

CHAPTER 2 FURTHER REVISIONARY MATHEMATICS

EXERCISE 11, Page 26

1. Express 60,000 Pa in engineering notation in prefix form.

$$60,000 \text{ Pa} = 60 \times 10^3 \text{ Pa} = 60 \text{ kPa}$$

2. Express 0.00015 W in engineering notation in prefix form.

$$0.00015 \text{ W} = 150 \times 10^{-6} \text{ W} = 150 \mu\text{W} \text{ or } 0.15 \text{ mW}$$

3. Express $5 \times 10^7 \text{ V}$ in engineering notation in prefix form.

$$5 \times 10^7 \text{ V} = 50 \times 10^6 \text{ V} = 50 \text{ MV}$$

4. Express $5.5 \times 10^{-8} \text{ F}$ in engineering notation in prefix form.

$$5.5 \times 10^{-8} \text{ F} = 55 \times 10^{-9} \text{ F} = 55 \text{ nF}$$

5. Express 100,000 N in engineering notation in prefix form.

$$100,000 \text{ N} = 100 \times 10^3 \text{ N} = 100 \text{ kN}$$

6. Express 0.00054 A in engineering notation in prefix form.

$$0.00054 \text{ A} = 0.54 \times 10^{-3} \text{ A} = 0.54 \text{ mA} \text{ or } 540 \times 10^{-6} \text{ A} = 540 \mu\text{A}$$

7. Express $15 \times 10^5 \Omega$ in engineering notation in prefix form.

$$15 \times 10^5 \Omega = 1500000 \Omega = 1.5 \times 10^6 \Omega = 1.5 \text{ M}\Omega$$

8. Express 225×10^{-4} V in engineering notation in prefix form.

$$225 \times 10^{-4} \text{ V} = 0.0225 \text{ V} = \mathbf{22.5 \times 10^{-3} \text{ V} = 22.5 \text{ mV}}$$

9. Express 35,000,000,000 Hz in engineering notation in prefix form.

$$35,000,000,000 \text{ Hz} = \mathbf{35 \times 10^9 \text{ Hz} = 35 \text{ GHz}}$$

10. Express 1.5×10^{-11} F in engineering notation in prefix form.

$$1.5 \times 10^{-11} \text{ F} = \mathbf{15 \times 10^{-12} \text{ F} = 15 \text{ pF}}$$

11. Express 0.000017 A in engineering notation in prefix form.

$$0.000017 \text{ A} = \mathbf{17 \times 10^{-6} \text{ A} = 17 \text{ }\mu\text{A}}$$

12. Express 46200Ω in engineering notation in prefix form.

$$46200 \Omega = \mathbf{46.2 \times 10^3 \Omega = 46.2 \text{ k}\Omega}$$

13. Rewrite 0.003 mA in μA .

$$\mathbf{0.003 \text{ mA} = 0.003 \times 10^{-3} \text{ A} = 0.000003 \text{ A} = 3 \times 10^{-6} = 3 \text{ }\mu\text{A}}$$

14. Rewrite 2025 kHz as MHz.

$$\mathbf{2025 \text{ kHz} = 2025,000 \text{ Hz} = 2.025 \times 10^6 \text{ Hz} = 2.025 \text{ MHz}}$$

15. Rewrite 5×10^4 N in kN.

$$\mathbf{5 \times 10^4 \text{ N} = 50000 \text{ N} = 50 \times 10^3 \text{ N} = 50 \text{ kN}}$$

16. Rewrite 300 pF in nF.

$$300 \text{ pF} = 300 \times 10^{-12} \text{ F} = 0.0000000003 \text{ F} = 0.3 \times 10^{-9} \text{ F} = 0.3 \text{ nF}$$

17. Rewrite 6250 cm in metres.

$$6250 \text{ cm} = \frac{6250 \text{ cm}}{100 \text{ cm / m}} = 62.50 \text{ m}$$

18. Rewrite 34.6 g in kg.

$$34.6 \text{ g} = \frac{34.6 \text{ g}}{1000 \text{ g / kg}} = 0.0346 \text{ kg}$$

19. The tensile stress acting on a rod is 5600000 Pa. Write this value in engineering notation.

$$\text{Tensile stress} = 5600000 \text{ Pa} = 5.6 \times 10^6 = 5.6 \text{ MPa}$$

20. The expansion of a rod is 0.0043 m. Write this in engineering notation.

$$\text{Expansion} = 0.0043 \text{ m} = 4.3 \times 10^{-3} \text{ m} = 4.3 \text{ mm}$$

EXERCISE 12, Page 29

1. Calculate the number of inches in 476 mm, correct to 2 decimal places.

$$476 \text{ mm} = 476 \times 0.03937 \text{ inches} = \mathbf{18.74 \text{ inches}}$$
 from Table 2.1, page 26

2. Calculate the number of inches in 209 cm, correct to 4 significant figures.

$$209 \text{ cm} = 209 \times 0.3937 \text{ inches} = \mathbf{82.28 \text{ inches}}$$
 from Table 2.1, page 26

3. Calculate the number of yards in 34.7 m, correct to 2 decimal places .

$$34.7 \text{ m} = 34.7 \times 1.0936 \text{ yards} = \mathbf{37.95 \text{ yds}}$$
 from Table 2.1, page 26

4. Calculate the number of miles in 29.55 km, correct to 2 decimal places.

$$29.55 \text{ km} = 29.55 \times 0.6214 \text{ miles} = \mathbf{18.36 \text{ miles}}$$
 from Table 2.1, page 26

5. Calculate the number of centimetres in 16.4 inches, correct to 2 decimal places.

$$16.4 \text{ inches} = 16.4 \times 2.54 \text{ cm} = \mathbf{41.66 \text{ cm}}$$
 from Table 2.2, page 26

6. Calculate the number of metres in 78 inches, correct to 2 decimal places.

$$78 \text{ inches} = \frac{78}{12} \text{ feet} = \frac{78}{12} \times 0.3048 \text{ m} = \mathbf{1.98 \text{ m}}$$
 from Table 2.2, page 26

7. Calculate the number of metres in 15.7 yards, correct to 2 decimal places.

$$15.7 \text{ yards} = 15.7 \times 0.9144 \text{ m} = \mathbf{14.36 \text{ m}}$$
 from Table 2.2, page 26

8. Calculate the number of kilometres in 3.67 miles, correct to 2 decimal places.

$$3.67 \text{ miles} = 3.67 \times 1.6093 \text{ km} = \mathbf{5.91 \text{ km}}$$
 from Table 2.2, page 26

9. Calculate the number of (a) yards (b) kilometres in 11.23 nautical miles.

(a) 11.23 nautical miles = 11.23×2025.4 yards = **22,745 yards** from Table 2.2, page 26

(b) 11.23 nautical miles = 11.23×1.853 km = **20.81 km** from Table 2.2, page 26

10. Calculate the number of square inches in 62.5 cm^2 , correct to 4 significant figures.

$$62.5 \text{ cm}^2 = 62.5 \times 0.1550 \text{ in}^2 = \mathbf{9.688 \text{ in}^2} \text{ from Table 2.3, page 27}$$

11. Calculate the number of square yards in 15.2 m^2 , correct to 2 decimal places.

$$15.2 \text{ m}^2 = 15.2 \times 1.1960 \text{ yd}^2 = \mathbf{18.18 \text{ yd}^2} \text{ from Table 2.3, page 27}$$

12. Calculate the number of acres in 12.5 hectares, correct to 2 decimal places.

$$12.5 \text{ hectares} = 12.5 \times 2.4711 \text{ acres} = \mathbf{30.89 \text{ acres}} \text{ from Table 2.3, page 27}$$

13. Calculate the number of square miles in 56.7 km^2 , correct to 2 decimal places.

$$56.7 \text{ km}^2 = 56.7 \times 0.3861 \text{ mile}^2 = \mathbf{21.89 \text{ mile}^2} \text{ from Table 2.3, page 27}$$

14. Calculate the number of square centimetres in 6.37 in^2 , correct to the nearest square centimetre.

$$6.37 \text{ in}^2 = 6.37 \times 6.4516 \text{ cm}^2 = \mathbf{41 \text{ cm}^2} \text{ from Table 2.4, page 27}$$

15. Calculate the number of square metres in 308.6 ft^2 , correct to 2 decimal places.

$$308.6 \text{ ft}^2 = 308.6 \times 0.0929 \text{ m}^2 = \mathbf{28.67 \text{ m}^2} \text{ from Table 2.4, page 27}$$

16. Calculate the number of square metres in 2.5 acres, correct to the nearest square metre.

$$2.5 \text{ acres} = 2.5 \times 4046.9 \text{ m}^2 = \mathbf{10117 \text{ m}^2}$$
 from Table 2.4, page 27

17. Calculate the number of square kilometres in 21.3 mile^2 , correct to 2 decimal places.

$$21.3 \text{ mile}^2 = 21.3 \times 2.59 \text{ km}^2 = \mathbf{55.17 \text{ km}^2}$$
 from Table 2.4, page 27

18. Calculate the number of cubic inches in 200.7 cm^3 , correct to 2 decimal places.

$$200.7 \text{ cm}^3 = 200.7 \times 0.0610 \text{ in}^3 = \mathbf{12.24 \text{ in}^3}$$
 from Table 2.5, page 28

19. Calculate the number of cubic feet in 214.5 dm^3 , correct to 3 decimal places.

$$214.5 \text{ dm}^3 = 214.5 \times 0.0353 \text{ ft}^3 = \mathbf{7.572 \text{ ft}^3}$$
 from Table 2.5, page 28

20. Calculate the number of cubic yards in 13.45 m^3 , correct to 4 significant figures.

$$13.45 \text{ m}^3 = 13.45 \times 1.3080 \text{ yd}^3 = \mathbf{17.59 \text{ yd}^3}$$
 from Table 2.5, page 28

21. Calculate the number of fluid pints in 15 litres, correct to 1 decimal place.

$$15 \text{ litre} = 15 \times 2.113 \text{ fluid pints} = \mathbf{31.7 \text{ fluid pints}}$$
 from Table 2.5, page 28

