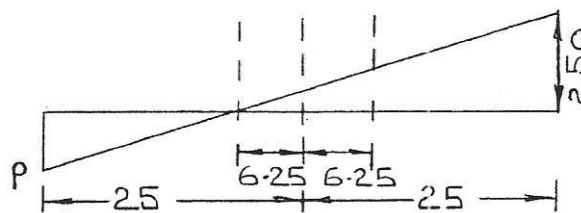


CHAPTER 13 THICK CYLINDERS AND SPHERES

EXERCISE 53, Page 291

1. Determine the maximum permissible internal pressure that a thick-walled cylinder of internal diameter 0.2 m and wall thickness 0.1 m can be subjected to, if the maximum permissible stress in this vessel is not to exceed 250 MPa.

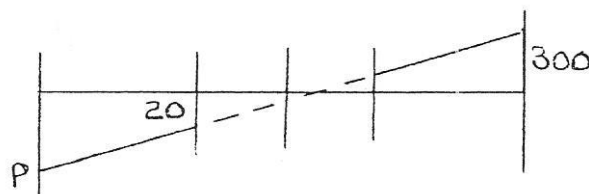


$$\frac{P}{25 - 6.25} = \frac{250}{25 + 6.25}$$

from which, **internal pressure, P** = $\frac{250 \times 18.75}{31.25}$

= 150 MPa

2. Determine the maximum permissible internal pressure that a thick-walled cylinder of internal diameter 0.2 m and wall thickness 0.1 m can be subjected to, if the cylinder is also subjected to an external pressure of 20 MPa. Assume that $\sigma_{yp} = 300$ MPa.



$$\frac{300 + P}{50} = \frac{P - 20}{25 - 6.25}$$

$$(300 + P) 18.75 = (P - 20) 50$$

$$5625 + 18.75 P = + 50 P - 1000$$

$$6625 = 31.25 P$$

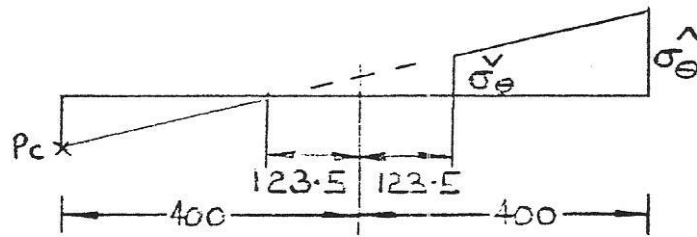
i.e. **internal pressure, $P = 212 \text{ MPa}$**

3. A steel ring of 9 cm external diameter and 5 cm internal diameter is to be shrunk into a solid bronze shaft, where the interference fit is $0.005 \times 10^{-2} \text{ m}$, based on the diameter. Determine the maximum tensile stress that is set up in the material given that:

$$\text{For steel} \quad E_s = 2 \times 10^{11} \text{ N/m}^2 \quad \text{and} \quad \nu_s = 0.3$$

$$\text{For bronze} \quad E_b = 1 \times 10^{11} \text{ N/m}^2 \quad \text{and} \quad \nu_b = 0.35$$

Steel ring



$$\frac{\sigma_{\theta_{\max}}}{400 + 123.5} = \frac{P_c}{400 - 123.5}$$

$$\sigma_{\theta_{\max}} = 1.893 P_c$$

For shaft

$$\sigma_{\theta_s} = -P_c$$

$$w_R = \frac{5 \times 10^{-2}}{2 \times 10^{11}} 2(1.893 P_c + 0.3 P_c) - 5.483 \times 10^{-13} P_c$$

$$w_s = \frac{5 \times 10^{-2}}{1 \times 10^{11}} 3(-P_c + 0.35 P) = -3.25 \times 10^{-13} P_c$$

