

# CHAPTER 1 REVISIONARY MATHEMATICS

## EXERCISE 1, Page 2

1. Convert the following angles to degrees correct to 3 decimal places (where necessary):

(a) 0.6 rad    (b) 0.8 rad    (c) 2 rad    (d) 3.14159 rad

$$(a) 0.6 \text{ rad} = 0.6 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{34.377^\circ}$$

$$(b) 0.8 \text{ rad} = 0.8 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{45.837^\circ}$$

$$(c) 2 \text{ rad} = 2 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{114.592^\circ}$$

$$(d) 3.14159 \text{ rad} = 3.14159 \text{ rad} \times \frac{180^\circ}{\pi \text{ rad}} = \mathbf{180^\circ}$$

2. Convert the following angles to radians correct to 4 decimal places:

(a)  $45^\circ$     (b)  $90^\circ$     (c)  $120^\circ$     (d)  $180^\circ$

$$(a) 45^\circ = 45^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{4} \text{ rad or } \mathbf{0.7854 \text{ rad}}$$

$$(b) 90^\circ = 90^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{\pi}{2} \text{ rad or } \mathbf{1.5708 \text{ rad}}$$

$$(c) 120^\circ = 120^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \frac{2\pi}{3} \text{ rad or } \mathbf{2.0944 \text{ rad}}$$

$$(d) 180^\circ = 180^\circ \times \frac{\pi \text{ rad}}{180^\circ} = \pi \text{ rad or } \mathbf{3.1416 \text{ rad}}$$

## EXERCISE 2, Page 3

1. Find the cosine, sine and tangent of the following angles, where appropriate each correct to 4 decimal places:

- (a)  $60^\circ$  (b)  $90^\circ$  (c)  $150^\circ$  (d)  $180^\circ$  (e)  $210^\circ$  (f)  $270^\circ$  (g)  $330^\circ$  (h)  $-30^\circ$  (i)  $420^\circ$  (j)  $450^\circ$   
(k)  $510^\circ$

(a)  $\cos 60^\circ = \mathbf{0.5000}$

$\sin 60^\circ = \mathbf{0.8660}$

$\tan 60^\circ = \mathbf{1.7321}$

(b)  $\cos 90^\circ = \mathbf{0}$

$\sin 90^\circ = \mathbf{1}$

$\tan 90^\circ = \infty$

(c)  $\cos 150^\circ = -\mathbf{0.8660}$

$\sin 150^\circ = \mathbf{0.5000}$

$\tan 150^\circ = -\mathbf{0.5774}$

(d)  $\cos 180^\circ = -\mathbf{1}$

$\sin 180^\circ = \mathbf{0}$

$\tan 180^\circ = \mathbf{0}$

(e)  $\cos 210^\circ = -\mathbf{0.8660}$

$\sin 210^\circ = -\mathbf{0.5000}$

$\tan 210^\circ = \mathbf{0.5774}$

(f)  $\cos 270^\circ = \mathbf{0}$

$\sin 270^\circ = -\mathbf{1}$

$\tan 270^\circ = -\infty$

(g)  $\cos 330^\circ = \mathbf{0.8660}$

$\sin 330^\circ = -\mathbf{0.5000}$

$\tan 330^\circ = -\mathbf{0.5774}$

(h)  $\cos -30^\circ = \mathbf{0.8660}$

$\sin -30^\circ = -\mathbf{0.5000}$

$\tan -30^\circ = -\mathbf{0.5774}$

(i)  $\cos 420^\circ = \mathbf{0.5000}$

$\sin 420^\circ = \mathbf{0.8660}$

$\tan 420^\circ = \mathbf{1.7321}$

(j)  $\cos 450^\circ = \mathbf{0}$

$\sin 450^\circ = \mathbf{1}$

$\tan 450^\circ = \infty$

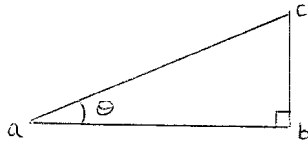
(k)  $\cos 510^\circ = -\mathbf{0.8660}$

$\sin 510^\circ = \mathbf{0.5000}$

$\tan 510^\circ = -\mathbf{0.5774}$

### EXERCISE 3, Page 4

1. If  $ab = 2.1$  m and  $bc = 1.5$  m, determine angle  $\theta$ .

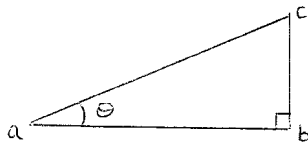


It is convenient to use the expression for  $\tan \theta$ , since sides  $ab$  and  $bc$  are given.

Hence, 
$$\tan \theta = \frac{bc}{ab} = \frac{1.5}{2.1} = 0.7142857\dots$$

from which, 
$$\theta = \tan^{-1}(0.7142857\dots) = \mathbf{35.54^\circ}$$

2. If  $ab = 2.3$  m and  $ac = 5.0$  m, determine angle  $\theta$ .

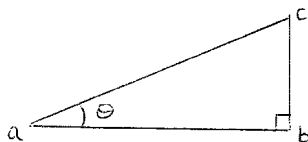


It is convenient to use the expression for  $\cos \theta$ , since sides  $ab$  and  $ac$  are given.

Hence, 
$$\cos \theta = \frac{ab}{ac} = \frac{2.3}{5.0} = 0.460$$

from which, 
$$\theta = \cos^{-1}(0.460) = \mathbf{62.61^\circ}$$

3. If  $bc = 3.1$  m and  $ac = 6.4$  m, determine angle  $\theta$ .

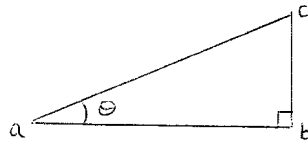


It is convenient to use the expression for  $\sin \theta$ , since sides  $bc$  and  $ac$  are given.