22. Calculate the number of cubic centimetres in 2.15 in^3 , correct to 2 decimal places.

$$2.15 \text{ in}^3 = 2.15 \times 16.387 \text{ cm}^3 = \mathbf{35.23 \text{ cm}^3}$$
 from Table 2.6, page 28

23. Calculate the number of cubic metres in 175 ft^3 , correct to 4 significant figures.

$$175 \text{ ft}^3 = 175 \times 0.02832 \text{ m}^3 = \mathbf{4.956 \text{ m}^3}$$
 from Table 2.6, page 28

24. Calculate the number of litres in 7.75 imperial pints, correct to 3 decimal places.

7.75 imperial pints = 7.75×0.4732 litres = **3.667 litres** from Table 2.6, page 28

25. Calculate the number of litres in 12.5 imperial gallons, correct to 2 decimal places.

12.5 imperial gallons = 12.5×3.7854 litre = **47.32 litre** from Table 2.6, page 28

26. Calculate the number of ounces in 980 g, correct to 2 decimal places.

980 g = 980×0.0353 oz = **34.59 oz** from Table 2.7, page 28

27. Calculate the mass, in pounds, in 55 kg, correct to 4 significant figures.

55 kg = 55×2.2046 lb = **121.3 lb** from Table 2.7, page 28

28. Calculate the number of short tons in 4000 kg, correct to 3 decimal places.

4000 kg = 4 t = 4×1.1023 short tons = **4.409 short tons** from Table 2.7, page 28

29. Calculate the number of grams in 7.78 oz, correct to 4 significant figures.

7.78 oz = 7.78×28.35 g = **220.6 g** from Table 2.8, page 29

30. Calculate the number of kilograms in 57.5 oz, correct to 3 decimal places.

$57.5 \text{ oz} = \frac{57.5}{16} \text{ lb} = \frac{57.5}{16} \times 0.4536 \text{ kg} = \mathbf{1.630 \text{ kg}}$ from Table 2.8, page 29

31. Convert 2.5 cwt into (a) pounds (b) kilograms.

(a) $2.5 \text{ cwt} = 2.5 \times 112 \text{ lb} = \mathbf{280 \text{ lb}}$ from Table 2.8, page 29

(b) $2.5 \text{ cwt} = 2.5 \times 50.892 \text{ kg} = \mathbf{127.2 \text{ kg}}$ from Table 2.8, page 29

32. Convert 55°C to degrees Fahrenheit.

$$F = \frac{9}{5}C + 32 \text{ hence } 55^{\circ}\text{C} = \frac{9}{5}(55) + 32 = 99 + 32 = \mathbf{131^{\circ}\text{F}}$$

33. Convert 167°F to degrees Celsius.

$$C = \frac{5}{9}(F - 32) \text{ hence } 167^{\circ}\text{F} = \frac{5}{9}(167 - 32) = \frac{5}{9}(135) = \mathbf{75^{\circ}\text{C}}$$

EXERCISE 13, Page 32

1. Corresponding values obtained experimentally for two quantities are:

x -5 -3 -1 0 2 4

y -13 -9 -5 -3 1 5

Plot a graph of y (vertically) against x (horizontally) to scales of 2 cm = 1 for the horizontal x-axis and 1 cm = 1 for the vertical y-axis. (This graph will need the whole of the graph paper with the origin somewhere in the centre of the paper).

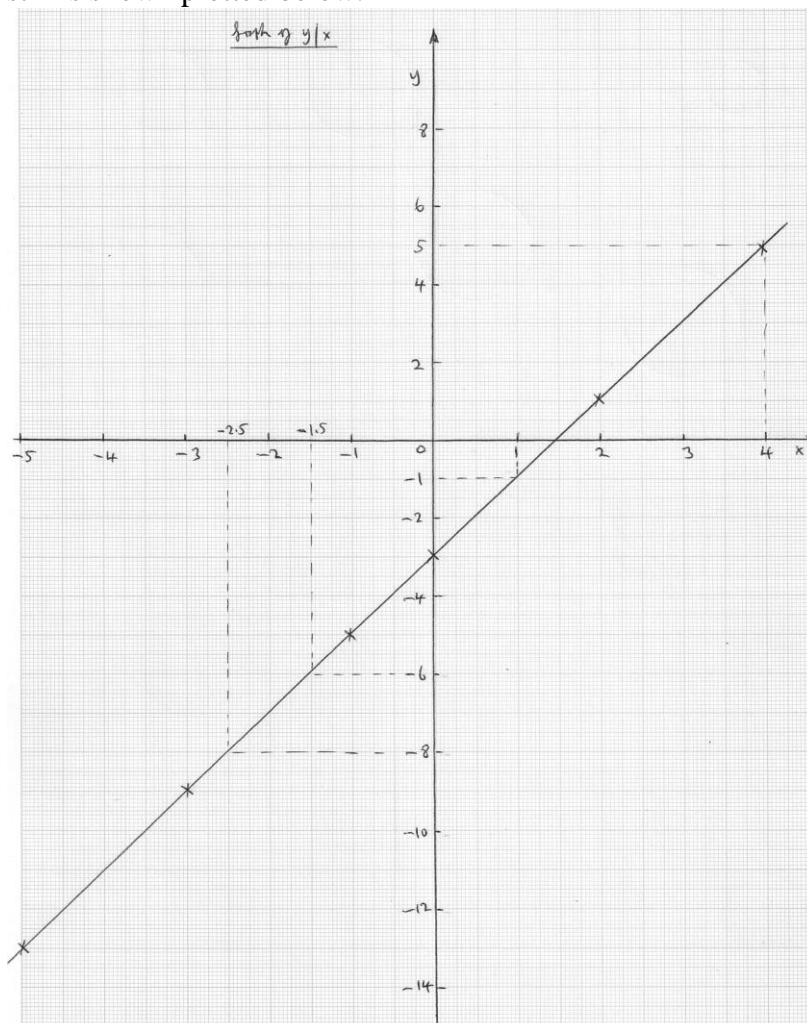
From the graph find: (a) the value of y when $x = 1$

(b) the value of y when $x = -2.5$

(c) the value of x when $y = -6$

(d) the value of x when $y = 5$

A graph of y against x is shown plotted below.



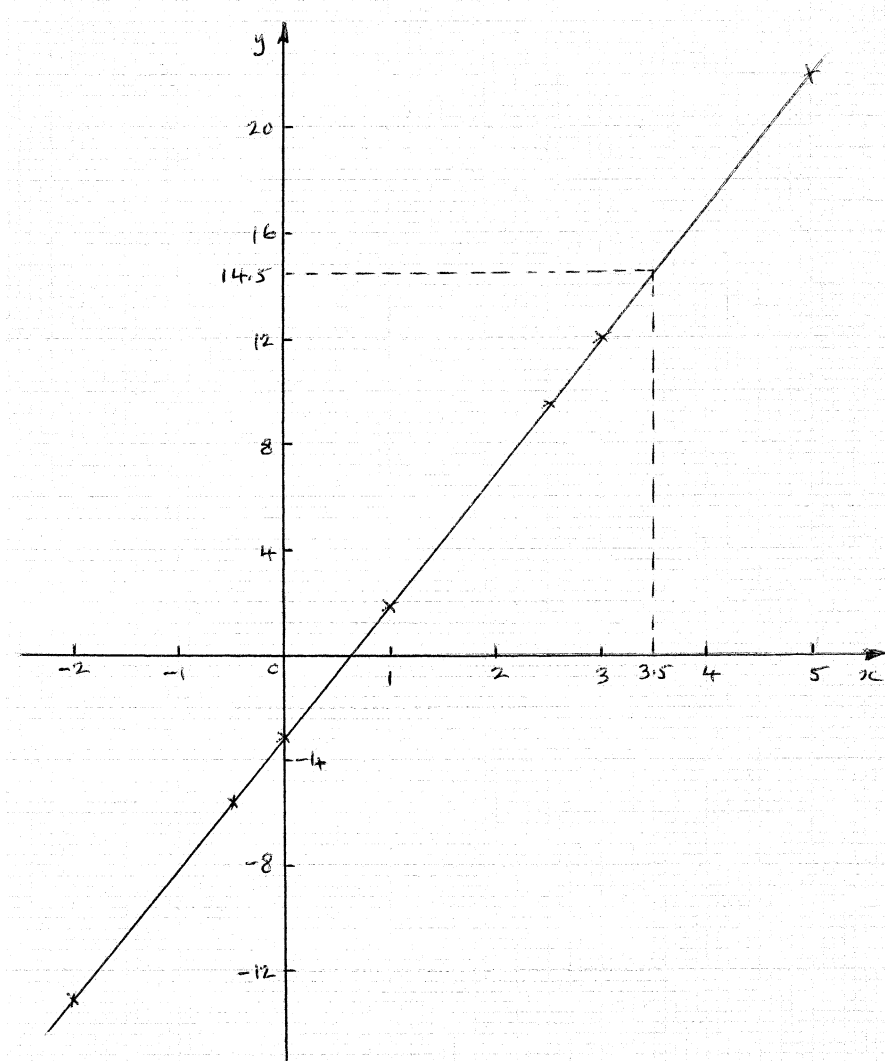
- (a) When $x = 1$, the value of $y = -1$
- (b) When $x = -2.5$, the value of $y = -8$
- (c) When $y = -6$, the value of $x = -1.5$
- (d) When $y = 5$, the value of $x = 4$

2. Corresponding values obtained experimentally for two quantities are:

x	-2.0	-0.5	0	1.0	2.5	3.0	5.0
y	-13.0	-5.5	-3.0	2.0	9.5	12.0	22.0

Use a horizontal scale for x of $1 \text{ cm} = \frac{1}{2}$ unit and a vertical scale for y of $1 \text{ cm} = 2$ units and draw a graph of x against y . Label the graph and each of its axes. By interpolation, find from the graph the value of y when x is 3.5

Graph of y/x



The graph of y against x is shown plotted above.

