

## CHAPTER 4 STRESS AND STRAIN

### EXERCISE 24, Page 95

**1.** If a solid stone is dropped into the sea and comes to rest at a depth of 5000 m below the surface of the sea, what will be the stress in the stone? Take the density of seawater =  $1020 \text{ kg/m}^3$  and  $g = 9.81 \text{ m/s}^2$ .

**Stress in stone,  $\sigma = \rho gh = 1020 \times 9.81 \times -5000$**   
 **$= -50 \text{ MN/m}^2$**

**2.** A solid bar of length 1 m consists of three shorter sections firmly joined together. Assuming the following apply, determine the change in length of the bar when it is subjected to an axial pull of 50 kN. Assume Young's modulus,  $E = 2 \times 10^{11} \text{ N/m}^2$ .

Section	Length (m)	Diameter (mm)
1	0.2	15
2	0.3	20
3	0.5	30

$$A_1 = \frac{\pi \times (15 \times 10^{-3})^2}{4} = 1.767 \times 10^{-4} \text{ m}^2, \quad A_2 = \frac{\pi \times (20 \times 10^{-3})^2}{4} = 3.142 \times 10^{-4} \text{ m}^2$$

$$\text{and } A_3 = \frac{\pi \times (30 \times 10^{-3})^2}{4} = 7.069 \times 10^{-4} \text{ m}^2$$

$$\sigma_1 = \frac{F}{A_1} = \frac{50 \times 10^3}{1.767 \times 10^{-4}} = 283 \text{ MN/m}^2, \quad \sigma_2 = \frac{F}{A_2} = \frac{50 \times 10^3}{3.142 \times 10^{-4}} = 159.1 \text{ MN/m}^2$$

$$\text{and } \sigma_3 = \frac{F}{A_3} = \frac{50 \times 10^3}{7.069 \times 10^{-4}} = 70.73 \text{ MN/m}^2$$

$$\delta = \frac{283 \times 10^6 \times 0.2 + 159.1 \times 10^6 \times 0.3 + 70.7 \times 10^6 \times 0.5}{2 \times 10^{11}} \text{ m}$$

i.e. the change in length of bar,  $\delta = 6.984 \times 10^{-4} \text{ m} = \mathbf{0.698 \text{ mm}}$

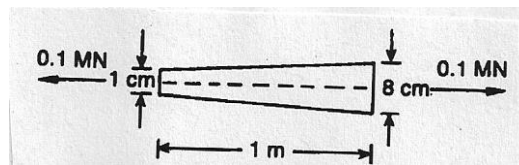
**3.** If the bar of Problem 2 were made from three different materials with the following elastic moduli, determine the change in length of the bar:

Section	E (N / m <sup>2</sup> )
1	$2 \times 10^{11}$
2	$7 \times 10^{10}$
3	$1 \times 10^{11}$

$$\delta = \frac{283 \times 10^6 \times 0.2}{2 \times 10^{11}} + \frac{159.1 \times 10^6 \times 0.3}{7 \times 10^{10}} + \frac{70.7 \times 10^6 \times 0.5}{1 \times 10^{11}}$$

i.e.  $\delta = 1.318 \times 10^{-3} \text{ m} = \mathbf{1.318 \text{ mm}}$

**4.** A circular-section solid bar of linear taper is subjected to an axial pull of 0.1 MN, as show. If E =  $2 \times 10^{11} \text{ N / m}^2$ , by how much will the bar extend?



$$u = \frac{Wl}{A_1 E} \left( \frac{d_1}{d_2} \right)$$

$$A_1 = \frac{\pi \times (1 \times 10^{-2})^2}{4} = 7.854 \times 10^{-5} \text{ m}^2$$

