

CHAPTER 16 SHEAR STRESSES IN BENDING AND SHEAR DEFLECTIONS

EXERCISE 60, Page 352

1. A beam of length 3 m is simply supported at its ends and subjected to a uniformly distributed load of 200 kN/m, spread over its entire length. If the beam has a uniform cross-section of depth 0.2 m and width 0.1 m, determine the position and value of the maximum shearing stress due to bending. What will be the value of the maximum shear stress at mid-span?

Maximum shearing force occurs at the ends,

where
$$F_{max} = \frac{wl}{2} = \frac{200 \times 3}{2}$$

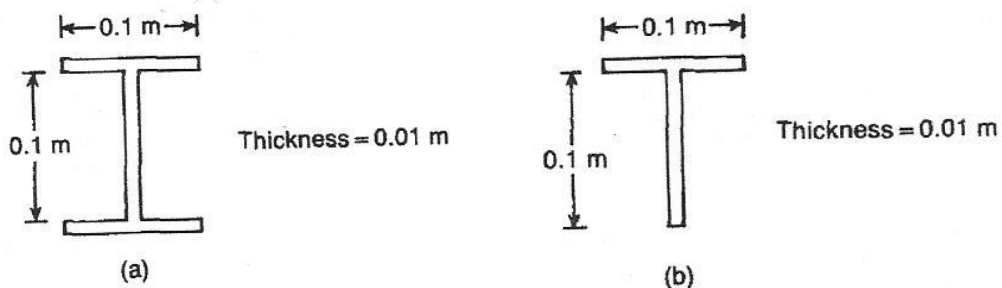
i.e.
$$F_{max} = 300 \text{ kN}$$

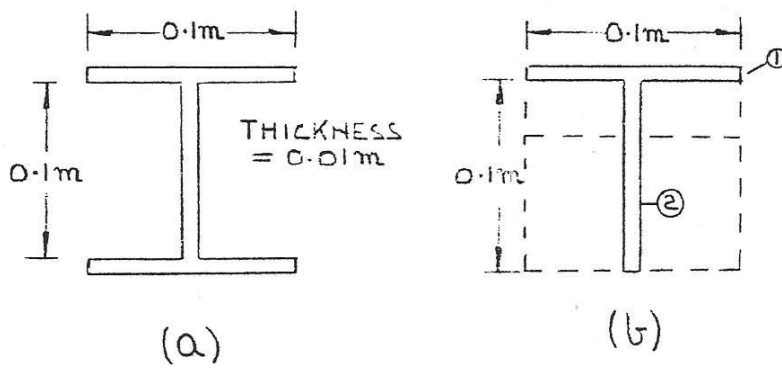
Maximum shearing stress
$$= \frac{F_{max} \times 1.5}{b \times d} = \frac{300 \times 10^3 \times 1.5}{0.1 \times 0.2} \text{ N/m}^2$$

i.e.
$$\tau = 22.5 \text{ MPa}$$

At mid-span $\tau = F = 0$

2. Determine the maximum values of shear stress due to bending in the web and flanges of the sections shown when they are subjected to vertical shearing forces of 100 kN.





(a)
$$I = \frac{0.1 \times 0.12^3}{12} - \frac{0.09 \times 0.1^3}{12} = 6.9 \times 10^{-6} \text{ m}^4$$

Flange

$$\hat{\tau}_F = \frac{0.045 \times 0.01 \times 0.055}{0.01 \times 6.9 \times 10^{-6}} \times 100 \times 10^3$$

$$= 35.87 \text{ MN/m}^2 = 35.87 \text{ MPa}$$

Web

$$\hat{\tau}_W = \frac{(0.1 \times 0.01 \times 0.055 + 0.05 \times 0.01 \times 0.025) \times 100 \times 10^3}{0.01 \times 6.9 \times 10^{-6}}$$

$$= 97.83 \text{ MN/m}^2 = 97.83 \text{ MPa}$$

(b)