From the graph, when $x = 3.5$, $y = 14.5$

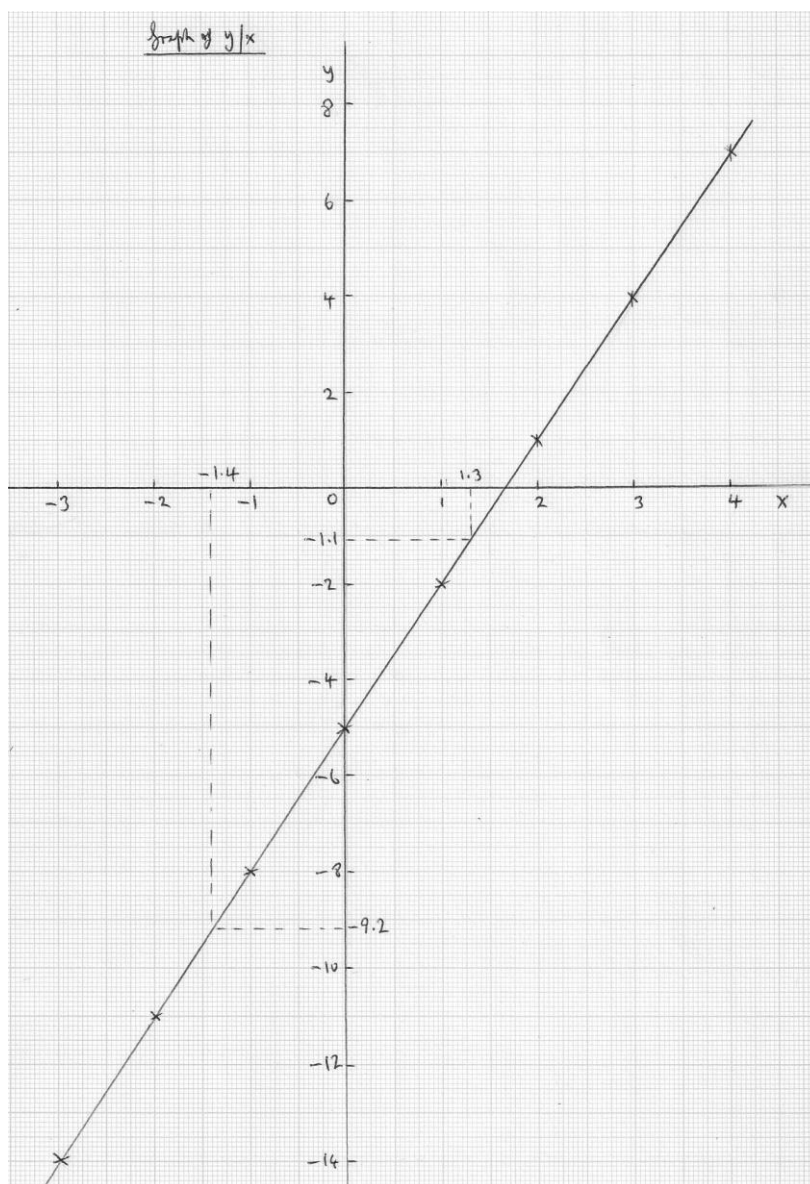
3. Draw a graph of $y - 3x + 5 = 0$ over a range of $x = -3$ to $x = 4$. Hence determine (a) the value of y when $x = 1.3$ and (b) the value of x when $y = -9.2$.

If $y - 3x + 5 = 0$ then $y = 3x - 5$

A table of values is shown below:

x	-3	-2	-1	0	1	2	3	4
y	-14	-11	-8	-5	-2	1	4	7

A graph of y against x is shown plotted below.



(a) When $x = 1.3$, the value of $y = -1.1$

(b) When $y = -9.2$, the value of $x = -1.4$

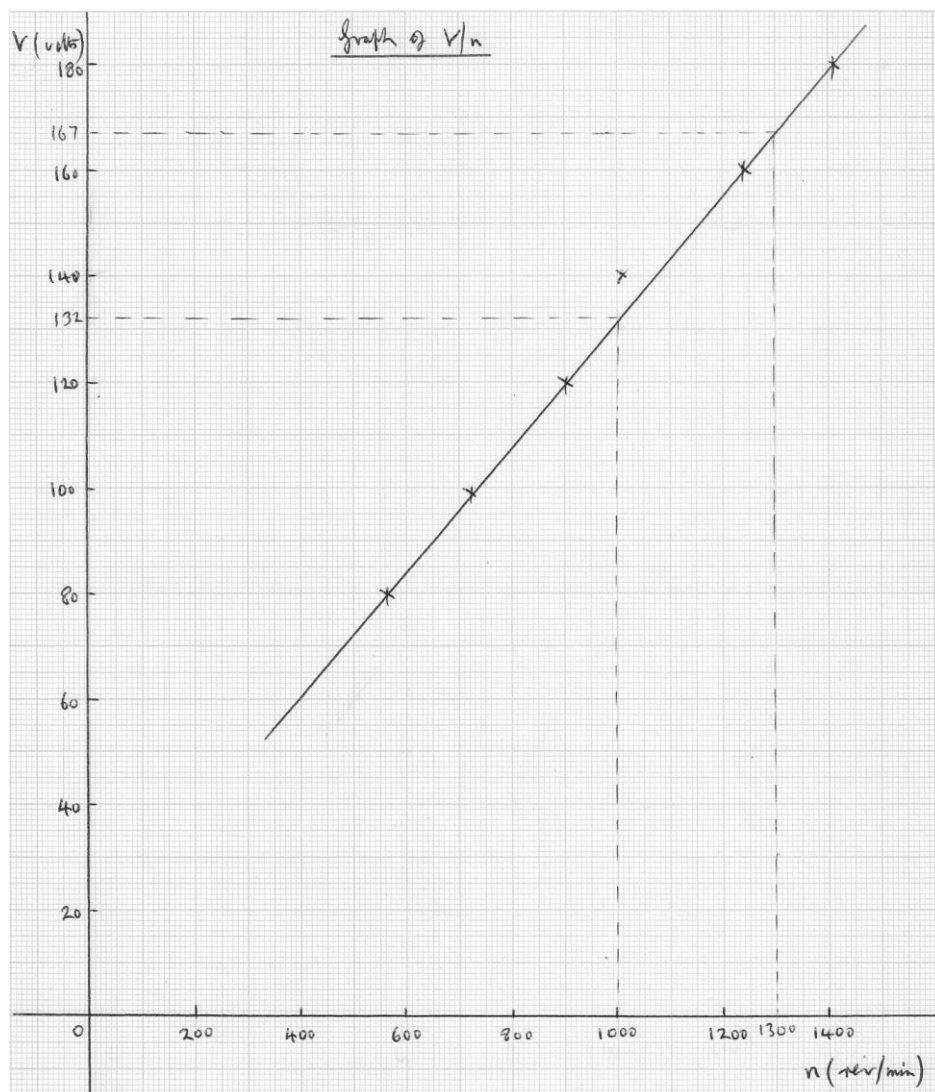
4. The speed n rev/min of a motor changes when the voltage V across the armature is varied.

The results are shown in the following table:

n (rev/min)	560	720	900	1010	1240	1410
V (volts)	80	100	120	140	160	180

It is suspected that one of the readings taken of the speed is inaccurate. Plot a graph of speed (horizontally) against voltage (vertically) and find this value. Find also (a) the speed at a voltage of 132 V, and (b) the voltage at a speed of 1300 rev/min.

A graph of voltage against speed is shown below.



The 1010 rev/min reading should be 1070 rev/min at 140 V.

(a) At a voltage of 132 V, the **speed = 1000 rev/min**

(b) At a speed of 1300 rev/min, the **voltage = 167 V**

EXERCISE 14, Page 34

1. The equation of a line is $4y = 2x + 5$. A table of corresponding values is produced and is shown below. Complete the table and plot a graph of y against x . Find the gradient of the graph.

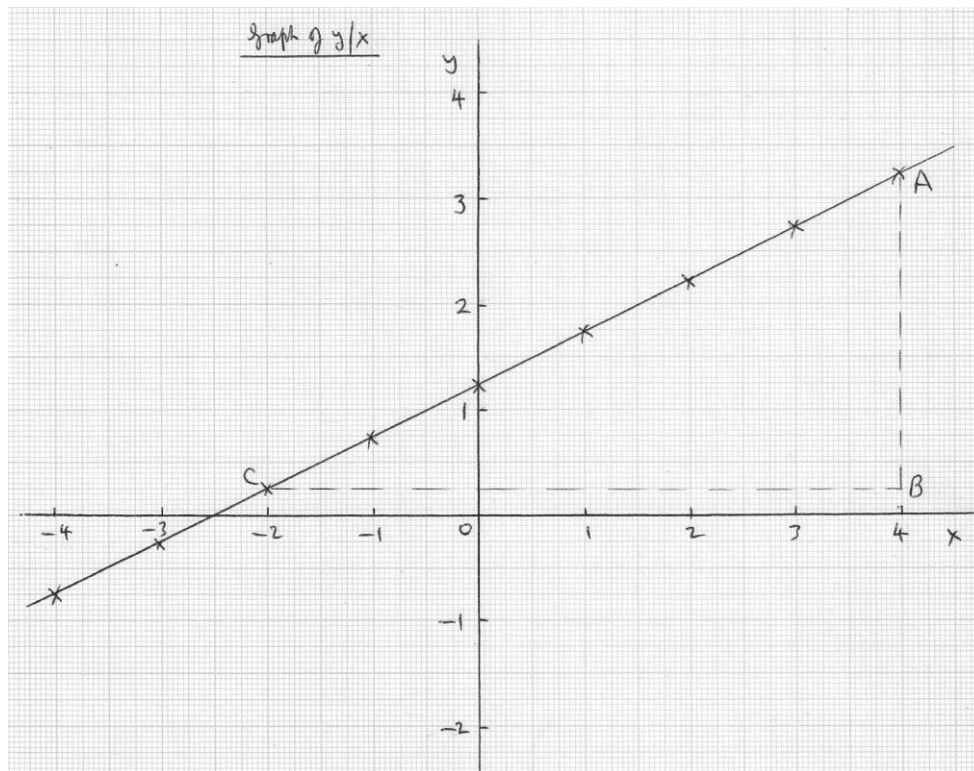
x	-4	-3	-2	-1	0	1	2	3	4
y		-0.25			1.25				3.25

$$4y = 2x + 5 \quad \text{hence} \quad y = 0.5x + 1.25$$

The missing values are shown in bold.