Hence, 
$$\sin \theta = \frac{bc}{ac} = \frac{3.1}{6.4} = 0.484375$$

from which, 
$$\theta = \sin^{-1}(0.484375) = \mathbf{28.97^\circ}$$

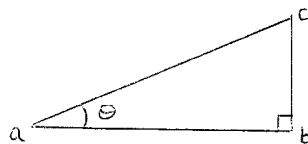
4. If  $ab = 5.7$  cm and  $bc = 4.2$  cm, determine the length  $ac$ .



$$\begin{aligned}\text{From Pythagoras, } ac^2 &= ab^2 + bc^2 \\ &= 5.7^2 + 4.2^2 = 32.49 + 17.64 \\ &= 50.13\end{aligned}$$

$$\text{from which, } ac = \sqrt{50.13} = \mathbf{7.08 \text{ m}}$$

5. If  $ab = 4.1$  m and  $ac = 6.2$  m, determine length  $bc$ .

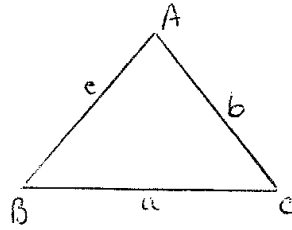


$$\begin{aligned}\text{From Pythagoras, } ac^2 &= ab^2 + bc^2 \\ \text{from which, } bc^2 &= ac^2 - ab^2 \\ &= 6.2^2 - 4.1^2 = 38.44 - 16.81 \\ &= 21.63\end{aligned}$$

$$\text{from which, } bc = \sqrt{21.63} = \mathbf{4.65 \text{ m}}$$

#### EXERCISE 4, Page 5

1. If  $b = 6$  m,  $c = 4$  m and  $B = 100^\circ$ , determine angles  $A$  and  $C$  and length  $a$ .



Using the sine rule,  $\frac{b}{\sin B} = \frac{c}{\sin C}$  i.e.  $\frac{6}{\sin 100^\circ} = \frac{4}{\sin C}$

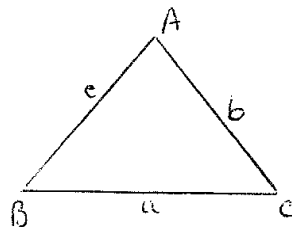
from which,  $\sin C = \frac{4 \sin 100^\circ}{6} = \frac{4 \times 0.98481}{6} = 0.65654$

and  $C = \sin^{-1}(0.65654) = 41.04^\circ$

Angle,  $A = 180^\circ - 100^\circ - 41.04^\circ = 38.96^\circ$

Using the sine rule again gives:  $\frac{a}{\sin A} = \frac{b}{\sin B}$  i.e.  $a = \frac{b \sin A}{\sin B} = \frac{6 \times \sin 38.96^\circ}{\sin 100^\circ} = 3.83$  m

2. If  $a = 15$  m,  $c = 23$  m and  $B = 67^\circ$ , determine length  $b$  and angles  $A$  and  $C$ .



From the cosine rule,  $b^2 = a^2 + c^2 - 2ac \cos B$

$$= 15^2 + 23^2 - 2 \times 15 \times 23 \times \cos 67^\circ$$
$$= 225 + 529 - 2(15)(23) \cos 67^\circ$$
$$= 484.3955$$

Hence, **length,  $b = \sqrt{484.3955} = 22.01$  m**

Using the sine rule:  $\frac{b}{\sin B} = \frac{c}{\sin C}$  i.e.  $\frac{22.01}{\sin 67^\circ} = \frac{23}{\sin C}$

from which,

$$22.01 \sin C = 23 \sin 67^\circ$$

and

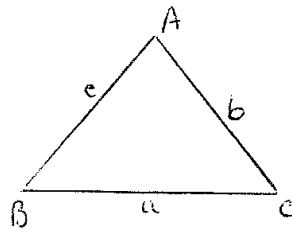
$$\sin C = \frac{23 \sin 67^\circ}{22.01} = 0.96191$$

and

$$C = \sin^{-1}(0.96191) = \mathbf{74.14^\circ}$$

Since  $A + B + C = 180^\circ$ , then  $A = 180^\circ - B - C = 180^\circ - 67^\circ - 74.14^\circ = \mathbf{38.86^\circ}$

3. If  $a = 4$  m,  $b = 8$  m and  $c = 6$  m, determine angle A.



Applying the cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$

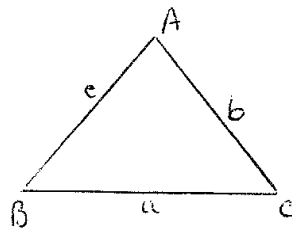
from which,  $2bc \cos A = b^2 + c^2 - a^2$

and

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8^2 + 6^2 - 4^2}{2(8)(6)} = 0.875$$

Hence,  $A = \cos^{-1} 0.875 = \mathbf{28.96^\circ}$

4. If  $a = 10.0$  cm,  $b = 8.0$  cm and  $c = 7.0$  cm, determine angles A, B and C.



Applying the cosine rule:  $a^2 = b^2 + c^2 - 2bc \cos A$

from which,  $2bc \cos A = b^2 + c^2 - a^2$

and

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{8.0^2 + 7.0^2 - 10.0^2}{2(8.0)(7.0)} = 0.11607$$

Hence,  $A = \cos^{-1} 0.11607 = \mathbf{83.33^\circ}$

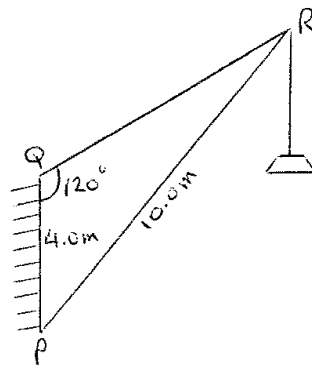
Applying the sine rule:  $\frac{10.0}{\sin 83.33^\circ} = \frac{8.0}{\sin B}$

from which,  $\sin B = \frac{8.0 \sin 83.33^\circ}{10.0} = 0.794585$

Hence,  $B = \sin^{-1} 0.794585 = 52.62^\circ$

and  $C = 180^\circ - 83.33^\circ - 52.62^\circ = 44.05^\circ$

**5.** PR represents the inclined jib of a crane and is 10.0 m long. PQ is 4.0 m long. Determine the inclination of the jib to the vertical (i.e. angle P) and the length of tie QR.