Section	a	y	ay	ay ²	i
1	1×10^{-3}	0.105	1.05×10^{-4}	1.03×10^{-5}	8.33×10^{-9}
2	1×10^{-3}	0.05	5×10^{-5}	2.5×10^{-6}	8.33×10^{-7}
3	2×10^{-3}	—	1.55×10^{-4}	1.353×10^{-5}	8.416×10^{-7}

$$\bar{y} = 0.0775 \text{ m}$$

$$I_{XX} = 1.437 \times 10^{-5} \text{ m}^4$$

$$I_{NA} = 2.358 \times 10^{-6} \text{ m}^4$$

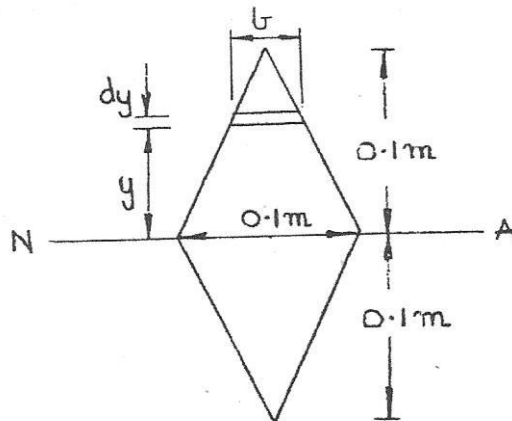
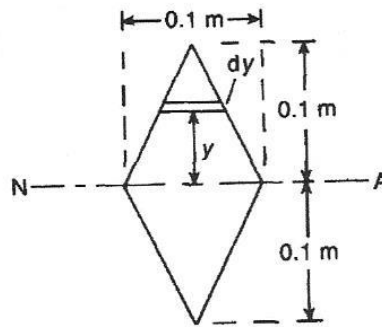
$$\hat{\tau}_F = \frac{100 \times 10^3 \times 0.045 \times 0.01 \times 0.0275}{0.01 \times 2.358 \times 10^{-6}}$$

$$= 52.48 \text{ MPa}$$

$$\hat{\tau}_w = \frac{100 \times 10^3 (0.1 \times 0.01 \times 0.0275 + 0.0225^2 / 2 \times 0.01) \times 100 \times 10^3}{0.01 \times 2.358 \times 10^{-6}}$$

$$= 127.36 \text{ MPa}$$

3. Determine an expression for the maximum shearing stress due to bending for the section shown, assuming that it is subjected to a shearing force of 0.5 MN acting through its centroid and in a perpendicular direction to NA.



$$I_{NA} = \frac{0.1 \times 0.1^3}{12} \times 2 = 1.667 \times 10^{-5} \text{ m}^4$$

$$b = 0.1 (1 - 10y)$$

$$\int y dA = \int b dy (y) = \int_y^{0.1} 0.1(1 - 10y) y dy$$

$$= 0.1 \left[\frac{y^2}{2} - \frac{10y^3}{3} \right]_y^{0.1} = \frac{0.1}{6} [3y^2 - 20y^3]_y^{0.1}$$

$$\begin{aligned}
&= \frac{0.1}{6} [0.01 - (3y^2 - 20y^3)] \\
\tau &= \frac{0.5 \times 0.1}{6 \times 1.667 \times 10^{-5}} \frac{[0.01 - (3y^2 - 20y^3)]}{0.1(1 - 10y)} \\
&= \frac{5000 [0.01 - (3y^2 - 20y^3)]}{(1 - 10y)}
\end{aligned}$$