x	-4	-3	-2	-1	0	1	2	3	4
y	-0.75	-0.25	0.25	0.75	1.25	1.75	2.25	2.75	3.25

A graph of y/x is shown below.



$$\text{Gradient of graph} = \frac{AB}{BC} = \frac{3.25 - 0.25}{4 - (-2)} = \frac{3}{6} = \frac{1}{2}$$

2. Determine the gradient and intercept on the y -axis for each of the following equations:

(a) $y = 4x - 2$

(b) $y = -x$

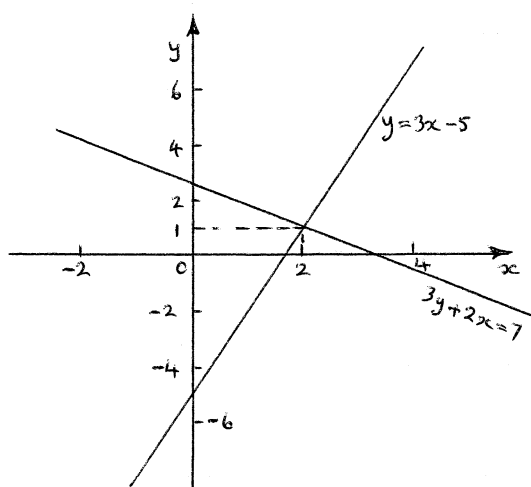
(c) $y = -3x - 4$

(d) $y = 4$

- (a) Since $y = 4x - 2$, then **gradient = 4** and **y-axis intercept = -2**
- (b) Since $y = -x$, then **gradient = -1** and **y-axis intercept = 0**
- (c) Since $y = -3x - 4$, then **gradient = -3** and **y-axis intercept = -4**
- (d) Since $y = 4$ i.e. $y = 0x + 4$, then **gradient = 0** and **y-axis intercept = 4**

3. Draw on the same axes the graphs of $y = 3x - 5$ and $3y + 2x = 7$. Find the co-ordinates of the point of intersection. Check the result obtained by solving the two simultaneous equations algebraically.

The graphs of $y = 3x - 5$ and $3y + 2x = 7$, i.e. $y = -\frac{2}{3}x + \frac{7}{3}$ are shown below.



The two graphs intersect at $x = 2$ and $y = 1$, i.e. the co-ordinate (2, 1)

Solving simultaneously gives:

$$y = 3x - 5 \quad \text{i.e.} \quad y - 3x = -5 \quad (1)$$

$$y = -\frac{2}{3}x + \frac{7}{3} \quad \text{i.e.} \quad 3y + 2x = 7 \quad (2)$$

$$3 \times (1) \text{ gives:} \quad 3y - 9x = -15 \quad (3)$$

$$(2) - (3) \text{ gives:} \quad 11x = 22 \quad \text{from which,} \quad x = 2$$

Substituting in (1) gives: $y - 6 = -5$ from which, $y = 1$ as obtained graphically above.

4. A piece of elastic is tied to a support so that it hangs vertically, and a pan, on which weights can be placed, is attached to the free end. The length of the elastic is measured as various weights are added to the pan and the results obtained are as follows:

Load, W (N)	5	10	15	20	25
Length, l (cm)	60	72	84	96	108

Plot a graph of load (horizontally) against length (vertically) and determine:

- (a) the value length when the load is 17 N, (b) the value of load when the length is 74 cm, (c) its gradient, and (d) the equation of the graph.

A graph of load against length is plotted as shown below.

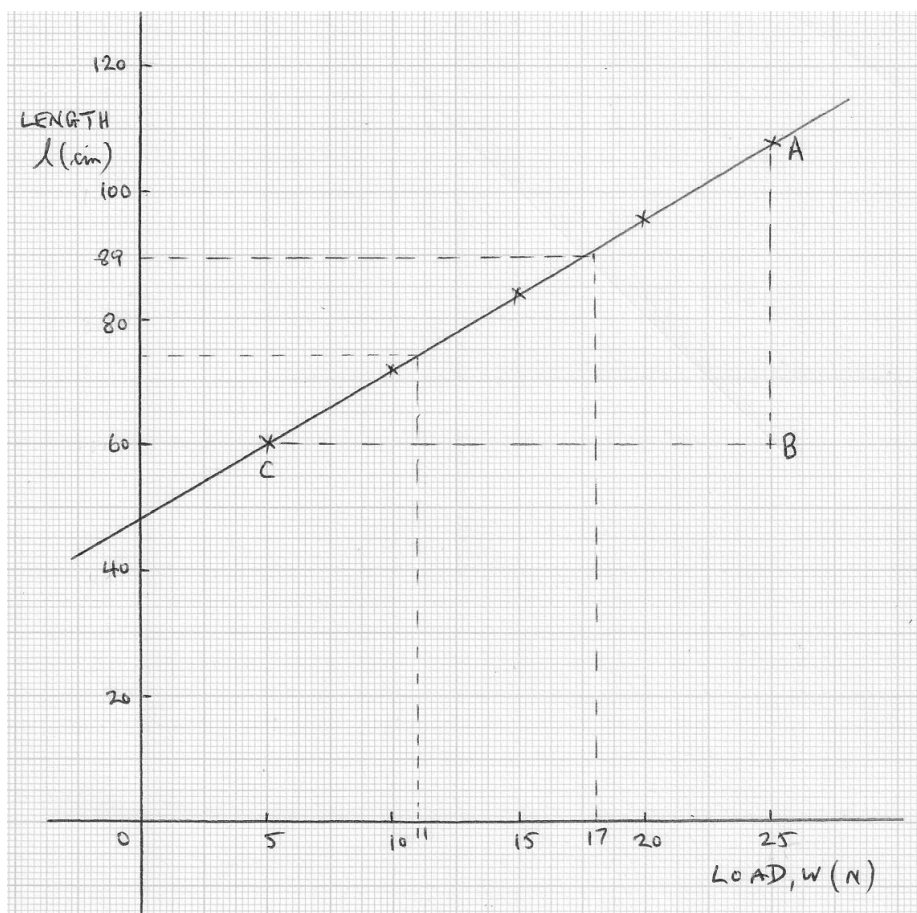
(a) When the load is 17 N, the **length = 89 cm**

(b) When the length is 74 cm, the **load = 11 N**

(c) **Gradient** = $\frac{AB}{BC} = \frac{108 - 60}{25 - 5} = \frac{48}{20} = 2.4$

(d) Vertical axis intercept = 48 cm

Hence, the equation of the graph is: $l = 2.4W + 48$



5. The following table gives the effort P to lift a load W with a small lifting machine:

W (N)	10	20	30	40	50	60
---------	----	----	----	----	----	----

P (N) 5.1 6.4 8.1 9.6 10.9 12.4

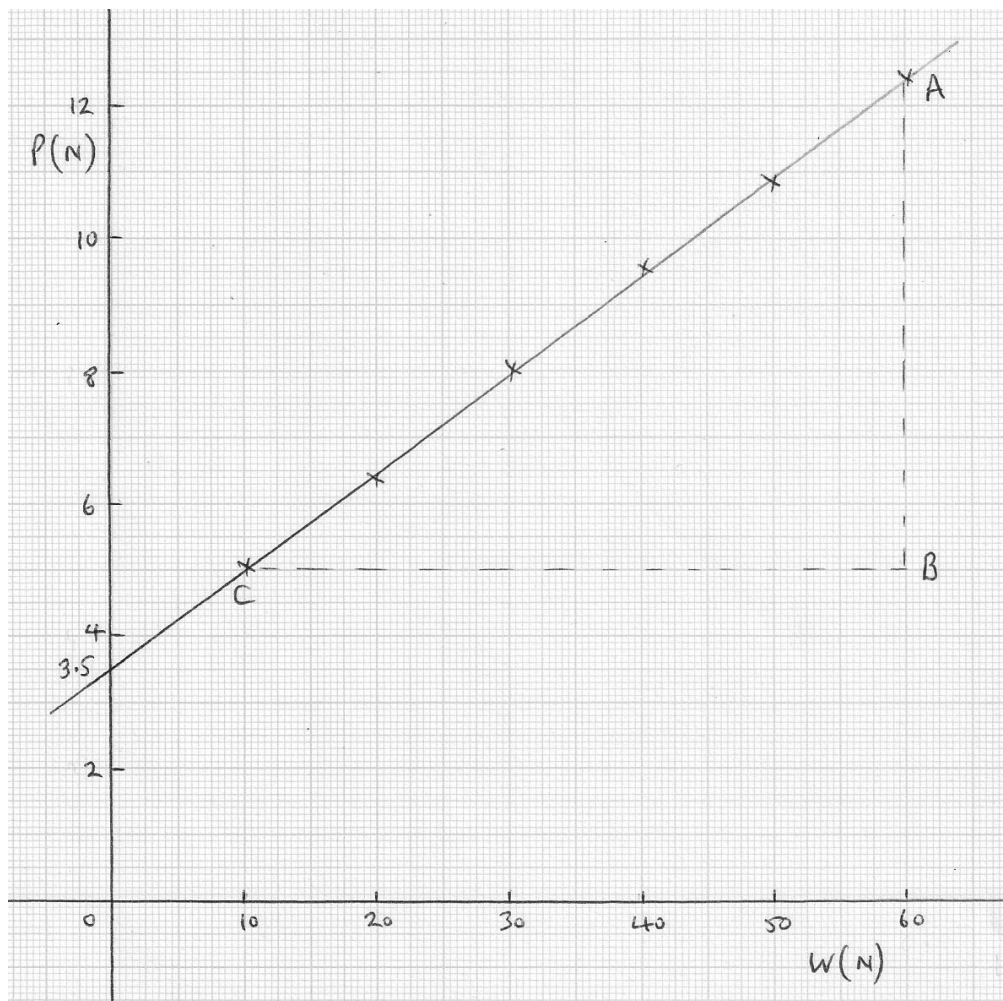
Plot W horizontally against P vertically and show that the values lie approximately on a straight line. Determine the probable relationship connecting P and W in the form $P = aW + b$.