Applying the sine rule:  $\frac{PR}{\sin 120^\circ} = \frac{PQ}{\sin R}$

from which,  $\sin R = \frac{PQ \sin 120^\circ}{PR} = \frac{(4.0) \sin 120^\circ}{10.0} = 0.3464$

Hence,  $\angle R = \sin^{-1} 0.3464 = 20.27^\circ$  (or  $159.73^\circ$ , which is not possible)

$\angle P = 180^\circ - 120^\circ - 20.27^\circ = 39.73^\circ$ , which is the inclination of the jib to the vertical.

Applying the sine rule:  $\frac{10.0}{\sin 120^\circ} = \frac{QR}{\sin 39.73^\circ}$

from which, **length of tie, QR** =  $\frac{10.0 \sin 39.73^\circ}{\sin 120^\circ} = 7.38 \text{ m}$

**EXERCISE 5, Page 6**

1. Show that: (a)  $\sin\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) = \sqrt{3} \cos x$

and (b)  $-\sin\left(\frac{3\pi}{2} - \phi\right) = \cos \phi$

$$\begin{aligned}
 \text{(a) LHS} &= \sin\left(x + \frac{\pi}{3}\right) + \sin\left(x + \frac{2\pi}{3}\right) \\
 &= \left(\sin x \cos \frac{\pi}{3} + \cos x \sin \frac{\pi}{3}\right) + \left(\sin x \cos \frac{2\pi}{3} + \cos x \sin \frac{2\pi}{3}\right) \text{ from compound angle formulae} \\
 &= \sin x \left(\frac{1}{2}\right) + \cos x \left(\frac{\sqrt{3}}{2}\right) + \sin x \left(-\frac{1}{2}\right) + \cos x \left(\frac{\sqrt{3}}{2}\right) \text{ by calculator} \\
 &= \cos x \left(\frac{\sqrt{3}}{2}\right) + \cos x \left(\frac{\sqrt{3}}{2}\right) \\
 &= \sqrt{3} \cos x = \text{RHS}
 \end{aligned}$$

$$\begin{aligned}
 \text{(b) LHS} &= -\sin\left(\frac{3\pi}{2} - \phi\right) = -\left[\sin \frac{3\pi}{2} \cos \phi - \cos \frac{3\pi}{2} \sin \phi\right] \text{ from compound angle formulae} \\
 &= -[(-1)\cos \phi - (0)\sin \phi] \\
 &= \cos \phi = \text{RHS}
 \end{aligned}$$

2. Prove that: (a)  $\sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \sqrt{2}(\sin \theta + \cos \theta)$

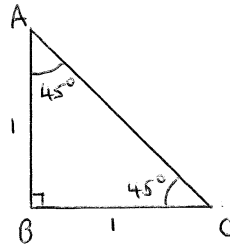
and (b)  $\frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \tan \theta$

$$\begin{aligned}
 \text{(a) L.H.S.} &= \sin\left(\theta + \frac{\pi}{4}\right) - \sin\left(\theta - \frac{3\pi}{4}\right) = \left(\sin \theta \cos \frac{\pi}{4} + \cos \theta \sin \frac{\pi}{4}\right) - \left(\sin \theta \cos \frac{3\pi}{4} - \cos \theta \sin \frac{3\pi}{4}\right) \\
 &= \left[\sin \theta \left(\frac{1}{\sqrt{2}}\right) + \cos \theta \left(\frac{1}{\sqrt{2}}\right)\right] - \left[\sin \theta \left(-\frac{1}{\sqrt{2}}\right) - \cos \theta \left(\frac{1}{\sqrt{2}}\right)\right] \\
 &= \frac{1}{\sqrt{2}}[\sin \theta + \cos \theta + \sin \theta + \cos \theta] = \frac{2}{\sqrt{2}}(\sin \theta + \cos \theta)
 \end{aligned}$$



$$= \sqrt{2}(\sin \theta + \cos \theta) = \text{R.H.S.}$$

The diagram below shows an isosceles triangle where  $AB = BC = 1$  and angles  $A$  and  $C$  are both  $45^\circ$ . By Pythagoras,  $AC = \sqrt{1^2 + 1^2} = \sqrt{2}$ . Hence,  $\sin \frac{\pi}{4} = \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$ .



$$\begin{aligned} \text{(b) L.H.S.} &= \frac{\cos(270^\circ + \theta)}{\cos(360^\circ - \theta)} = \frac{\cos 270^\circ \cos \theta - \sin 270^\circ \sin \theta}{\cos 360^\circ \cos \theta + \sin 360^\circ \sin \theta} = \frac{0 - (-1) \sin \theta}{(1) \cos \theta + 0} \\ &= \frac{\sin \theta}{\cos \theta} = \tan \theta = \text{R.H.S.} \end{aligned}$$

3. Prove the following identities:

$$\text{(a) } 1 - \frac{\cos 2\phi}{\cos^2 \phi} = \tan^2 \phi$$

$$\text{(b) } \frac{1 + \cos 2t}{\sin^2 t} = 2 \cot^2 t$$

$$\text{(c) } \frac{(\tan 2x)(1 + \tan x)}{\tan x} = \frac{2}{1 - \tan x}$$

$$\text{(d) } 2 \operatorname{cosec} 2\theta \cos 2\theta = \cot \theta - \tan \theta$$

$$\begin{aligned} \text{(a) L.H.S.} &= 1 - \frac{\cos 2\phi}{\cos^2 \phi} = 1 - \left( \frac{\cos^2 \phi - \sin^2 \phi}{\cos^2 \phi} \right) = 1 - \left( \frac{\cos^2 \phi}{\cos^2 \phi} - \frac{\sin^2 \phi}{\cos^2 \phi} \right) \\ &= 1 - (1 - \tan^2 \phi) = \tan^2 \phi = \text{R.H.S.} \end{aligned}$$

$$\text{(b) L.H.S.} = \frac{1 + \cos 2t}{\sin^2 t} = \frac{1 + (2 \cos^2 t - 1)}{\sin^2 t} = \frac{2 \cos^2 t}{\sin^2 t} = 2 \cot^2 t = \text{R.H.S.}$$