For τ , $\frac{d\tau}{dy} = 0$

$$(1 - 10y)[- (6y - 60y^2)] - (-10) [0.01 - (3y^2 - 20y^3)] = 0$$

i.e. $-6y + 60y^2 + 60y^2 - 600y^3 + 0.1 - 30y^2 + 200y^3 = 0$

i.e. $-400y^3 + 90y^2 - 6y + 0.1 = 0$

and $+400y^3 - 90y^2 + 6y - 0.1 = 0 = \Psi$

$$1200y^2 - 180y + 6 = \frac{d\Psi}{dy} = 0$$

$$y_1 = 0 + \frac{0.1}{6} = 0.017$$

$$y_2 = 0.017 + \frac{0.022}{3.29} = \mathbf{0.024}$$

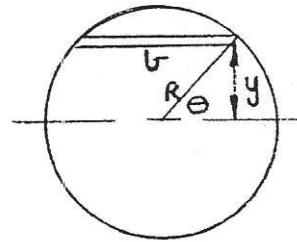
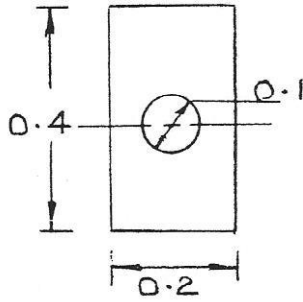
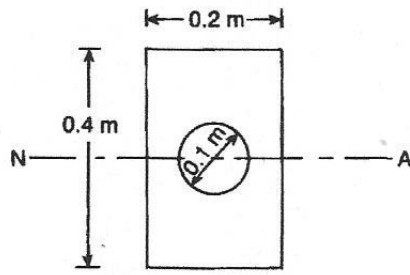
$$y_3 = 0.024 + \frac{2.31 \times 10^{-3}}{2.37} = \mathbf{0.025}$$

$$y_4 = 0.025 - 0$$

i.e. $y = \pm \mathbf{0.025 \text{ m perpendicular to NA}}$ (\pm because it is a symmetrical beam)

$$\hat{\tau} = \pm \frac{5000 \times 8.4375 \times 10^{-3}}{0.75} = \pm \mathbf{56.25 \text{ MN/m}^2}$$

4. Determine the value of the maximum shear stress for the cross-section shown, assuming that it is subjected to a shearing force of magnitude 0.5 MN acting through its centroid and in a perpendicular direction to NA.



$$b = 2R \cos \theta$$

$$y = R \sin \theta$$

$$dy = R \cos \theta d\theta$$

$$\int y dA = \int b dy(y)$$

$$I_{NA} = \frac{0.2 \times 0.4^3}{12} - \frac{\pi \times 0.1^4}{64} = 1.062 \times 10^{-3} \text{ m}^4$$

$$\int y dA = 0.2 \times 0.2 \times 0.1 - \int_0^{\pi/2} 2R \cos \theta R \sin \theta R \cos \theta d\theta$$

$$= 4 \times 10^{-3} + 2R^3 \int_0^{\pi/2} \cos^2 \theta d(\cos \theta)$$

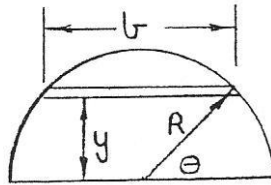
$$= 4 \times 10^{-3} + 2.5 \times 10^{-4} \left[\frac{\cos^3 \theta}{3} \right]_0^{\pi/2}$$

$$= 4 \times 10^{-3} + 2.5 \times 10^{-4} \left[0 - \frac{1}{3} \right]$$

Hence, $\int y dA = 3.917 \times 10^{-3} \text{ m}$ 3

Maximum shear stress, $\tau = \frac{0.5 \times 3.917 \times 10^{-3}}{0.1 \times 1.062 \times 10^{-3}} = 18.44 \text{ MPa}$

Proof of $\int y dA$



$$b = 2R \cos \theta$$

$$y = R \sin \theta$$

$$dy = R \cos \theta d\theta$$

$$\int y dA = \int_0^{\pi/2} 2R \cos \theta R \sin \theta R \cos \theta d\theta$$

$$= 2R^3 \int_0^{\pi/2} \cos^2 \theta \sin \theta d\theta$$

$$= -2R^3 \int_0^{\pi/2} \cos^2 \theta d(\cos \theta)$$

$$= -2R^3 \left[\frac{\cos^3 \theta}{3} \right]_0^{\pi/2}$$

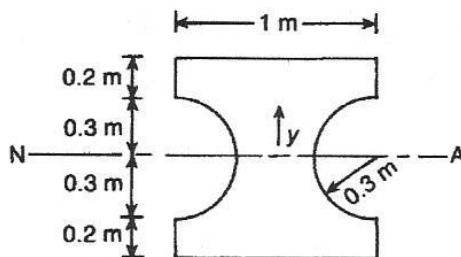
$$= \frac{2R^3}{3}$$

$$dA = \frac{\pi R^2}{2}$$

Therefore,

$$\bar{y} = \frac{2R^3}{3} \times \frac{2}{\pi R^2} = \frac{4R}{3\pi}$$

5. A simply supported beam, with a cross-section as shown, is subjected to a centrally placed concentrated load of 100 MN, acting through its centroid and perpendicular to NA. Determine the values of the vertical shearing stress at intervals of 0.1 m from NA.