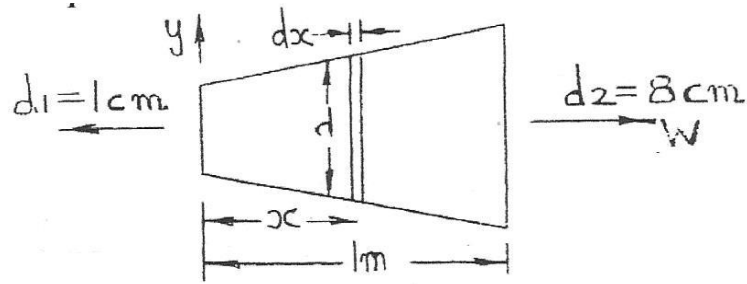
Hence,

$$u = \frac{0.1 \times 10^6 \times 1}{7.854 \times 10^{-5} \times 2 \times 10^{11}} \left( \frac{1}{8} \right)$$

$$= 7.958 \times 10^{-4} \text{ m} = u_1 \quad (1)$$

i.e. bar extension,  $u = \mathbf{0.796 \text{ mm}}$

*Proof of formula for this problem*



At  $x$ ,  $d = 1 \times 10^{-2} + 7 \times 10^{-2}x$

i.e.  $d = 1 \times 10^{-2}(1 + 7x)$

At  $x$ , 
$$\sigma = \frac{W}{\pi d^2 / 4} = \frac{W \times 4}{\pi [1 \times 10^{-2}(1 + 7x)]^2}$$

i.e. 
$$\sigma = \frac{12732W}{(1 + 7x)^2}$$

At  $x$ , 
$$\varepsilon = \frac{12732W}{(1 + 7x)^2 \times 2 \times 10^{11}}$$
  

$$= \frac{6.366 \times 10^{-8}W}{(1 + 7x)^2}$$

and 
$$\delta u = \frac{6.366 \times 10^{-8}W \delta x}{(1 + 7x)^2}$$

Thus, 
$$u = 6.366 \times 10^{-8}W \int_0^1 \frac{1}{(1 + 7x)^2} dx$$
  

$$= 6.366 \times 10^{-8} \times 0.1 \times 10^6 \left[ \frac{-1}{7(1 + 7x)} \right]_0^1$$
  

$$= 6.366 \times 10^{-8} \times 0.1 \times 10^6 \left[ \left( \frac{-1}{56} \right) - \left( \frac{-1}{7} \right) \right]$$

i.e.  $u = 7.96 \times 10^{-4} \text{ m} = \mathbf{0.796 \text{ mm}}$

**5.** If a solid stone is dropped into the sea and comes to rest at a depth of 11000 m below the surface of the sea, what will be the stress in the stone? Assume that density of sea water =  $1020 \text{ kg/m}^3$  and  $g = 9.81 \text{ m/s}^2$ .

**Stress in stone,  $\sigma = \rho gh = 1020 \times 9.81 \times -11000$**

$$= -110 \text{ MPa}$$

**6.** A solid bar of length 0.7 m consists of three shorter sections firmly joined together. Assuming the following apply, determine the change in length of the bar when it is subjected to an axial pull of 30 kN. Assume that Young's modulus,  $E = 2 \times 10^{11} \text{ N/m}^2$ .

Section	Length (m)	Diameter (mm)
1	0.1	10
2	0.2	15
3	0.4	20

$$A_1 = \frac{\pi \times (10 \times 10^{-3})^2}{4} = 7.854 \times 10^{-5} \text{ m}^2, \quad A_2 = \frac{\pi \times (15 \times 10^{-3})^2}{4} = 1.767 \times 10^{-4} \text{ m}^2$$

$$\text{and } A_3 = \frac{\pi \times (20 \times 10^{-3})^2}{4} = 3.142 \times 10^{-4} \text{ m}^2$$

$$\sigma_1 = \frac{F}{A_1} = \frac{30 \times 10^3}{7.854 \times 10^{-5}} = 382 \text{ MN/m}^2, \quad \sigma_2 = \frac{F}{A_2} = \frac{30 \times 10^3}{1.767 \times 10^{-4}} = 169.8 \text{ MN/m}^2$$

$$\text{and } \sigma_3 = \frac{F}{A_3} = \frac{30 \times 10^3}{3.142 \times 10^{-4}} = 95.48 \text{ MN/m}^2$$

$$\delta = \frac{382 \times 10^6 \times 0.1 + 169.8 \times 10^6 \times 0.2 + 95.48 \times 10^6 \times 0.4}{2 \times 10^{11}} \text{ m}$$

i.e. the change in length of bar,  $\delta = 5.5176 \times 10^{-4} \text{ m} = 0.552 \text{ mm}$

**7.** If the bar of question 6 were made from three different materials with the following elastic moduli, determine the change in length of the bar.