$$\begin{aligned} \text{(c) L.H.S.} &= \frac{(\tan 2x)(1 + \tan x)}{\tan x} = \frac{\left( \frac{2 \tan x}{1 - \tan^2 x} \right)(1 + \tan x)}{\tan x} = \frac{(2 \tan x)(1 + \tan x)}{(1 - \tan x)(1 + \tan x)} \\ &= \frac{2 \tan x}{(1 - \tan x)} = \frac{2 \tan x}{\tan x (1 - \tan x)} = \frac{2}{1 - \tan x} = \text{R.H.S.} \end{aligned}$$

$$(d) \text{ L.H.S.} = 2 \operatorname{cosec} 2\theta \cos 2\theta = \left( \frac{2}{\sin 2\theta} \right) (\cos 2\theta) = 2 \cot 2\theta = \frac{2}{\tan 2\theta} = \frac{2}{\frac{2 \tan \theta}{1 - \tan^2 \theta}}$$

$$= \frac{2(1 - \tan^2 \theta)}{2 \tan \theta} = \frac{1 - \tan^2 \theta}{\tan \theta} = \frac{1}{\tan \theta} - \tan \theta = \cot \theta - \tan \theta = \text{R.H.S.}$$

**4.** Express as a sum or difference:  $2 \sin 7t \cos 2t$

$$\sin 7t \cos 2t = \frac{1}{2} [\sin(7t + 2t) + \sin(7t - 2t)]$$

$$\text{Hence, } 2 \sin 7t \cos 2t = \mathbf{[\sin 9t + \sin 5t]}$$

**5.** Express as a sum or difference:  $4 \cos 8x \sin 2x$

$$\cos 8x \sin 2x = \frac{1}{2} [\sin(8x + 2x) - \sin(8x - 2x)]$$

$$\text{Hence, } 4 \cos 8x \sin 2x = \mathbf{2[\sin 10x - \sin 6x]}$$

**6.** Express as a sum or difference:  $2 \sin 7t \sin 3t$

$$2 \sin 7t \sin 3t = (2) \left( -\frac{1}{2} \right) [\cos(7t + 3t) - \cos(7t - 3t)]$$

$$= \mathbf{-[\cos 10t - \cos 4t]} \quad \text{or} \quad \mathbf{\cos 4t - \cos 10t}$$

**7.** Express as a sum or difference:  $6 \cos 3\theta \cos \theta$

$$6 \cos 3\theta \cos \theta = 6 \left\{ \frac{1}{2} [\cos(3\theta + \theta) + \cos(3\theta - \theta)] \right\}$$

$$= \mathbf{3[\cos 4\theta + \cos 2\theta]}$$

**8.** Express as a product:  $\sin 3x + \sin x$

$$\sin 3x + \sin x = 2 \sin \left( \frac{3x + x}{2} \right) \cos \left( \frac{3x - x}{2} \right)$$

$$= 2 \sin 2x \cos x$$

**9.** Express as a product:  $\frac{1}{2}(\sin 9\theta - \sin 7\theta)$

$$\begin{aligned}\frac{1}{2}(\sin 9\theta - \sin 7\theta) &= \frac{1}{2}(2)\cos\left(\frac{9\theta+7\theta}{2}\right)\sin\left(\frac{9\theta-7\theta}{2}\right) \\ &= \cos 8\theta \sin \theta\end{aligned}$$

**10.** Express as a product:  $\cos 5t + \cos 3t$

$$\begin{aligned}\cos 5t + \cos 3t &= 2\cos\left(\frac{5t+3t}{2}\right)\cos\left(\frac{5t-3t}{2}\right) \\ &= 2 \cos 4t \cos t\end{aligned}$$


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## EXERCISE 6, Page 7

1. Evaluate A given  $A = 3(2 + 1 + 4)$

$$A = 3(2 + 1 + 4) = 3(7) = 3 \times 7 = \mathbf{21}$$

2. Evaluate A given  $A = 4[5(2 + 1) - 3(6 - 7)]$

$$\begin{aligned} A &= 4[5(2 + 1) - 3(6 - 7)] = 4[5(3) - 3(-1)] \\ &= 4[15 + 3] = 4[18] \\ &= 4 \times 18 = \mathbf{72} \end{aligned}$$

3. Expand the brackets:  $2(x - 2y + 3)$

$$\begin{aligned} 2(x - 2y + 3) &= 2(x) - 2(2y) + 2(3) \\ &= \mathbf{2x - 4y + 6} \end{aligned}$$

4. Expand the brackets:  $(3x - 4y) + 3(y - z) - (z - 4x)$

$$\begin{aligned} (3x - 4y) + 3(y - z) - (z - 4x) &= 3x - 4y + 3y - 3z - z + 4x \\ &= 3x + 4x - 4y + 3y - 3z - z \\ &= \mathbf{7x - y - 4z} \end{aligned}$$

5. Expand the brackets:  $2x + [y - (2x + y)]$

$$\begin{aligned} 2x + [y - (2x + y)] &= 2x + [y - 2x - y] \\ &= 2x + [-2x] = 2x - 2x = \mathbf{0} \end{aligned}$$

6. Expand the brackets:  $24a - [2\{3(5a - b) - 2(a + 2b)\} + 3b]$

$$24a - [2\{3(5a - b) - 2(a + 2b)\} + 3b] = 24a - [2\{15a - 3b - 2a - 4b\} + 3b]$$

$$= 24a - [2\{13a - 7b\} + 3b]$$

$$= 24a - [26a - 14b + 3b]$$

$$= 24a - [26a - 11b]$$

$$= 24a - 26a + 11b$$

$$= -2a + 11b \quad \text{or} \quad 11b - 2a$$

**7.** Expand the brackets:  $ab[c + d - e(f - g + h\{i + j\})]$

$$ab[c + d - e(f - g + h\{i + j\})] = ab[c + d - e(f - g + hi + hj)]$$

$$= ab[c + d - ef + eg - ehi - ehj]$$

$$= abc + abd - abef + abeg - abehi - abehj$$


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## EXERCISE 7, Page 9

1. Evaluate  $\frac{1}{3} + \frac{1}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is  $3 \times 4 = 12$ .

The two fractions can now be made equivalent, i.e.  $\frac{1}{3} = \frac{4}{12}$  and  $\frac{1}{4} = \frac{3}{12}$

so that they can be easily added together, as follows:  $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{4+3}{12} = \frac{7}{12}$

2. Evaluate  $\frac{1}{5} + \frac{1}{4}$

A common denominator can be obtained by multiplying the two denominators together, i.e. the common denominator is  $5 \times 4 = 20$ .