$$I = \frac{1 \times 1^3}{12} - \frac{\pi \times 0.6^4}{64} = 0.08333 - 6.362 \times 10^{-3}$$

i.e. $I = 0.077 \text{ m}^4$

At $y = 0.5$, $\tau = 0$

At $y = 0.4 \text{ m}$, $\tau_{0.4} = \frac{50 \times 1 \times 0.1 \times 0.45}{1 \times 0.077} = 29.22 \text{ MPa}$

At $y = 0.3 \text{ m}$, $\tau_{0.3} = \frac{50 \times 1 \times 0.2 \times 0.4}{1 \times 0.077} = 51.95 \text{ MPa}$

$$b = 2R \cos \phi \quad y = R \sin \phi \quad dy = R \cos \phi d\phi$$

$$\int y dA = \int y b dy = \int 2R^3 \cos^2 \phi d(\cos \phi)$$

$$= + \frac{2R^3}{3} [\cos^3 \phi]_0^\phi$$

At $y = 0.2 \text{ m}$, $\phi = \sin^{-1} (0.2/0.3) = 41.81^\circ$

$$\int y dA = 1 \times 0.3 \times 0.35 - \frac{2}{3} \times 0.3^3 \cos^3 (41.81^\circ)$$

$$= 0.105 - 7.454 \times 10^{-3} = 0.0975 \text{ m}^3$$

$b = 0.553 \text{ m}$, $\tau = 114.49 \text{ MPa}$

At $y = 0.1 \text{ m}$, $\phi = \sin^{-1} (0.1/0.3) = 19.47^\circ$

$$\int y dA = 1 \times 0.4 \times 0.3 - \frac{2}{3} \times 0.3^3 \cos^3 (19.47^\circ)$$

$$= 0.12 - 0.0151 = 0.1049 \text{ m}^3$$

$b = 0.434 \text{ m}$, $\tau = 156.95 \text{ MPa}$

At $y = 0$, $\phi = 0$,

$$\int y dA = 1 \times 0.5 \times 0.25 - \frac{2}{3} \times 0.3^3 \cos^3 (0^\circ)$$

$$= 0.107 \text{ m}^3$$

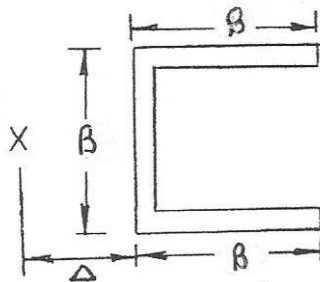
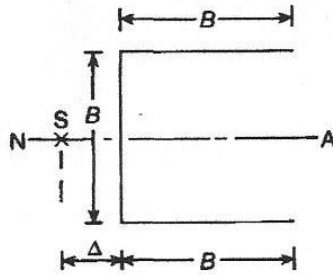
$b = 1 - 2R = 0.4 \text{ m}$, $\tau = 173.70 \text{ MPa}$

Summarising, at $y = 0$, $\tau_0 = 173.7 \text{ MPa}$, $\tau_{0.1} = 156.95 \text{ MPa}$, $\tau_{0.2} = 114.49 \text{ MPa}$, $\tau_{0.3} = 51.95 \text{ MPa}$,

$$\tau_{0.4} = 29.22 \text{ MPa; at } y = 0.5, \tau_{0.5} = 0$$

EXERCISE 61, Page 356

1. Determine the position of the shear centre for the thin-walled section shown.



$$I = \frac{tB^3}{12} + B \times t \times \left(\frac{B}{2}\right)^2 \times 2$$

i.e. $I = 0.5833 t B^3$

$$\hat{\tau} = \frac{F}{tI} \times \frac{B}{2} = \frac{0.5tB^2F}{tI} = \frac{0.5B^2F}{0.5833tB^3}$$