A graph of W against P is shown plotted below.

$$\text{Gradient of graph} = \frac{AB}{BC} = \frac{12.5 - 5}{60 - 10} = \frac{7.5}{50} = \mathbf{0.15}$$

Vertical axis intercept = 3.5

Hence, the equation of the graph is: $P = 0.15W + 3.5$



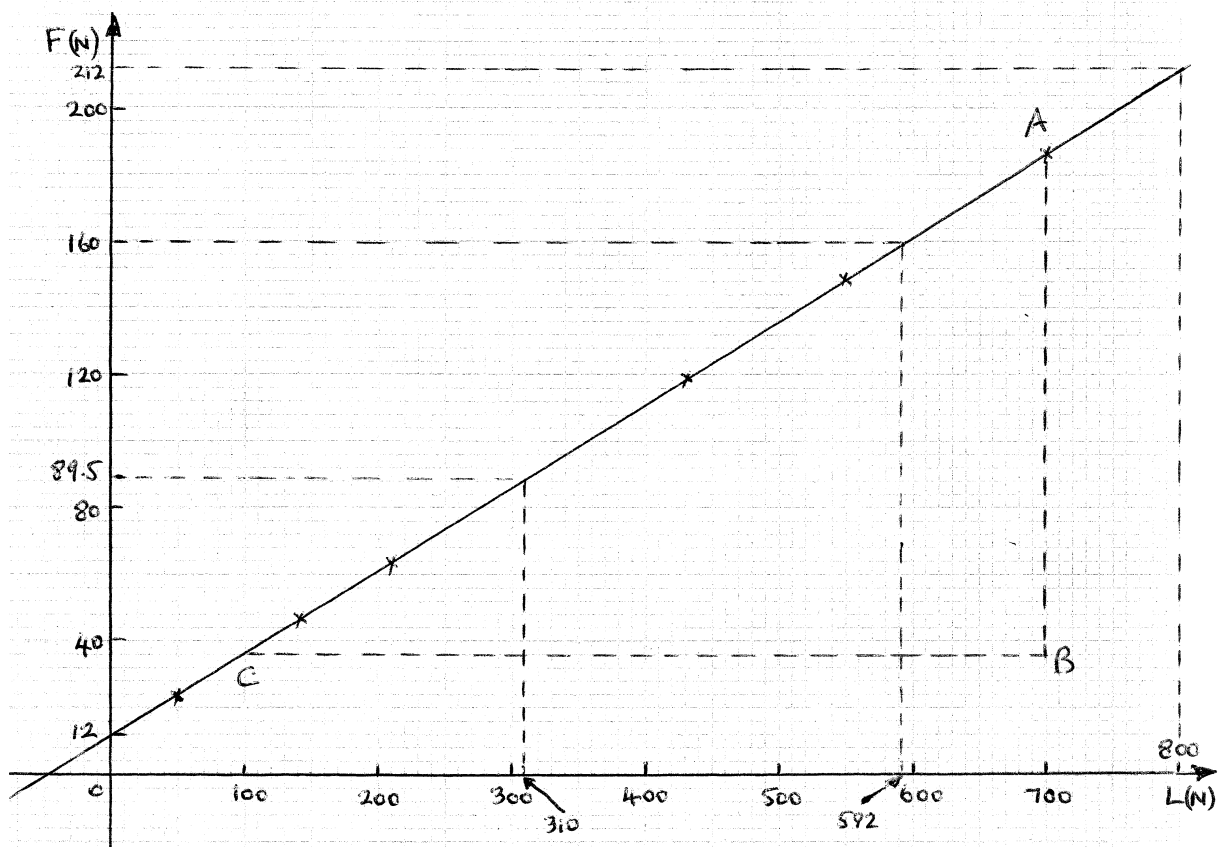
EXERCISE 15, Page 36

1. The following table gives the force F Newtons which, when applied to a lifting machine, overcomes a corresponding load of L Newtons.

Force F Newtons	25	47	64	120	149	187
Load L Newtons	50	140	210	430	550	700

Choose suitable scales and plot a graph of F (vertically) against L (horizontally). Draw the best straight line through the points. Determine from the graph (a) the gradient, (b) the F -axis intercept, (c) the equation of the graph, (d) the force applied when the load is 310 N, and (e) the load that a force of 160 N will overcome. (f) If the graph were to continue in the same manner, what value of force will be needed to overcome a 800 N load?

A graph of F against L is shown below.



From the graph:

(a) the **gradient** = $\frac{AB}{BC} = \frac{187 - 37}{700 - 100} = \frac{150}{600} = 0.25$

(b) the F -axis intercept = **12 N**

(c) the equation of the graph is: **$F = 0.25L + 12$**

(d) the force applied when the load is 310 N is **89.5 N**

(e) the load that a force of 160 N will overcome is **592 N**

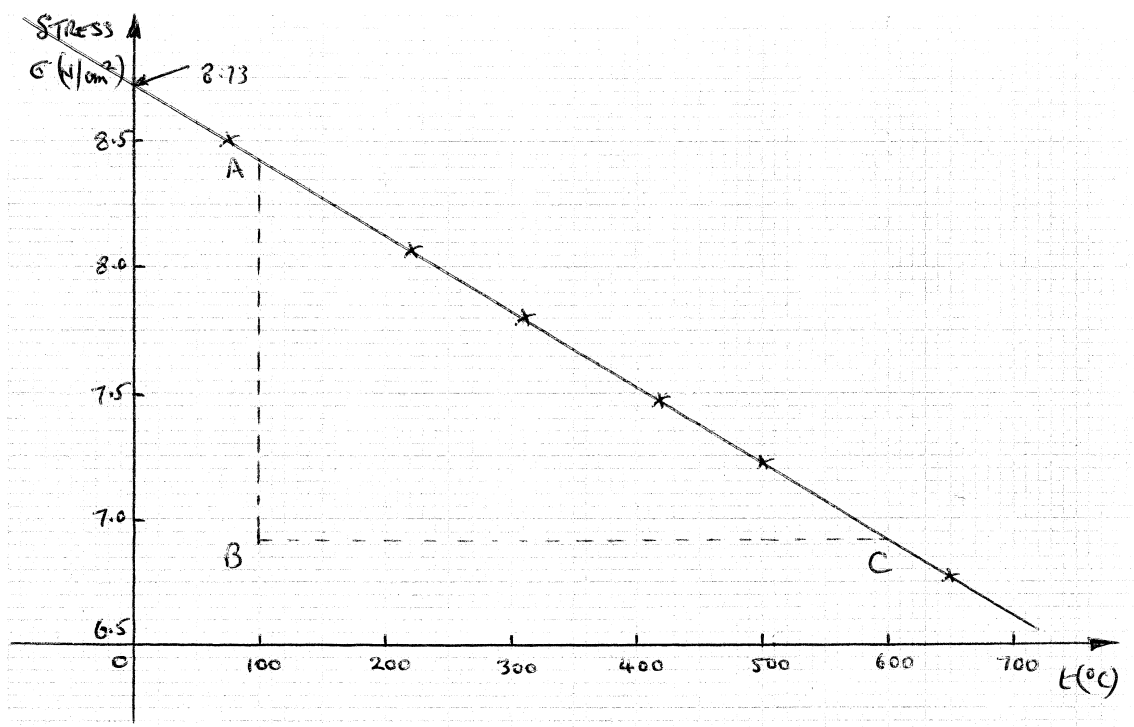
(f) If the graph were to continue in the same manner the force needed to overcome a 800 N load is **212 N**. From the equation of the graph, $F = 0.25L + 12 = 0.25(800) + 12 = 200 + 12 = \mathbf{212\text{ N}}$.

2. The following table gives the results of tests carried out to determine the breaking stress σ of rolled copper at various temperatures, t :

Stress σ (N/cm ²)	8.51	8.07	7.80	7.47	7.23	6.78
Temperature t (°C)	75	220	310	420	500	650

Plot a graph of stress (vertically) against temperature (horizontally). Draw the best straight line through the plotted co-ordinates. Determine the slope of the graph and the vertical axis intercept.

A graph of stress σ against temperature t is shown below.



$$\text{The slope of graph} = \frac{AB}{BC} = \frac{8.45 - 6.95}{100 - 600} = \frac{1.50}{-500} = -0.003$$