The two fractions can now be made equivalent, i.e.  $\frac{1}{5} = \frac{4}{20}$  and  $\frac{1}{4} = \frac{5}{20}$

so that they can be easily added together, as follows:  $\frac{1}{5} + \frac{1}{4} = \frac{4}{20} + \frac{5}{20} = \frac{4+5}{20} = \frac{9}{20}$

3. Evaluate  $\frac{1}{6} + \frac{1}{2} - \frac{1}{5}$

$$\frac{1}{6} + \frac{1}{2} - \frac{1}{5} = \frac{5+15-6}{30} = \frac{14}{30} = \frac{7}{15}$$

4. Use a calculator to evaluate  $\frac{1}{3} - \frac{3}{4} \times \frac{8}{21}$

$$\begin{aligned} \frac{1}{3} - \frac{3}{4} \times \frac{8}{21} &= \frac{1}{3} - \frac{1}{1} \times \frac{2}{7} \text{ by cancelling} \\ &= \frac{1}{3} - \frac{2}{7} = \frac{7-6}{21} = \frac{1}{21} \end{aligned}$$

5. Use a calculator to evaluate  $\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9}$

$$\begin{aligned}\frac{3}{4} \times \frac{4}{5} - \frac{2}{3} \div \frac{4}{9} &= \frac{3}{1} \times \frac{1}{5} - \frac{2}{3} \times \frac{9}{4} \\ &= \frac{3}{1} \times \frac{1}{5} - \frac{1}{1} \times \frac{3}{2} = \frac{3}{5} - \frac{3}{2} = \frac{6-15}{10} = -\frac{9}{10}\end{aligned}$$

6. Evaluate  $\frac{3}{8} + \frac{5}{6} - \frac{1}{2}$  as a decimal, correct to 4 decimal places.

$$\frac{3}{8} + \frac{5}{6} - \frac{1}{2} = \frac{9+20-12}{24} = \frac{17}{24} = \mathbf{0.7083} \text{ correct to 4 decimal places}$$

7. Evaluate  $8\frac{8}{9} \div 2\frac{2}{3}$  as a mixed number.

$$8\frac{8}{9} \div 2\frac{2}{3} = \frac{80}{9} \div \frac{8}{3} = \frac{80}{9} \times \frac{3}{8} = \frac{10}{3} \times \frac{1}{1} = \frac{10}{3} = \mathbf{3\frac{1}{3}}$$

8. Evaluate  $3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10}$  as a decimal, correct to 3 decimal places.

$$\begin{aligned}3\frac{1}{5} \times 1\frac{1}{3} - 1\frac{7}{10} &= \frac{16}{5} \times \frac{4}{3} - \frac{17}{10} = \frac{64}{15} - \frac{17}{10} \\ &= \frac{128-51}{30} = \frac{77}{30} = 2\frac{17}{30} = \mathbf{2.567} \text{ correct to 3 decimal places}\end{aligned}$$

9. Determine  $\frac{2}{x} + \frac{3}{y}$  as a single fraction.

$$\frac{2}{x} = \frac{2y}{xy} \quad \text{and} \quad \frac{3}{y} = \frac{3x}{xy}$$

$$\text{Hence, } \frac{2}{x} + \frac{3}{y} = \frac{2y}{xy} + \frac{3x}{xy} = \frac{\mathbf{2y+3x}}{\mathbf{xy}} \text{ or } \frac{\mathbf{3x+2y}}{\mathbf{xy}}$$

## EXERCISE 8, Page 10

1. Express 0.057 as a percentage.

$$0.057 = 0.057 \times 100\% = \mathbf{5.7\%}$$

2. Express 0.374 as a percentage.

$$0.374 = 0.374 \times 100\% = \mathbf{37.4\%}$$

3. Express 20% as a decimal number.

$$20\% = \frac{20}{100} = \mathbf{0.20}$$

4. Express  $\frac{11}{16}$  as a percentage.

$$\frac{11}{16} = \frac{11}{16} \times 100\% = \frac{1100}{16}\% = \mathbf{68.75\%}$$

5. Express  $\frac{5}{13}$  as a percentage, correct to 3 decimal places.

$$\frac{5}{13} = \frac{5}{13} \times 100\% = \frac{500}{13}\% = 38.461538.... \text{ by calculator}$$

= **38.462%** correct to 3 decimal places

6. Place the following in order of size, the smallest first, expressing each as percentages, correct to 1 decimal place: (a)  $\frac{12}{21}$  (b)  $\frac{9}{17}$  (c)  $\frac{5}{9}$  (d)  $\frac{6}{11}$

$$(a) \frac{12}{21} = \frac{12}{21} \times 100\% = \frac{1200}{21}\% = 57.1\%$$

$$(b) \frac{9}{17} = \frac{9}{17} \times 100\% = \frac{900}{17}\% = 52.9\%$$



$$(c) \frac{5}{9} = \frac{5}{9} \times 100\% = \frac{500}{9}\% = 55.6\%$$

$$(d) \frac{6}{11} = \frac{6}{11} \times 100\% = \frac{600}{11}\% = 54.6\%$$

Placing them in order of size, the smallest first, gives: **(b), (d), (c) and (a)**

**7.** Express 65% as a fraction in its simplest form.

$$65\% = \frac{65}{100} \text{ and by dividing the numerator and denominator by 5 gives: } 65\% = \frac{65}{100} = \frac{13}{20}$$

**8.** Calculate 43.6% of 50 kg.

$$43.6\% \text{ of } 50 \text{ kg} = \frac{43.6}{100} \times 50 \text{ kg} = \mathbf{21.8 \text{ kg}}$$