$$0.005 \times 10^{-2} = (5.483 \times 10^{-13} + 3.25 \times 10^{-13}) P_c$$

i.e. **$P_c = 57.25 \text{ MPa}$**

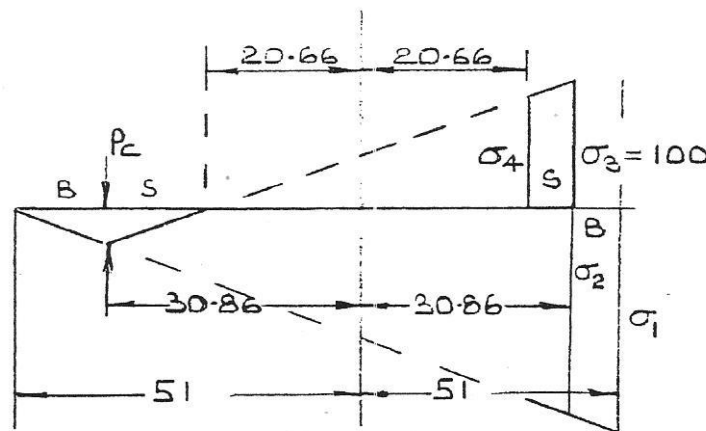
$$\sigma_{\theta_{\max}} = 1.893 \times 57.25$$

$$= \mathbf{108.3 \text{ MPa}}$$

EXERCISE 54, Page 296

1. A compound cylinder is manufactured by shrinking a steel cylinder of external diameter 22 cm and internal diameter 18 cm onto another steel cylinder of internal diameter 14 cm, the dimensions being nominal. If the maximum tensile stress in the outer cylinder is 100 MPa, determine the radial compressive stress at the common surface and the interference fit at the common diameter. Determine also the maximum stress in the inner cylinder. Assume that:

$$E_s = 2 \times 10^{11} \text{ N/m}^2 \quad \text{and} \quad \nu_s = 0.3$$



Outer cylinder

$$\frac{\sigma_3}{30.86 + 20.66} = \frac{P_c}{30.86 - 20.66}$$

from which, **radial compressive stress, P_c** = $\frac{100 \times 10.2}{51.52}$

= 19.8 MPa

Inner cylinder

$$\frac{\sigma_2}{51 + 30.86} = \frac{-P_c}{51 - 30.86}$$

from which, $\sigma_2 = \frac{-81.86 \times 19.8}{20.14}$

= 80.48 MPa

$$w_E = \frac{18 \times 10^{-2}}{2 \times 10^{11}} 4(100 \times 10^6 + 0.3 \times 19.8 \times 10^6) = 9.535 \times 10^{-5} \text{ m}$$

$$w_I = \frac{18 \times 10^{-2}}{2 \times 10^{11}} 5(-80.48 \times 10^6 + 0.3 \times 19.8 \times 10^6) = -6.709 \times 10^{-5} \text{ m}$$

i.e. **the interference fit, $\delta = w_E - w_I = 1.624 \times 10^{-4} \text{ m}$**

$$= \mathbf{0.16 \text{ mm}}$$

$$\frac{\sigma_1}{51 \times 2} = \frac{-19.8}{51 - 30.86}$$

i.e. $\sigma_1 = \frac{51 \times 2 \times -19.8}{51 - 30.86}$

$$= \mathbf{-100 \text{ MPa}}$$

2. If the inner cylinder of Problem 1 were made from bronze, what would be the value of δ ?

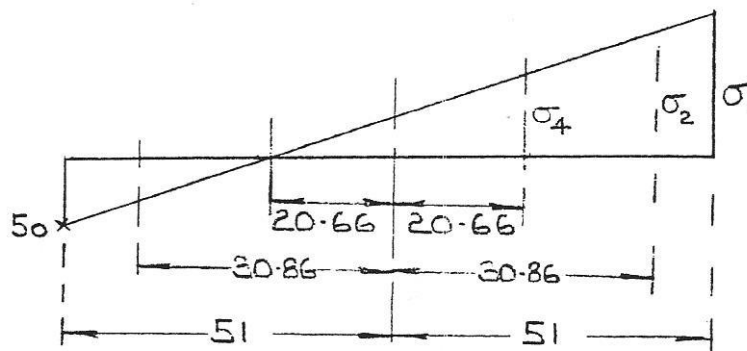
For bronze: $E_b = 1 \times 10^{11} \text{ N/m}^2$ and $\nu_b = 0.4$

$$w_E = 9.535 \times 10^{-5}$$

$$w_I = \frac{18 \times 10^{-2}}{1 \times 10^{11}} 6(-80.48 \times 10^6 + 0.4 \times 19.8 \times 10^6) = -1.306 \times 10^{-4} \text{ m}$$

i.e. **$\delta = 0.226 \text{ mm}$**

3. If the compound cylinder of Problem 1 were subjected to an internal pressure of 50 MPa, what would be the value of the maximum resultant stress?