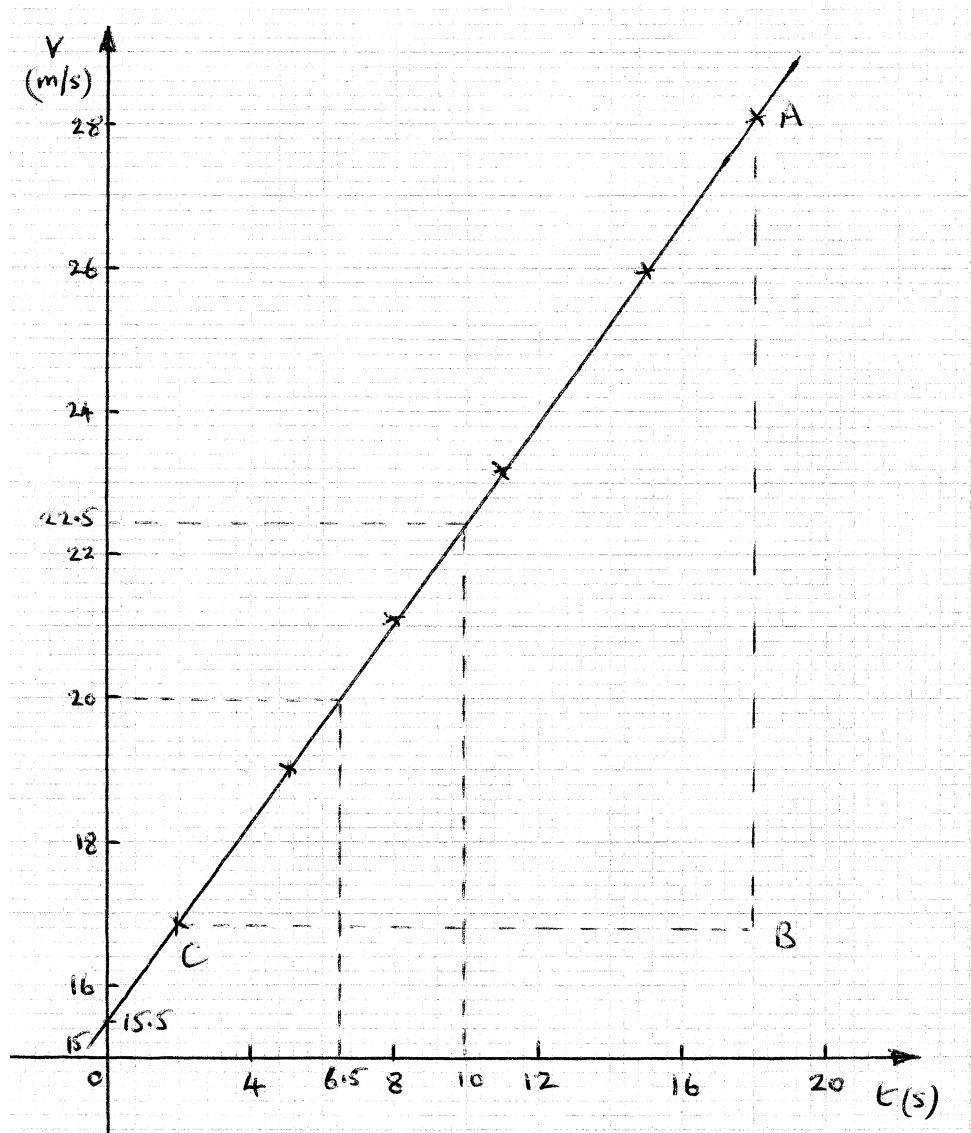
$$\text{Vertical axis intercept} = 8.73 \text{ N/cm}^2$$

3. The velocity v of a body after varying time intervals t was measured as follows:

t (seconds)	2	5	8	11	15	18
v (m/s)	16.9	19.0	21.1	23.2	26.0	28.1

Plot v vertically and t horizontally and draw a graph of velocity against time. Determine from the graph (a) the velocity after 10 s, (b) the time at 20 m/s and (c) the equation of the graph.

A graph of velocity v against time t is shown below.



From the graph:

(a) After 10 s, the velocity = **22.5 m/s**

(b) At 20 m/s, the time = **6.5 s**

(c) Gradient of graph = $\frac{AB}{BC} = \frac{28.1 - 16.9}{18 - 2} = \frac{11.2}{16} = 0.7$

Vertical axis intercept at $t = 0$, is $v = 15.5 \text{ m/s}$

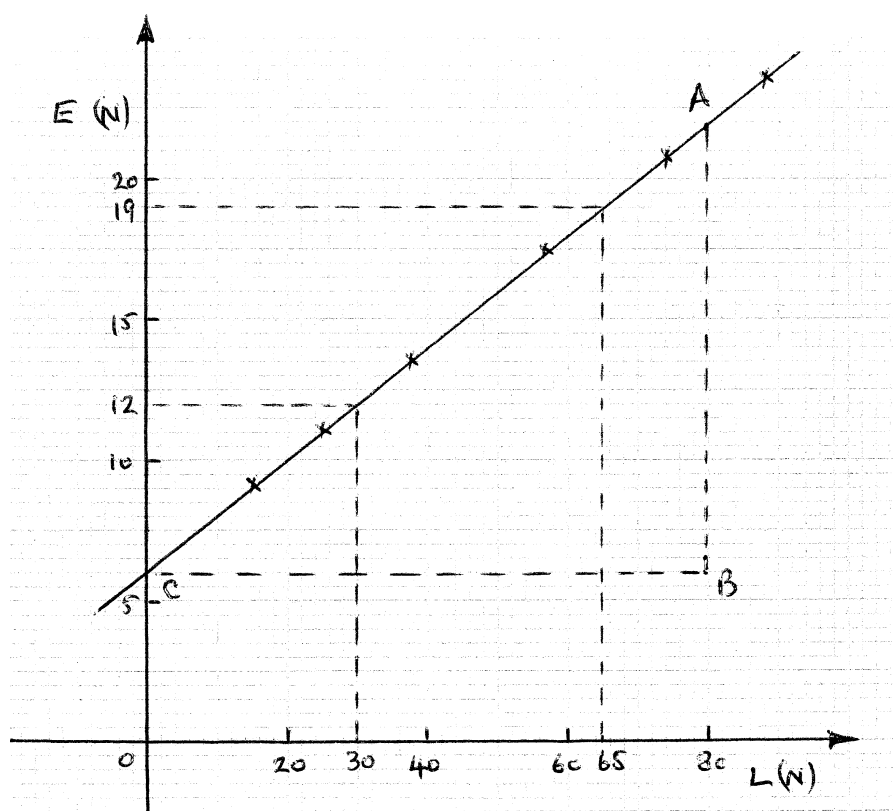
Hence, the equation of the graph is: $v = 0.7t + 15.5$

4. An experiment with a set of pulley blocks gave the following results:

Effort, E (newtons)	9.0	11.0	13.6	17.4	20.8	23.6
Load, L (newtons)	15	25	38	57	74	88

Plot a graph of effort (vertically) against load (horizontally) and determine (a) the gradient, (b) the vertical axis intercept, (c) the law of the graph, (d) the effort when the load is 30 N and (e) the load when the effort is 19 N.

A graph of effort E against load L is shown below.



(a) Gradient of straight line = $\frac{AB}{BC} = \frac{22-6}{80-0} = \frac{16}{80} = \frac{1}{5}$ or 0.2

(b) Vertical axis intercept = 6

(c) The law of the graph is: $E = 0.2L + 6$

(d) From the graph, when the load is 30 N, effort, $E = 12 \text{ N}$

(e) From the graph, when the effort is 19 N, load, $L = 65 \text{ N}$

EXERCISE 16, Page 38

1. Differentiate with respect to x : (a) $5x^5$ (b) $\frac{1}{x}$

(a) If $y = 5x^5$ then $\frac{dy}{dx} = (5)(5x^4) = \mathbf{25x^4}$

(b) If $y = \frac{1}{x} = x^{-1}$ then $\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$

2. Differentiate with respect to x : (a) $\frac{-4}{x^2}$ (b) 6

(a) If $y = \frac{-4}{x^2} = -4x^{-2}$ then $\frac{dy}{dx} = (-4)(-2x^{-3}) = 8x^{-3}$ or $\frac{8}{x^3}$

(b) If $y = 6$ then $\frac{dy}{dx} = \mathbf{0}$

3. Differentiate with respect to x : (a) $2x$ (b) $2\sqrt{x}$

(a) If $y = 2x$ then $\frac{dy}{dx} = \mathbf{2}$

(b) If $y = 2\sqrt{x} = 2x^{\frac{1}{2}}$ then $\frac{dy}{dx} = (2)\left(\frac{1}{2}x^{-\frac{1}{2}}\right) = x^{-\frac{1}{2}} = \frac{1}{x^{\frac{1}{2}}} = \frac{1}{\sqrt{x}}$

4. Differentiate with respect to x : (a) $3\sqrt[3]{x^5}$ (b) $\frac{4}{\sqrt{x}}$

(a) If $y = 3\sqrt[3]{x^5} = 3x^{\frac{5}{3}}$ then $\frac{dy}{dx} = (3)\left(\frac{5}{3}x^{\frac{2}{3}}\right) = 5x^{\frac{2}{3}} = \mathbf{5\sqrt[3]{x^2}}$

(b) If $y = \frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}} = 4x^{-\frac{1}{2}}$ then $\frac{dy}{dx} = (4)\left(-\frac{1}{2}x^{-\frac{3}{2}}\right) = -2x^{-\frac{3}{2}} = -\frac{2}{x^{\frac{3}{2}}} = -\frac{2}{\sqrt{x^3}}$

5. Differentiate with respect to x : (a) $2 \sin 3x$ (b) $-4 \cos 2x$

(a) If $y = 2 \sin 3x$ then $\frac{dy}{dx} = (2)(3 \cos 3x) = \mathbf{6 \cos 3x}$

(b) If $y = -4 \cos 2x$ then $\frac{dy}{dx} = (-4)(-2 \sin 2x) = \mathbf{8 \sin 2x}$

6. Differentiate with respect to x : (a) $2e^{6x}$ (b) $4 \ln 9x$

(a) If $y = 2e^{6x}$ then $\frac{dy}{dx} = (2)(6e^{6x}) = \mathbf{12e^{6x}}$

(b) If $y = 4 \ln 9x$ then $\frac{dy}{dx} = (4)\left(\frac{1}{x}\right) = \mathbf{\frac{4}{x}}$

7. Find the gradient of the curve $y = 2t^4 + 3t^3 - t + 4$ at the points $(0, 4)$ and $(1, 8)$.

If $y = 2t^4 + 3t^3 - t + 4$, then gradient, $\frac{dy}{dt} = 8t^3 + 9t^2 - 1$

At $(0, 4)$, $t = 0$, hence gradient $= 8(0)^3 + 9(0)^2 - 1 = \mathbf{-1}$

At $(1, 8)$, $t = 1$, hence gradient $= 8(1)^3 + 9(1)^2 - 1 = \mathbf{16}$

8. (a) Differentiate $y = \frac{2}{\theta^2} + 2 \ln 2\theta - 2(\cos 5\theta + 3 \sin 2\theta) - \frac{2}{e^{3\theta}}$

(b) Evaluate $\frac{dy}{d\theta}$ when $\theta = \frac{\pi}{2}$, correct to 4 significant figures.