**9.** Determine 36% of 27 m.

$$36\% \text{ of } 27 \text{ m} = \frac{36}{100} \times 27 \text{ m} = \mathbf{9.72 \text{ m}}$$

**10.** Calculate correct to 4 significant figures:

(a) 18% of 2758 tonnes    (b) 47% of 18.42 grams    (c) 147% of 14.1 seconds

$$(a) 18\% \text{ of } 2758 \text{ t} = \frac{18}{100} \times 2758 \text{ t} = \mathbf{496.4 \text{ t}}$$

$$(b) 47\% \text{ of } 18.42 \text{ g} = \frac{47}{100} \times 18.42 \text{ g} = \mathbf{8.657 \text{ g}}$$

$$(c) 147\% \text{ of } 14.1 \text{ s} = \frac{147}{100} \times 14.1 \text{ s} = \mathbf{20.73 \text{ s}}$$

**11.** Express: (a) 140 kg as a percentage of 1 t    (b) 47 s as a percentage of 5 min

(c) 13.4 cm as a percentage of 2.5 m

It is essential when expressing one quantity as a percentage of another that both quantities are in the same units.

(a) 1 tonne = 1000 kg, hence 140 kg as a percentage of 1 t =  $\frac{140}{1000} \times 100\% = \mathbf{14\%}$

(b) 5 minutes =  $5 \times 60 = 300$  s, hence 47 s as a percentage of 5 minutes =  $\frac{47}{300} \times 100\% = \mathbf{15.67\%}$

(c) 2.5 m =  $2.5 \times 100 = 250$  cm, hence 13.4 cm as a percentage of 2.5 m =  $\frac{13.4}{250} \times 100\% = \mathbf{5.36\%}$

**12.** A computer is advertised on the Internet at £520, exclusive of VAT. If VAT is payable at 20%, what is the total cost of the computer?

$$\text{VAT} = 20\% \text{ of } £520 = \frac{20}{100} \times 520 = £104$$

$$\text{Total cost of computer} = £520 + £104 = \mathbf{£624}$$

**13.** Express 325 mm as a percentage of 867 mm, correct to 2 decimal places.

$$325 \text{ mm as a percentage of } 867 \text{ mm} = \frac{325}{867} \times 100\% = \mathbf{37.49\%}$$

**14.** When signing a new contract, a Premiership footballer's pay increases from £15,500 to £21,500 per week. Calculate the percentage pay increase, correct to 3 significant figures.

Percentage change is given by:  $\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$

$$\text{i.e.} \quad \% \text{ increase} = \frac{21500 - 15500}{15500} \times 100\% = \frac{6000}{15500} \times 100\% = \mathbf{38.7\%}$$

**15.** A metal rod 1.80 m long is heated and its length expands by 48.6 mm. Calculate the percentage increase in length.

$$\% \text{ increase} = \frac{48.6}{1.80 \times 1000} \times 100\% = \frac{48.6}{1800} \times 100\% = \mathbf{2.7\%}$$

### EXERCISE 9, Page 12

1. Evaluate  $2^2 \times 2 \times 2^4$

$$2^2 \times 2 \times 2^4 = 2^{2+1+4} = 2^7 \quad \text{by law 1 of indices}$$
$$= \mathbf{128}$$

2. Evaluate  $3^5 \times 3^3 \times 3$  in index form.

$$3^5 \times 3^3 \times 3 = 3^{5+3+1} = \mathbf{3^9} \quad \text{by law 1 of indices}$$

3. Evaluate  $\frac{2^7}{2^3}$

$$\frac{2^7}{2^3} = 2^{7-3} = 2^4 \quad \text{by law 2 of indices}$$
$$= \mathbf{16}$$

4. Evaluate  $\frac{3^3}{3^5}$

$$\frac{3^3}{3^5} = 3^{3-5} = 3^{-2} \quad \text{by law 2 of indices}$$
$$= \frac{1}{3^2} \quad \text{by law 5 of indices}$$
$$= \mathbf{\frac{1}{9}}$$

5. Evaluate  $7^0$

$$7^0 = \mathbf{1} \quad \text{by law 4 of indices}$$

6. Evaluate  $\frac{2^3 \times 2 \times 2^6}{2^7}$

$$\frac{2^3 \times 2 \times 2^6}{2^7} = \frac{2^{3+1+6}}{2^7} = \frac{2^{10}}{2^7} = 2^{10-7} = 2^3 = \mathbf{8} \quad \text{by laws 1 and 2 of indices}$$

**7. Evaluate**  $\frac{10 \times 10^6}{10^5}$

$$\frac{10 \times 10^6}{10^5} = 10^{1+6-5} = 10^2 = \mathbf{100} \quad \text{by laws 1 and 2 of indices}$$

**8. Evaluate**  $10^4 \div 10$

$$10^4 \div 10 = \frac{10^4}{10^1} = 10^{4-1} = 10^3 = \mathbf{1000} \quad \text{by law 2 of indices}$$

**9. Evaluate**  $\frac{10^3 \times 10^4}{10^9}$

$$\frac{10^3 \times 10^4}{10^9} = 10^{3+4-9} = 10^{-2} = \frac{1}{10^2} = \frac{1}{100} = \mathbf{0.01} \quad \text{by law 2 of indices}$$

**10. Evaluate**  $5^6 \times 5^2 \div 5^7$

$$5^6 \times 5^2 \div 5^7 = \frac{5^6 \times 5^2}{5^7} = 5^{6+2-7} = 5^1 = \mathbf{5} \quad \text{by laws 1 and 2 of indices}$$

**11. Evaluate**  $(7^2)^3$  in index form.

$$(7^2)^3 = 7^{2 \times 3} = \mathbf{7^6} \quad \text{by law 3 of indices}$$

**12. Evaluate**  $(3^3)^2$

$$(3^3)^2 = 3^{3 \times 2} = \mathbf{3^6} = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = \mathbf{729}$$