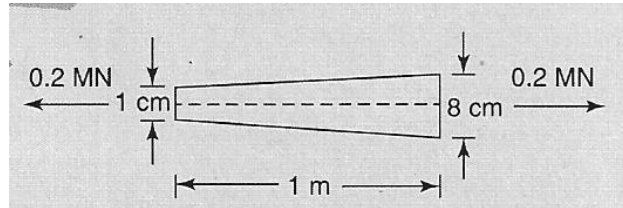
Section	$E (\text{N/m}^2)$
1	$2 \times 10^{11}$
2	$7 \times 10^{10}$
3	$1 \times 10^{11}$

$$\delta = \frac{382 \times 10^6 \times 0.1}{2 \times 10^{11}} + \frac{169.8 \times 10^6 \times 0.2}{7 \times 10^{10}} + \frac{95.48 \times 10^6 \times 0.4}{1 \times 10^{11}}$$

i.e.  $\delta = 1.058 \times 10^{-3} \text{ m} = \mathbf{1.058 \text{ mm}}$

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**8.** A circular-section solid bar of linear taper is subjected to an axial pull of 0.2 MN, as shown. If  $E = 2 \times 10^{11} \text{ N/m}^2$ , by how much will the bar expand?



From equation (1) of Problem 4,

$$u = \frac{WL}{A_1 E} \left( \frac{d_1}{d_2} \right)$$

$$A_1 = \frac{\pi \times (1 \times 10^{-2})^2}{4} = 7.854 \times 10^{-5} \text{ m}^2$$

Hence,

$$u = \frac{0.2 \times 10^6 \times 1}{7.854 \times 10^{-5} \times 2 \times 10^{11}} \left( \frac{1}{8} \right)$$

$$= 1.592 \times 10^{-3} \text{ m}$$

i.e. **bar extension,  $u = 1.592 \text{ mm}$**

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### EXERCISE 25, Page 103

1. If the bar of question 4 in Practise Exercise 24, page 95, were prevented from moving axially by two rigid walls and subjected to a temperature rise of  $10^{\circ}\text{C}$ , what would be the maximum stress in the bar? Assume the  $0.1\text{ MN}$  load is not acting. Assume that coefficient of linear expansion,  $\alpha = 15 \times 10^{-6} / ^{\circ}\text{C}$ .

$$\text{Free expansion} = \alpha l T = 15 \times 10^{-6} \times 1 \times 10 = \mathbf{1.5 \times 10^{-4} \text{ m}}$$

From equation (1) in the solution to Problem 4, Exercise 24,

$$u = 1.5 \times 10^{-4} = u_1 \times \frac{W_T}{W}$$

$$\text{so that} \quad 1.5 \times 10^{-4} = 7.96 \times 10^{-4} \times \left( \frac{W_T}{0.1 \times 10^6} \right)$$

$$\text{from which,} \quad \mathbf{W_T = 18844 \text{ N}}$$

At the smaller end,

$$\text{maximum stress in bar, } \sigma = \frac{-18844}{7.854 \times 10^{-5}} = -\mathbf{240 \text{ MN/m}^2}$$

2. If the bar in Problem 4, Exercise 24, were prevented from moving axially by two rigid walls and subjected to a temperature rise of  $10^{\circ}\text{C}$ , what would be the maximum stress in the bar? Assume the  $0.2\text{ MN}$  load is not acting, and  $\alpha = 15 \times 10^{-6} / ^{\circ}\text{C}$ .

$$\text{Free expansion} = \alpha l T = 15 \times 10^{-6} \times 1 \times 10 = \mathbf{1.5 \times 10^{-4} \text{ m}}$$

From equation (1) in the solution to Problem 4, Exercise 24,

$$u = 1.5 \times 10^{-4} = u_1 \times \frac{W_T}{W}$$

$$\text{so that} \quad 1.5 \times 10^{-4} = 7.96 \times 10^{-4} \times \left( \frac{W_T}{0.2 \times 10^6} \right)$$

$$\text{from which,} \quad \mathbf{W_T = 37688 \text{ N}}$$

At the smaller end,

$$\text{maximum stress in bar, } \sigma = \frac{-37688}{7.854 \times 10^{-5}} = -480 \text{ MN/m}^2$$

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## EXERCISE 26, Page 108

1. An electrical cable consists of a copper core surrounded co-axially by a steel sheath, so that the two can be assumed to act as a compound bar. If the cable hangs down a vertical mineshaft, determine the maximum permissible length of the cable, assuming the following apply:

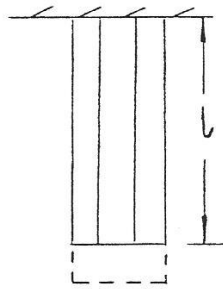
$A_c = 1 \times 10^{-4} \text{ m}^2$  = sectional area of copper,  $E_c = 1 \times 10^{11} \text{ N/m}^2$  = elastic modulus of copper,

$\rho_c = 8960 \text{ kg/m}^3$  = density of copper, maximum permissible stress in copper =  $30 \text{ MN/m}^2$ ,

$A_s = 0.2 \times 10^{-4} \text{ m}^2$  = sectional area of steel,  $E_s = 2 \times 10^{11} \text{ N/m}^2$  = elastic modulus of steel,

$\rho_s = 7860 \text{ kg/m}^3$  = density of steel, maximum permissible stress in steel =  $100 \text{ MN/m}^2$

$g = 9.81 \text{ m/s}^2$



$$\begin{aligned} W &= (\rho_c A_c + \rho_s A_s) g l \\ &= (0.896 + 0.1572) \times 9.81 l \end{aligned}$$

i.e.  $W = 10.332 l$

*Compatibility*

$$\delta_c = \delta_s$$

$$l \times \epsilon_c = l \times \epsilon_s$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

from which,  $\sigma_c = \frac{E_c}{E_s} \sigma_s = \frac{1 \times 10^{11}}{2 \times 10^{11}} \sigma_s$

i.e.  $\sigma_c = 0.5 \sigma_s$



$\sigma_c$  is the design criterion, i.e.  $\sigma_c = 30 \text{ MN/m}^2$

and  $\sigma_s = 60 \text{ MN/m}^2$

*Equilibrium*

$$W = \sigma_c A_c + \sigma_s A_s$$

or  $10.332 l = (30 \times 10^6 \times 1 \times 10^{-4} + 60 \times 10^6 \times 0.2 \times 10^{-4})$

i.e.  $l = \frac{(30 \times 10^6 \times 1 \times 10^{-4} + 60 \times 10^6 \times 0.2 \times 10^{-4})}{10.332}$

i.e. **the maximum permissible length of the cable,  $l = 406.5 \text{ m}$**

**2. How much will the cable of question 1 stretch, owing to self-weight?**

Average stress in copper =  $15 \text{ MN/m}^2$

Therefore, average strain =  $\frac{15 \times 10^6}{1 \times 10^{11}} = 1.5 \times 10^{-4}$

Hence, **cable stretch,  $\delta = 1.5 \times 10^{-4} \times 406.5 = 0.06098 \text{ m}$**

**= 61 mm**

*Check:*  $\sigma_{AV} \text{ (steel)} = 30 \text{ MN/m}^2$

$\epsilon_{AV} \text{ (steel)} = 1.5 \times 10^{-4}$

and  **$\delta = 1.5 \times 10^{-4} \times 406.5 = 0.061 \text{ m} = 61 \text{ mm}$**

**3. If a weight of 100kN were lowered into the sea, via a steel cable of cross-sectional area  $8 \times 10^{-4} \text{ m}^2$ , what would be the maximum permissible depth that the weight could be lowered if the following apply?**

Density of steel =  $7860 \text{ kg/m}^3$ , density of sea water =  $1020 \text{ kg/m}^3$ ,

maximum permissible stress in steel =  $200 \text{ MN/m}^2$ ,  $g = 9.81 \text{ m/s}^2$

Any buoyancy acting on the weight itself may be neglected.

$$W = (100,000 + (7860 - 1020) \times 8 \times 10^{-4} l) g$$

i.e.  $W = 100,000 + 53.68 l$  (1)

However,  $W = \sigma A = 200 \times 10^6 \times 8 \times 10^{-4}$   
 $= 160,000 \text{ N}$  (2)

Equating equations (1) and (2) gives:

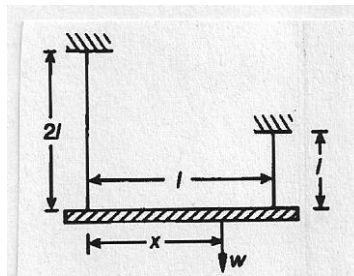
$$160,000 = 100,000 + 53.68 l$$

i.e. **the maximum permissible depth,  $l = 60000/53.68 = 1118 \text{ m}$**

**4.** A weightless rigid horizontal beam is supported by two vertical wires, as shown. If the following apply, determine the position from the left that a weight  $W$  can be suspended, so that the bar will remain horizontal when the wires stretch.