(a) $y = \frac{2}{\theta^2} + 2 \ln 2\theta - 2(\cos 5\theta + 3 \sin 2\theta) - \frac{2}{e^{3\theta}}$
 $= 2\theta^{-2} + 2 \ln 2\theta - 2 \cos 5\theta - 6 \sin 2\theta - 2e^{-3\theta}$

Hence, $\frac{dy}{d\theta} = -4\theta^{-3} + \frac{2}{\theta} - 2(-5 \sin 5\theta) - 6(2 \cos 2\theta) - 2(-3e^{-3\theta})$
 $= -\frac{4}{\theta^3} + \frac{2}{\theta} + 10 \sin 5\theta - 12 \cos 2\theta + \frac{6}{e^{3\theta}}$

$$\begin{aligned}
 \text{(b) When } \theta = \frac{\pi}{2}, \quad \frac{dy}{d\theta} &= -\frac{4}{\left(\frac{\pi}{2}\right)^3} + \frac{2}{\left(\frac{\pi}{2}\right)} + 10\sin\frac{5\pi}{2} - 12\cos\frac{2\pi}{2} + \frac{6}{e^{3\left(\frac{\pi}{2}\right)}} \\
 &= -1.032049 + 1.2732395 + 10 + 12 + 0.0538997 \\
 &= \mathbf{22.30}, \text{ correct to 4 significant figures}
 \end{aligned}$$

EXERCISE 17, Page 40

1. Determine: (a) $\int 4 \, dx$ (b) $\int 7x \, dx$

$$(a) \int 4 \, dx = 4x + c$$

$$(b) \int 7x \, dx = \frac{7x^2}{2} + c$$

2. Determine: (a) $\int 5x^3 \, dx$ (b) $\int 3t^7 \, dt$

$$(a) \int 5x^3 \, dx = (5) \frac{x^4}{4} + c = \frac{5}{4}x^4 + c$$

$$(b) \int 3t^7 \, dt = (3) \frac{t^8}{8} + c = \frac{3}{8}t^8 + c$$

3. Determine: (a) $\int \frac{2}{5}x^2 \, dx$ (b) $\int \frac{5}{6}x^3 \, dx$

$$(a) \int \frac{2}{5}x^2 \, dx = \left(\frac{2}{5}\right) \frac{x^3}{3} + c = \frac{2}{15}x^3 + c$$

$$(b) \int \frac{5}{6}x^3 \, dx = \left(\frac{5}{6}\right) \frac{x^4}{4} + c = \frac{5}{24}x^4 + c$$

4. Determine: (a) $\int (2x^4 - 3x) \, dx$ (b) $\int (2 - 3t^3) \, dt$

$$(a) \int (2x^4 - 3x) \, dx = (2) \frac{x^5}{5} - (3) \frac{x^2}{2} + c = \frac{2}{5}x^5 - \frac{3}{2}x^2 + c$$

$$(b) \int (2 - 3t^3) \, dt = 2t - (3) \frac{t^4}{4} + c = 2t - \frac{3}{4}t^4 + c$$

5. Determine: (a) $\int \frac{4}{3x^2} dx$ (b) $\int \frac{3}{4x^4} dx$

$$(a) \int \frac{4}{3x^2} dx = \frac{4}{3} \int x^{-2} dx = \left(\frac{4}{3}\right) \left(\frac{x^{-2+1}}{-2+1}\right) + c = \left(\frac{4}{3}\right) \left(\frac{x^{-1}}{-1}\right) + c = -\frac{4}{3} x^{-1} + c = -\frac{4}{3x} + c$$

$$(b) \int \frac{3}{4x^4} dx = \frac{3}{4} \int x^{-4} dx = \left(\frac{3}{4}\right) \left(\frac{x^{-4+1}}{-4+1}\right) + c = \left(\frac{3}{4}\right) \left(\frac{x^{-3}}{-3}\right) + c = -\frac{1}{4} x^{-3} + c = -\frac{1}{4x^3} + c$$

6. Determine: (a) $2 \int \sqrt{x^3} dx$ (b) $\int \frac{1}{4} \sqrt[4]{x^5} dx$

$$(a) 2 \int \sqrt{x^3} dx = 2 \int x^{\frac{3}{2}} dx = (2) \left(\frac{x^{\frac{3}{2}+1}}{\frac{3}{2}+1}\right) + c = (2) \left(\frac{x^{\frac{5}{2}}}{\frac{5}{2}}\right) + c = (2) \left(\frac{2}{5} x^{\frac{5}{2}}\right) + c = \frac{4}{5} \sqrt{x^5} + c$$

$$(b) \int \frac{1}{4} \sqrt[4]{x^5} dx = \frac{1}{4} \int x^{\frac{5}{4}} dx = \left(\frac{1}{4}\right) \left(\frac{x^{\frac{5}{4}+1}}{\frac{5}{4}+1}\right) + c = \frac{1}{4} \left(\frac{x^{\frac{9}{4}}}{\frac{9}{4}}\right) + c = \left(\frac{1}{4}\right) \left(\frac{4}{9} x^{\frac{9}{4}}\right) + c = \frac{1}{9} \sqrt[4]{x^9} + c$$

7. Determine: (a) $\int 3 \cos 2x dx$ (b) $\int 7 \sin 3\theta d\theta$

$$(a) \int 3 \cos 2x dx = (3) \left(\frac{1}{2} \sin 2x\right) + c = \frac{3}{2} \sin 2x + c$$

$$(b) \int 7 \sin 3\theta d\theta = (7) \left(-\frac{1}{3} \cos 3\theta\right) + c = -\frac{7}{3} \cos 3\theta + c$$

8. Determine: (a) $\int 3 \sin \frac{1}{2} x dx$ (b) $\int 6 \cos \frac{1}{3} x dx$

$$(a) \int 3 \sin \frac{1}{2} x dx = (3) \left(-\frac{1}{\frac{1}{2}} \cos \frac{1}{2} x\right) + c = -6 \cos \frac{1}{2} x + c$$

$$(b) \int 6 \cos \frac{1}{3} x \, dx = (6) \left(\frac{1}{\frac{1}{3}} \sin \frac{1}{3} x \right) + c = \mathbf{18 \sin \frac{1}{3} x + c}$$

9. Determine: (a) $\int \frac{3}{4} e^{2x} \, dx$ (b) $\int \frac{2}{3x} \, dx$

$$(a) \int \frac{3}{4} e^{2x} \, dx = \frac{3}{4} \left(\frac{1}{2} e^{2x} \right) + c = \mathbf{\frac{3}{8} e^{2x} + c}$$

$$(b) \int \frac{2}{3x} \, dx = \left(\frac{2}{3} \right) \int \frac{1}{x} \, dx = \mathbf{\frac{2}{3} \ln x + c}$$

EXERCISE 18, Page 41

1. Evaluate: (a) $\int_1^2 x \, dx$ (b) $\int_1^2 (x-1) \, dx$

$$(a) \int_1^2 x \, dx = \left[\frac{x^2}{2} \right]_1^2 = \left[\frac{2^2}{2} \right] - \left[\frac{1^2}{2} \right] = (2) - (0.5) = \mathbf{1.5}$$

$$(b) \int_1^2 (x-1) \, dx = \left[\frac{x^2}{2} - x \right]_1^2 = \left[\left(\frac{(2)^2}{2} - 2 \right) - \left(\frac{(1)^2}{2} - 1 \right) \right] = [(0) - (-0.5)] = \mathbf{0.5}$$

2. Evaluate: (a) $\int_1^4 5x^2 \, dx$ (b) $\int_{-1}^1 -\frac{3}{4}t^2 \, dt$

$$(a) \int_1^4 5x^2 \, dx = \left[\frac{5x^3}{3} \right]_1^4 = \left[\frac{5(4)^3}{3} \right] - \left[\frac{5(1)^3}{3} \right] = \left(\frac{320}{3} \right) - \left(\frac{5}{3} \right) = \frac{315}{3} = \mathbf{105}$$

$$(b) \int_{-1}^1 -\frac{3}{4}t^2 \, dt = -\frac{3}{4} \left[\frac{t^3}{3} \right]_{-1}^1 = -\frac{3}{4} \left[\left(\frac{(1)^3}{3} - \frac{(-1)^3}{3} \right) \right] = -\frac{3}{4} \left[\frac{1}{3} - -\frac{1}{3} \right] = -\frac{3}{4} \left(\frac{2}{3} \right) = -\frac{1}{2} \text{ or } \mathbf{-0.5}$$

3. Evaluate: (a) $\int_{-1}^2 (3-x^2) \, dx$ (b) $\int_1^3 (x^2-4x+3) \, dx$

$$(a) \int_{-1}^2 (3-x^2) \, dx = \left[3x - \frac{x^3}{3} \right]_{-1}^2 = \left[3(2) - \frac{2^3}{3} \right] - \left[3(-1) - \frac{(-1)^3}{3} \right] = \left(6 - \frac{8}{3} \right) - \left(-3 - -\frac{1}{3} \right)$$

$$= 3\frac{1}{3} - \left(-2\frac{2}{3} \right) = \mathbf{6}$$

$$(b) \int_1^3 (x^2-4x+3) \, dx = \left[\frac{x^3}{3} - \frac{4x^2}{2} + 3x \right]_1^3 = (9-18+9) - \left(\frac{1}{3} - 2 + 3 \right) = (0) - \left(1\frac{1}{3} \right)$$

$$= \mathbf{-1\frac{1}{3} \text{ or } -1.333}$$

4. Evaluate: (a) $\int_0^4 2\sqrt{x} \, dx$ (b) $\int_2^3 \frac{1}{x^2} \, dx$

$$(a) \int_0^4 2\sqrt{x} \, dx = \int_0^4 2x^{\frac{1}{2}} \, dx = \left[\frac{2x^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^4 = \left[\frac{4}{3} \sqrt{x^3} \right]_0^4 = \left[\frac{4}{3} \sqrt{4^3} \right] - \left[\frac{4}{3} \sqrt{0^3} \right] = \left(\frac{32}{3} \right) - (0) = \mathbf{10.67}$$

$$(b) \int_2^3 \frac{1}{x^2} \, dx = \int_2^3 x^{-2} \, dx = \left[\frac{x^{-1}}{-1} \right]_2^3 = - \left[\frac{1}{x} \right]_2^3 = - \left[\left(\frac{1}{3} - \frac{1}{2} \right) \right] = - \left[-\frac{1}{6} \right] = \frac{1}{6} = \mathbf{0.1667}$$