$$\frac{\sigma_2}{30.86 + 20.66} = \frac{50}{51 - 30.86}$$

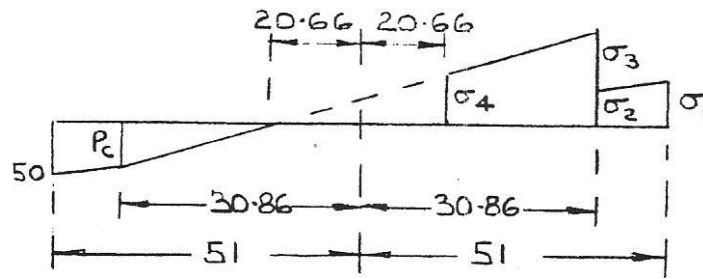
i.e.
$$\sigma_2 = \frac{50(30.86 + 20.66)}{51 - 30.86}$$

$$= 127.9 \text{ MPa}$$

σ_{\max} (on internal surface of outer cylinder)

$$= 227.9 \text{ MPa}$$

4. If the compound cylinder of Problem 2 were subjected to an internal pressure of 50 MPa, what would be the value of the maximum resultant stress?



$$\frac{\sigma_3}{30.86 + 20.66} = \frac{P_c}{30.86 - 20.66}$$

from which,
$$\sigma_3 = \frac{P_c (30.86 + 20.66)}{30.86 - 20.66} = 5.05 P_c \quad (1)$$

$$\frac{\sigma_2 + 50}{51 + 30.86} = \frac{P_c}{51 - 30.86}$$

from which,
$$\sigma_2 + 50 = \frac{P_c (51 + 30.86)}{51 - 30.86} = 4.065 P_c$$

i.e.
$$\sigma_2 = 4.065 P_c - 50$$

$$w_E = \frac{18 \times 10^{-2}}{2 \times 10^5} 7(5.05 P_c + 0.3 P_c) = 4.815 \times 10^{-6} P_c$$

$$w_I = \frac{18 \times 10^{-2}}{1 \times 10^5} 8(4.065 P_c - 50 + 0.4 P_c) = 8.037 \times 10^{-6} P_c - 9 \times 10^{-5}$$

but due to pressure above,

$$w_E = w_I$$

hence, $4.815 \times 10^{-6} P_c = 8.037 \times 10^{-6} P_c - 9 \times 10^{-5}$

i.e. $9 \times 10^{-5} = 8.037 \times 10^{-6} P_c - 4.815 \times 10^{-6} P_c$

i.e. $9 \times 10^{-5} = 3.222 \times 10^{-6} P_c$

from which, $P_c = \frac{9 \times 10^{-5}}{3.222 \times 10^{-6}} = 27.93 \text{ MPa}$

From equation (1), $\sigma_3 = 5.05 P_c = 5.05 \times 27.93 = 141.0 \text{ MPa}$

σ_{\max} (on inner surface of outer cylinder)

$$= 241.1 \text{ MPa}$$

EXERCISE 55, Page 301

1. A thick compound cylinder consists of a brass cylinder of internal diameter 0.1 m and external diameter 0.2 m, surrounded by a steel cylinder of external diameter 0.3 m and of the same length. If the compound cylinder is subjected to a compressive axial load of 5 MN, and the axial strain is constant for both cylinders, determine the pressure at the common surface and the longitudinal stresses in the two cylinders due to this load.

The following assumptions may be made:

(a) σ_{Ls} = a longitudinal stress in steel cylinder

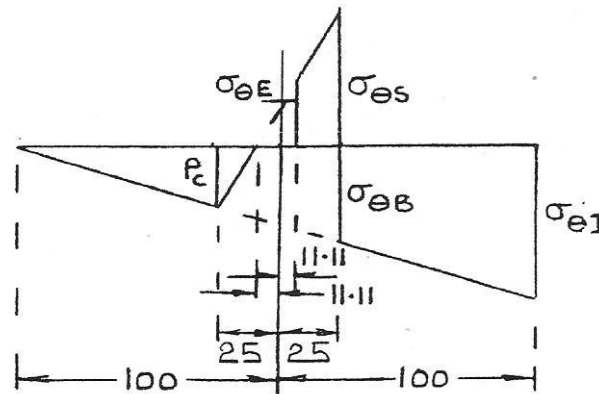
= a constant

(b) σ_{Lb} = longitudinal stress in brass cylinder

= a constant

For steel $E_s = 2 \times 10^{11} \text{ N/m}^2$ and $\nu_s = 0.3$

For brass $E_b = 1 \times 10^{11} \text{ N/m}^2$ and $\nu_b = 0.4$



$$A_B = 0.0236 \text{ m}^2$$

$$A_s = 0.0393 \text{ m}^2$$

At inner and outer surfaces $\sigma_r = 0$

and $\epsilon_{LS} = \epsilon_{LB}$

At outer surface

$$\varepsilon_{LS} = \frac{1}{E_s} (\sigma_{LS} - \nu_s \sigma_{\theta E})$$

At inner surface, $\varepsilon_{LB} = \frac{1}{E_B} (\sigma_{LB} - \nu_B \sigma_{\theta I})$

$$\frac{1}{E_s} (\sigma_{LS} - \nu_s \sigma_{\theta E}) = \frac{1}{E_B} (\sigma_{LB} - \nu_B \sigma_{\theta I})$$

$$\sigma_{LS} - 3 \sigma_{\theta E} = 2 \sigma_{LB} - 0.86 \sigma_{\theta I}$$

$$\sigma_{LS} - 2 \sigma_{LB} - 0.3 \sigma_{\theta E} - 0.86 \sigma_{\theta I} = 0 \quad (1)$$

Due to axial load, $P = \sigma_{LS} A_s + \sigma_{LB} A_B$

$$-5 = \sigma_{LS} \times 0.0393 + \sigma_{LB} \times 0.0236$$

$$\sigma_{LS} = -127.2 - 0.6 \sigma_{LB} \quad (2)$$

From Lamé line,

$$\frac{-P_C}{100 - 25} = \frac{\sigma_{\theta I}}{200}$$

$$\sigma_{\theta I} = -2.667 P_c$$

$$\frac{\sigma_{\theta E}}{11.11 \times 2} = \frac{P_C}{25 - 11.11}$$

$$\sigma_{\theta E} = 1.6 P_c$$

$$\frac{\sigma_{\theta B}}{125} = \frac{-P_C}{75}$$

$$\sigma_{\theta B} = -1.667 P_c$$

$$\frac{\sigma_{\theta S}}{25 + 11.11} = \frac{P_C}{25 - 11.11}$$

$$\sigma_{\theta S} = 2.6 P_c$$

At the common surface

$$\frac{w}{R} = \frac{1}{E_B} (\sigma_{\theta B} - \nu_B \sigma_{LB} + \nu_B P_C) = \frac{1}{E_s} (\sigma_{\theta S} - \nu_s \sigma_{LS} + \nu_s P_C)$$

or $2 \sigma_{\theta B} - 0.8 \sigma_{LB} + 0.8 P_c = \sigma_{\theta S} - 0.3 \sigma_{LS} + 0.3 P_c$

$$\begin{aligned}
-3.333 P_c - 0.8 \sigma_{LB} + 0.8 P_c &= 2.6 P_c - 0.3 \sigma_{LS} + 0.3 P_c \\
-2.533 P_c - 0.8 \sigma_{LB} &= 2.9 P_c - 0.3 \sigma_{LS} \\
\sigma_{LS} &= 18.11 P_c + 2.667 \sigma_{LB}
\end{aligned} \tag{3}$$

From equation (1),

$$\begin{aligned}
\sigma_{LS} - 2\sigma_{LB} - 0.3 \times 1.6 P_c + 0.8 \times (-2.667 P_c) &= 0 \\
\sigma_{LB} - 2\sigma_{LS} - 2.6136 P_c &= 0
\end{aligned} \tag{4}$$

Rewriting equations (2), (3) and (4) gives:

$$\begin{aligned}
\sigma_{LS} + 0.6 \sigma_{LB} + 0 P_c &= -127.2 \\
\sigma_{LS} - 2.667 \sigma_{LB} - 18.11 P_c &= 0 \\
\sigma_{LS} - 2\sigma_{LB} - 2.6136 P_c &= 0
\end{aligned}$$

These three simultaneous equations in three unknowns may be solved either either a CASIO fx-991ES PLUS calculator or by using a smart phone.

Solving the above gives:,

$$\begin{aligned}
\sigma_{LS} &= -96.52 \text{ MPa} \\
\sigma_{LB} &= -51.14 \text{ MPa} \\
P_c &= 2.20 \text{ MPa}
\end{aligned}$$