**13.** Evaluate  $\frac{3^7 \times 3^4}{3^5}$  in index form.

$$\frac{3^7 \times 3^4}{3^5} = 3^{7+4-5} = \mathbf{3^6} \quad \text{by laws 1 and 2 of indices}$$

**14.** Evaluate  $\frac{(9 \times 3^2)^3}{(3 \times 27)^2}$  in index form.

$$\frac{(9 \times 3^2)^3}{(3 \times 27)^2} = \frac{(3^2 \times 3^2)^3}{(3 \times 3^3)^2} = \frac{(3^4)^3}{(3^4)^2} = \frac{3^{12}}{3^8} = 3^{12-8} = \mathbf{3^4} \quad \text{by laws 1, 2 and 3 of indices}$$

**15.** Evaluate  $\frac{(16 \times 4)^2}{(2 \times 8)^3}$

$$\frac{(16 \times 4)^2}{(2 \times 8)^3} = \frac{(2^4 \times 2^2)^2}{(2 \times 2^3)^3} = \frac{(2^6)^2}{(2^4)^3} = \frac{2^{12}}{2^{12}} = \mathbf{1} \quad \text{by laws 1, 2 and 3 of indices}$$

**16.** Evaluate  $\frac{5^{-2}}{5^{-4}}$

$$\frac{5^{-2}}{5^{-4}} = 5^{-2-(-4)} = 5^{-2+4} = 5^2 = \mathbf{25} \quad \text{by law 2 of indices}$$

**17.** Evaluate  $\frac{3^2 \times 3^{-4}}{3^3}$

$$\frac{3^2 \times 3^{-4}}{3^3} = 3^{2+(-4)-3} = 3^{2-4-3} = 3^{-5} = \frac{1}{3^5} = \frac{\mathbf{1}}{\mathbf{243}} \quad \text{by laws 1, 2 and 5 of indices}$$

**18.** Evaluate  $\frac{7^2 \times 7^{-3}}{7 \times 7^{-4}}$

$$\frac{7^2 \times 7^{-3}}{7 \times 7^{-4}} = \frac{7^{2-3}}{7^{1-4}} = \frac{7^{-1}}{7^{-3}} = 7^{-1-(-3)} = 7^{-1+3} = 7^2 = \mathbf{49} \quad \text{by laws 1 and 2 of indices}$$

**19.** Simplify, giving the answer as a power:  $z^2 \times z^6$

$$z^2 \times z^6 = z^{2+6} = \mathbf{z^8} \quad \text{by law 1 of indices}$$

**20.** Simplify, giving the answer as a power:  $a \times a^2 \times a^5$

$$a \times a^2 \times a^5 = a^{1+2+5} = \mathbf{a^8} \quad \text{by law 1 of indices}$$

**21.** Simplify, giving the answer as a power:  $n^8 \times n^{-5}$

$$n^8 \times n^{-5} = n^{8-5} = \mathbf{n^3} \quad \text{by law 1 of indices}$$

**22.** Simplify, giving the answer as a power:  $b^4 \times b^7$

$$b^4 \times b^7 = b^{4+7} = \mathbf{b^{11}} \quad \text{by law 1 of indices}$$

**23.** Simplify, giving the answer as a power:  $b^2 \div b^5$

$$b^2 \div b^5 = \frac{b^2}{b^5} = b^{2-5} = \mathbf{b^{-3}} \quad \text{or} \quad \frac{1}{b^3} \quad \text{by laws 2 and 5 of indices}$$

**24.** Simplify, giving the answer as a power:  $c^5 \times c^3 \div c^4$

$$c^5 \times c^3 \div c^4 = \frac{c^5 \times c^3}{c^4} = \frac{c^{5+3}}{c^4} = \frac{c^8}{c^4} = c^{8-4} = \mathbf{c^4} \quad \text{by laws 1 and 2 of indices}$$

**25.** Simplify, giving the answer as a power:  $\frac{m^5 \times m^6}{m^4 \times m^3}$

$$\frac{m^5 \times m^6}{m^4 \times m^3} = \frac{m^{5+6}}{m^{4+3}} = \frac{m^{11}}{m^7} = m^{11-7} = \mathbf{m^4} \quad \text{by laws 1 and 2 of indices}$$

**26.** Simplify, giving the answer as a power:  $\frac{(x^2)(x)}{x^6}$

$$\frac{(x^2)(x)}{x^6} = \frac{x^{2+1}}{x^6} = \frac{x^3}{x^6} = x^{-3} \text{ or } \frac{1}{x^3} \quad \text{by laws 1, 2 and 5 of indices}$$

**27.** Simplify, giving the answer as a power:  $(x^3)^4$

$$(x^3)^4 = x^{3 \times 4} = x^{12} \quad \text{by law 3 of indices}$$

**28.** Simplify, giving the answer as a power:  $(y^2)^{-3}$

$$(y^2)^{-3} = y^{2 \times -3} = y^{-6} \text{ or } \frac{1}{y^6} \quad \text{by laws 3 and 5 of indices}$$

**29.** Simplify, giving the answer as a power:  $(t \times t^3)^2$

$$(t \times t^3)^2 = (t^{1+3})^2 = (t^4)^2 = t^8 \quad \text{by laws 1 and 3 of indices}$$

**30.** Simplify, giving the answer as a power:  $(c^{-7})^{-2}$

$$(c^{-7})^{-2} = c^{-7 \times -2} = c^{14} \quad \text{by law 3 of indices}$$

**31.** Simplify, giving the answer as a power:  $\left(\frac{a^2}{a^5}\right)^3$

$$\left(\frac{a^2}{a^5}\right)^3 = (a^{2-5})^3 = (a^{-3})^3 = a^{-9} \text{ or } \frac{1}{a^9} \quad \text{by laws 3 and 5 of indices}$$

**32.** Simplify, giving the answer as a power:  $\left(\frac{1}{b^3}\right)^4$

$$\left(\frac{1}{b^3}\right)^4 = (b^{-3})^4 = b^{-12} \quad \text{or} \quad \frac{1}{b^{12}} \quad \text{by laws 3 and 4 of indices}$$