5. Evaluate: (a) $\int_0^{\pi} \frac{3}{2} \cos \theta \, d\theta$ (b) $\int_0^{\pi/2} 4 \cos \theta \, d\theta$

$$(a) \int_0^{\pi} \frac{3}{2} \cos \theta \, d\theta = \frac{3}{2} [\sin \theta]_0^{\pi} = \frac{3}{2} [\sin \pi - \sin 0] = \frac{3}{2} (0 - 0) = \mathbf{0}$$

$$(b) \int_0^{\pi/2} 4 \cos \theta \, d\theta = 4 [\sin \theta]_0^{\pi/2} = 4 \left[\sin \frac{\pi}{2} - \sin 0 \right] = 4(1 - 0) = \mathbf{4}$$

6. Evaluate: (a) $\int_{\pi/6}^{\pi/3} 2 \sin 2\theta \, d\theta$ (b) $\int_0^2 3 \sin t \, dt$

$$(a) \int_{\pi/6}^{\pi/3} 2 \sin 2\theta \, d\theta = -\frac{2}{2} [\cos 2\theta]_{\pi/6}^{\pi/3} = - \left[\cos \frac{2\pi}{3} - \cos \frac{2\pi}{6} \right] \quad (\text{note that } \frac{2\pi}{3} \text{ and } \frac{2\pi}{6} \text{ are in radians})$$

$$= - [-0.5 - 0.5] = - [-1] = \mathbf{1}$$

$$(b) \int_0^2 3 \sin t \, dt = -3 [\cos t]_0^2 = -3 [\cos 2 - \cos 0] \quad (\text{note that 2 is 2 radians})$$

$$= -3 [-0.41615 - 1] = \mathbf{4.248}$$

7. Evaluate: (a) $\int_0^1 3e^{3t} \, dt$ (b) $\int_2^3 \frac{2}{3x} \, dx$

$$(a) \int_0^1 3e^{3t} \, dt = 3 \left[\frac{1}{3} e^{3t} \right]_0^1 = [e^{3t}]_0^1 = (e^3 - e^0) = (20.0855 - 1) = \mathbf{19.09}$$

$$(b) \int_2^3 \frac{2}{3x} \, dx = \frac{2}{3} \int_2^3 \frac{1}{x} \, dx = \frac{2}{3} [\ln x]_2^3 = \frac{2}{3} (\ln 3 - \ln 2) = \mathbf{0.2703}$$

EXERCISE 19, Page 43

1. Calculate the value of $\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix}$

$$\begin{vmatrix} 2 & -3 \\ 5 & 1 \end{vmatrix} = (2)(1) - (-3)(5) = 2 - -15 = \mathbf{17}$$

2. Evaluate $\begin{vmatrix} 1 & 2 \\ -5 & -7 \end{vmatrix}$

$$\begin{vmatrix} 1 & 2 \\ -5 & -7 \end{vmatrix} = (1)(-7) - (2)(-5) = -7 - -10 = \mathbf{3}$$

3. Find the value of $\begin{vmatrix} 2 & 3 & -1 \\ -5 & 0 & 4 \\ 1 & -4 & -2 \end{vmatrix}$

$$\begin{aligned} \begin{vmatrix} 2 & 3 & -1 \\ -5 & 0 & 4 \\ 1 & -4 & -2 \end{vmatrix} &= 2 \begin{vmatrix} 0 & 4 \\ -4 & -2 \end{vmatrix} - 3 \begin{vmatrix} -5 & 4 \\ 1 & -2 \end{vmatrix} + (-1) \begin{vmatrix} -5 & 0 \\ 1 & -4 \end{vmatrix} \\ &= 2(0 - -16) - 3(10 - 4) - (20 - 0) \\ &= 32 - 18 - 20 = \mathbf{-6} \end{aligned}$$

4. Find the value of $\begin{vmatrix} -3 & 2 & 5 \\ 7 & 1 & -2 \\ 4 & 0 & -1 \end{vmatrix}$

$$\begin{aligned} \begin{vmatrix} -3 & 2 & 5 \\ 7 & 1 & -2 \\ 4 & 0 & -1 \end{vmatrix} &= -3 \begin{vmatrix} 1 & -2 \\ 0 & -1 \end{vmatrix} - 2 \begin{vmatrix} 7 & -2 \\ 4 & -1 \end{vmatrix} + 5 \begin{vmatrix} 7 & 1 \\ 4 & 0 \end{vmatrix} \\ &= -3(-1 - -0) - 2(-7 - -8) + 5(0 - 4) \end{aligned}$$

$$= 3 - 2 - 20 = -19$$

5. Find the scalar product $\mathbf{a} \bullet \mathbf{b}$ when $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$

If $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = 2\mathbf{i} + 5\mathbf{j} + \mathbf{k}$

then $\mathbf{a} \bullet \mathbf{b} = (3)(2) + (2)(5) + (-1)(1) = 6 + 10 - 1 = 15$

6. Find the scalar product $\mathbf{p} \bullet \mathbf{q}$ when $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{q} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

If $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$ and $\mathbf{q} = \mathbf{i} + 2\mathbf{j} + 5\mathbf{k}$

then $\mathbf{p} \bullet \mathbf{q} = (2)(1) + (-3)(2) + (4)(5) = 2 - 6 + 20 = 16$

7. Given $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{q} = 4\mathbf{j} - \mathbf{k}$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$, determine the quantities:

(a) $\mathbf{p} \bullet \mathbf{q}$ (b) $\mathbf{p} \bullet \mathbf{r}$ (c) $\mathbf{q} \bullet \mathbf{r}$

If $\mathbf{p} = 2\mathbf{i} - 3\mathbf{j}$, $\mathbf{q} = 4\mathbf{j} - \mathbf{k}$ and $\mathbf{r} = \mathbf{i} + 2\mathbf{j} - 3\mathbf{k}$,

(a) $\mathbf{p} \bullet \mathbf{q} = (2)(0) + (-3)(4) + (0)(-1) = 0 - 12 - 0 = -12$

(b) $\mathbf{p} \bullet \mathbf{r} = (2)(1) + (-3)(2) + (0)(-3) = 2 - 6 - 0 = -4$

(c) $\mathbf{q} \bullet \mathbf{r} = (0)(1) + (4)(2) + (-1)(-3) = 0 + 8 + 3 = 11$

8. Calculate the work done by a force $\mathbf{F} = (-2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k})$ N when its point of application moves from point $(-\mathbf{i} - 5\mathbf{j} + 2\mathbf{k})$ m to the point $(4\mathbf{i} - \mathbf{j} + 8\mathbf{k})$ m.

Work done = $\mathbf{F} \bullet \mathbf{d}$ where $\mathbf{d} = (4\mathbf{i} - \mathbf{j} + 8\mathbf{k}) - (-\mathbf{i} - 5\mathbf{j} + 2\mathbf{k}) = 5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

Hence, **work done** = $(-2\mathbf{i} + 3\mathbf{j} + 5\mathbf{k}) \bullet (5\mathbf{i} + 4\mathbf{j} + 6\mathbf{k})$

$$= -10 + 12 + 30 = 32 \text{ N m}$$

9. Given that $\mathbf{p} = 3\mathbf{i} + 2\mathbf{k}$, $\mathbf{q} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r} = -4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$, determine

(a) $\mathbf{p} \times \mathbf{q}$ (b) $\mathbf{q} \times \mathbf{r}$

If $\mathbf{p} = 3\mathbf{i} + 2\mathbf{k}$, $\mathbf{q} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{r} = -4\mathbf{i} + 3\mathbf{j} - \mathbf{k}$

$$\begin{aligned} \text{(a) } \mathbf{p} \times \mathbf{q} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 2 \\ 1 & -2 & 3 \end{vmatrix} = \mathbf{i} \begin{vmatrix} 0 & 2 \\ -2 & 3 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 3 & 0 \\ 1 & -2 \end{vmatrix} \\ &= \mathbf{i}(0 - -4) - \mathbf{j}(9 - 2) + \mathbf{k}(-6 - 0) \\ &= \mathbf{4i} - \mathbf{7j} - \mathbf{6k} \end{aligned}$$

$$\begin{aligned} \text{(b) } \mathbf{q} \times \mathbf{r} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 3 \\ -4 & 3 & -1 \end{vmatrix} = \mathbf{i} \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} - \mathbf{j} \begin{vmatrix} 1 & 3 \\ -4 & -1 \end{vmatrix} + \mathbf{k} \begin{vmatrix} 1 & -2 \\ -4 & 3 \end{vmatrix} \\ &= \mathbf{i}(2 - 9) - \mathbf{j}(-1 - -12) + \mathbf{k}(3 - 8) \\ &= -\mathbf{7i} - \mathbf{11j} - \mathbf{5k} \end{aligned}$$

10. A force of $(4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k})$ newtons acts on a line through point P having co-ordinates (2, 1, 3) metres. Determine the moment vector about point Q having co-ordinates (5, 0, -2) metres.

Position vector, $\mathbf{r} = (2\mathbf{i} + \mathbf{j} + 3\mathbf{k}) - (5\mathbf{i} + 0\mathbf{j} - 2\mathbf{k}) = -3\mathbf{i} + \mathbf{j} + 5\mathbf{k}$

$$\begin{aligned} \text{Moment, } \mathbf{M} = \mathbf{r} \times \mathbf{F} \quad \text{where } \mathbf{M} &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 1 & 5 \\ 4 & -3 & 2 \end{vmatrix} = (2 + 15)\mathbf{i} - (-6 - 20)\mathbf{j} + (9 - 4)\mathbf{k} \\ &= \mathbf{(17i + 26j + 5k) N m} \end{aligned}$$