*Left wire:* cross-sectional area =  $2A$ , elastic modulus =  $E$ , length =  $2l$

*Right wire:* cross-sectional area =  $A$ , elastic modulus =  $3E$ , length =  $l$



$$\delta_1 = \delta_2$$

$$2 l \varepsilon_1 = l \varepsilon_2$$

$$2 \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad 3$$

or  $\sigma_1 = \frac{\sigma_2 E_1}{2 E_2} \quad 4$  (1)

Taking moments about the right wire gives:

$$F_1 l = W(l - x)$$

i.e.  $F_1 = \frac{W(l - x)}{l} \quad 5$

$$\sigma_1 A_1 = \frac{W(l-x)}{l} 6$$

$$\sigma_1 = \frac{W(l-x)}{A_1 l} 7 \quad (2)$$

$$F_1 + F_2 = W$$

$$\sigma_1 A_1 + \sigma_2 A_2 = W$$

$$\frac{W(l-x)}{A_1 l} \times A_1 + \sigma_1 \times \frac{2E_2}{E_1} \times A_2 = W \quad \text{from equations (1) and (2)}$$

$$\text{i.e.} \quad \frac{W(l-x)}{l} + \frac{W(l-x)}{A_1 l} \times \frac{2 \times 3E}{E} \times 0.5 \times A_1 = W$$

$$\text{i.e.} \quad (l-x) + 3(l-x) = l$$

$$\text{i.e.} \quad 4l - 4x = l$$

$$\text{i.e.} \quad 4x = 3l$$

$$\text{and} \quad x = 0.75 l$$

**5.** An electrical cable consists of a copper core surrounded co-axially by a steel sheath, so that the two can be assumed to act as a compound bar. If the cable hangs down a vertical mineshaft, determine the maximum permissible length of the cable, assuming the following apply:

$A_c = 2 \times 10^{-4} \text{ m}^2$  = sectional area of copper,  $E_c = 1 \times 10^{11} \text{ N / m}^2$  = elastic modulus of copper,

$\rho_c = 8960 \text{ kg / m}^3$  = density of copper, maximum permissible stress in copper =  $30 \text{ MN / m}^2$ ,

$A_s = 0.1 \times 10^{-4} \text{ m}^2$  = sectional area of steel,  $E_s = 2 \times 10^{11} \text{ N / m}^2$  = elastic modulus of steel,

$\rho_s = 7860 \text{ kg / m}^3$  = density of steel, maximum permissible stress in steel =  $100 \text{ MN / m}^2$ ,

$g = 9.81 \text{ m/s}^2$

$$\begin{aligned} W &= (\rho_c A_c + \rho_s A_s) g l \\ &= (1.792 + 0.0786) \times 9.81 l \end{aligned}$$

$$\text{i.e.} \quad W = 18.351 l$$

*Compatibility*

$$\delta_c = \delta_s$$

$$l \times \varepsilon_c = l \times \varepsilon_s$$

$$\frac{\sigma_c}{E_c} = \frac{\sigma_s}{E_s}$$

from which, 
$$\sigma_c = \frac{E_c}{E_s} \sigma_s = \frac{1 \times 10^{11}}{2 \times 10^{11}} \sigma_s$$

i.e. 
$$\sigma_c = 0.5 \sigma_s$$

$\sigma_c$  is the design criterion, i.e.  $\sigma_c = 30 \text{ MN/m}^2$

and 
$$\sigma_s = 60 \text{ MN/m}^2$$

*Equilibrium*

$$W = \sigma_c A_c + \sigma_s A_s$$

or 
$$18.351 l = (30 \times 10^6 \times 2 \times 10^{-4} + 60 \times 10^6 \times 0.1 \times 10^{-4})$$

i.e. 
$$l = \frac{(30 \times 10^6 \times 2 \times 10^{-4} + 60 \times 10^6 \times 0.1 \times 10^{-4})}{18.351}$$

i.e. the maximum permissible length of the cable,  $l = 359.7 \text{ m}$

**6.** How much will the cable of question 5 stretch, owing to self-weight?

Average stress in copper =  $15 \text{ MN/m}^2$

Therefore, average strain = 
$$\frac{15 \times 10^6}{1 \times 10^{11}} = 1.5 \times 10^{-4}$$