**33.** Simplify, giving the answer as a power:  $\left(\frac{b^2}{b^7}\right)^{-2}$

$$\left(\frac{b^2}{b^7}\right)^{-2} = (b^{2-7})^{-2} = (b^{-5})^{-2} = b^{10} \quad \text{by laws 2 and of indices}$$

**34.** Simplify, giving the answer as a power:  $\frac{1}{(s^3)^3}$

$$\frac{1}{(s^3)^3} = \frac{1}{s^{3 \times 3}} = \frac{1}{s^9} \quad \text{or} \quad s^{-9} \quad \text{by laws 3 and 5 of indices}$$

**35.** Simplify, giving the answer as a power:  $p^3qr^2 \times p^2q^5r \times pqr^2$

$$p^3qr^2 \times p^2q^5r \times pqr^2 = p^{3+2+1} \times q^{1+5+1} \times r^{2+1+2} = p^6 \times q^7 \times r^5 = p^6q^7r^5$$

**36.** Simplify, giving the answer as a power:  $\frac{x^3y^2z}{x^5yz^3}$

$$\frac{x^3y^2z}{x^5yz^3} = x^{3-5}y^{2-1}z^{1-3} = x^{-2}yz^{-2} \quad \text{or} \quad \frac{y}{x^2z^2}$$



## EXERCISE 10, Page 14

1. If 5 apples and 3 bananas cost £1.45 and 4 apples and 6 bananas cost £2.42, determine how much an apple and a banana each cost.

Let an apple = A and a banana = B, then:

$$5A + 3B = 145 \quad (1)$$

$$4A + 6B = 242 \quad (2)$$

From equation (1),  $5A = 145 - 3B$

and 
$$A = \frac{145 - 3B}{5} = 29 - 0.6B \quad (3)$$

From equation (2),  $4A = 242 - 6B$

and 
$$A = \frac{242 - 6B}{4} = 60.5 - 1.5B \quad (4)$$

Equating (3) and (4) gives:  $29 - 0.6B = 60.5 - 1.5B$

i.e.  $1.5B - 0.6B = 60.5 - 29$

and  $0.9B = 31.5$

and 
$$B = \frac{31.5}{0.9} = 35$$

Substituting in (3) gives:  $A = 29 - 0.6(35) = 29 - 21 = 8$

Hence, **an apple costs 8p and a banana costs 35p**

2. If 7 apples and 4 oranges cost £2.64 and 3 apples and 3 oranges cost £1.35, determine how much an apple and an orange each cost.

Let an apple = A and an orange = R, then:

$$7A + 4R = 264 \quad (1)$$

$$3A + 3R = 135 \quad (2)$$

Multiplying equation (1) by 3 gives:  $21A + 12R = 792 \quad (3)$

Multiplying equation (2) by 4 gives:  $12A + 12R = 540$  (4)

Equation (3) – equations (4) gives:  $9A = 252$

from which,  $A = \frac{252}{9} = 28$

Substituting in (3) gives:  $21(28) + 12R = 792$

i.e.  $12R = 792 - 21(28)$

i.e.  $12R = 204$

and  $B = \frac{204}{12} = 17$

Hence, **an apple costs 28p and an orange costs 17p**

**3.** Three new cars and four new vans supplied to a dealer together cost £93000, and five new cars and two new vans of the same models cost £99000. Find the respective costs of a car and a van.

Let a car = C and a van = V, then working in £1000's:

$$3C + 4V = 93 \quad (1)$$

$$5C + 2V = 99 \quad (2)$$

Multiplying equation (2) by 2 gives:  $10C + 4V = 198$  (3)

Equation (3) – equations (1) gives:  $7C = 105$

from which,  $C = \frac{105}{7} = 15$

Substituting in (1) gives:  $3(15) + 4V = 93$

i.e.  $4V = 93 - 3(15)$

i.e.  $4V = 48$

and  $V = \frac{48}{4} = 12$

Hence, **a car costs £15000 and a van costs £12000**

4. In a system of forces, the relationship between two forces  $F_1$  and  $F_2$  is given by:

$$5F_1 + 3F_2 = -6$$

$$3F_1 + 5F_2 = -18$$

Solve for  $F_1$  and  $F_2$

$$5F_1 + 3F_2 = -6 \quad (1)$$

$$3F_1 + 5F_2 = -18 \quad (2)$$

Multiplying equation (1) by 5 gives:  $25F_1 + 15F_2 = -30 \quad (3)$

Multiplying equation (2) by 3 gives:  $9F_1 + 15F_2 = -54 \quad (4)$

Equation (3) – equation (4) gives:  $16F_1 = -30 - (-54) = -30 + 54 = 24$

from which,  $F_1 = \frac{24}{16} = 1.5$

Substituting in (1) gives:  $5(1.5) + 3F_2 = -6$

i.e.  $3F_2 = -6 - 5(1.5)$

i.e.  $3F_2 = -13.5$

and  $F_2 = \frac{-13.5}{3} = -4.5$

Hence,  $F_1 = 1.5$  and  $F_2 = -4.5$

5. Solve the simultaneous equations:  $a + b = 7$

$$a - b = 3$$

$$a + b = 7 \quad (1)$$

$$a - b = 3 \quad (2)$$

Adding equations (1) and (2) gives:  $2a = 10$

from which,  $a = \frac{10}{2} = 5$

Substituting in (1) gives:  $5 + b = 7$

i.e.  $b = 7 - 5 = 2$

Hence, **a = 5 and b = 2**

**6. Solve the simultaneous equations:**  $8a - 3b = 51$   
 $3a + 4b = 14$

$$8a - 3b = 51 \quad (1)$$

$$3a + 4b = 14 \quad (2)$$

Multiplying equation (1) by 4 gives:  $32a - 12b = 204 \quad (3)$

Multiplying equation (2) by 3 gives:  $9a + 12b = 42 \quad (4)$

Equation (3) + equations (4) gives:  $41a = 246$

from which,  $a = \frac{246}{41} = 6$

Substituting in (1) gives:  $48 - 3b = 51$

i.e.  $48 - 51 = 3b$

i.e.  $-3 = 3b$

and  $b = -1$

Hence, **a = 6 and b = -1**

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