Hence, cable stretch,  $\delta = 1.5 \times 10^{-4} \times 359.7 = 0.0539 \text{ m}$

$$= 53.9 \text{ mm}$$

*Check:*  $\sigma_{AV} (\text{steel}) = 30 \text{ MN/m}^2$

$$\varepsilon_{AV} (\text{steel}) = 1.5 \times 10^{-4}$$

and 
$$\delta = 1.5 \times 10^{-4} \times 359.7 = 0.0539 \text{ m} = 53.9 \text{ mm}$$

7. If a weight of 95 kN were lowered into the sea via a steel cable of cross-sectional area  $8 \times 10^{-4} \text{ m}^2$  what would be the maximum permissible depth that the weight could be lowered if the following apply?

Density of steel =  $7860 \text{ kg/m}^3$       Density of sea water =  $1020 \text{ kg/m}^3$

Maximum permissible stress in steel =  $200 \text{ MN/m}^2$        $g = 9.81 \text{ m/s}^2$

Any buoyancy acting on the weight itself may be neglected.

$$W = (95000 + (7860 - 1020) \times 8 \times 10^{-4} l g)$$

i.e.  $W = 95000 + 53.68 l$  (1)

However,  $W = \sigma A = 200 \times 10^6 \times 8 \times 10^{-4}$

$$= 160,000 \text{ N} \quad (2)$$

Equating equations (1) and (2) gives:

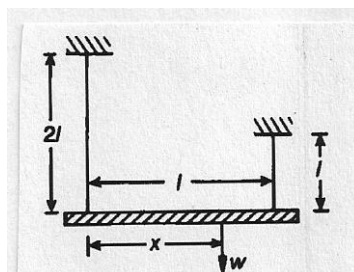
$$160,000 = 95000 + 53.68 l$$

i.e. the maximum permissible depth,  $l = 65000/53.68 = 1211 \text{ m}$

8. A weightless rigid horizontal beam is supported by two vertical wires, as shown. If the following apply, determine the position from the left that a weight  $W$  can be suspended, so that the bar will remain horizontal when the wires stretch.

Left wire: cross-sectional area =  $3A$ , elastic modulus =  $E$ , length =  $2l$

Right wire: cross-sectional area =  $1.5A$ , elastic modulus =  $3E$ , length =  $l$



$$\delta_1 = \delta_2$$

$$2 l \varepsilon_1 = l \varepsilon_2$$

$$2 \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2} \quad 8$$

or

$$\sigma_1 = \frac{\sigma_2 E_1}{2 E_2} \quad 9 \quad (1)$$

Taking moments about the right wire gives:

$$F_1 l = W(l - x)$$

i.e.

$$F_1 = \frac{W(l - x)}{l} \quad 10$$

$$\sigma_1 A_1 = \frac{W(l - x)}{l} \quad 11$$

$$\sigma_1 = \frac{W(l - x)}{A_1 l} \quad 12 \quad (2)$$

Resolving vertically gives:

$$F_1 + F_2 = W$$

$$\sigma_1 A_1 + \sigma_2 A_2 = W$$

$$\frac{W(l - x)}{A_1 l} \times A_1 + \sigma_1 \times \frac{2 E_2}{E_1} \times A_2 = W \quad \text{from equations (1) and (2)}$$

i.e.

$$\frac{W(l - x)}{l} + \frac{W(l - x)}{A_1 l} \times \frac{2 \times 3 E}{E} \times 0.5 \times A_1 = W$$

or

$$\frac{(l - x)}{l} + \frac{(l - x)}{0.5 l} \times 2 \times 3 \times 0.5 \times 0.5 = 1$$

i.e.

$$\frac{(l - x)}{l} + \frac{(l - x)}{0.5 l} \times 2 \times 3 \times 0.5 \times 0.5 = 1$$

i.e.

$$(l - x) + 3(l - x) = l$$

i.e.

$$4l - 4x = l$$

i.e.

$$4x = 4l - l = 3l$$

and

$$x = 0.75 l$$