i.e. $\hat{\tau} = \frac{0.8572F}{Bt}$

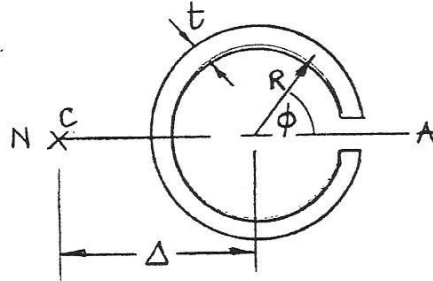
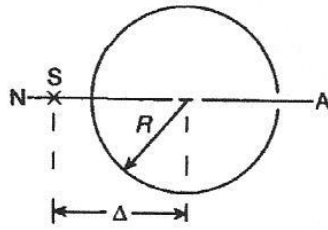
$$F_F = \left(\frac{0.429F}{Bt}\right) \times Bt = 0.429F$$

Taking moments about the centre of the web gives:

$$F\Delta = 0.429F \times \frac{B}{2} \times 2$$

from which, **shear centre, $\Delta = 0.429B$**

2. Determine the position of the shear centre for the thin-walled section shown.



$$I_{NA} = \int_0^{2\pi} (t R d\phi) (R \sin \phi)^2$$

$$= t R^3 \int_0^{2\pi} \left(\frac{1 - \cos 2\phi}{2} \right) d\phi$$

$$= \frac{t R^3}{2} \left[\phi - \frac{\sin 2\phi}{2} \right]_0^{2\pi}$$

from which,

$$I_{NA} = \pi t R^3$$

$$\int_0^\phi y dA = \int_0^\phi t R d\phi R \sin \phi = t R^2 [-\cos \phi]_0^\phi$$

$$= t R^2 [-\cos \phi + 1] = t R^2 (1 - \cos \phi)$$

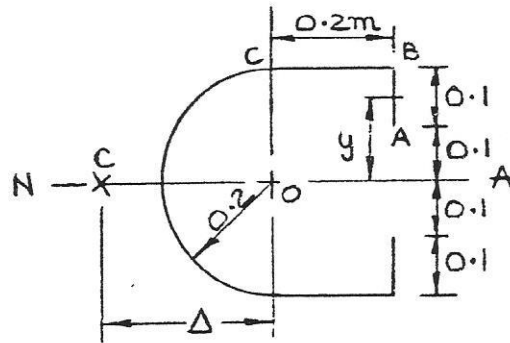
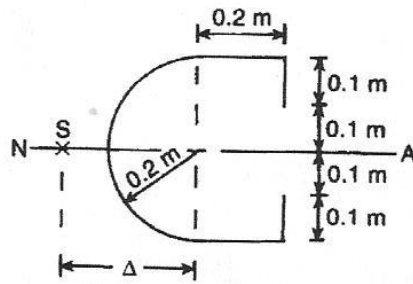
$$\tau_\phi = \frac{F}{\pi t^2 R^3} \times t R^2 (1 - \cos \phi) = \frac{F (1 - \cos \phi)}{\pi t R}$$

$$F \Delta = \int_0^{2\pi} \tau_\phi t R^2 d\phi = \frac{F t R^2}{\pi t R} [\phi - \sin \phi]_0^{2\pi}$$

$$= \frac{F t R^2}{\pi t R} (2\pi) = 2FR$$

Hence, the shear centre, $\Delta = \frac{2FR}{F} = 2R$

3. Determine the shear centre position for the thin-walled section shown.



$$I_{NA} = 0.2t \times 0.2^2 \times 2 + \frac{t \times 0.1^3}{12} \times 2 + 0.1 \times t \times 0.15^2 \times 2$$

$$= \int_0^\pi (0.2 \times \cos \phi)^2 (t \times 0.2 d\phi)$$

$$= 0.016t + 1.667 \times 10^{-4}t + 4.5 \times 10^{-3}t + 8 \times 10^{-3}t \int_0^\pi \left(\frac{1 + \cos 2\phi}{2} \right) d\phi$$

$$= 0.0207t + 0.01257t$$

i.e.

$$I_{NA} = 0.0333t$$

AB

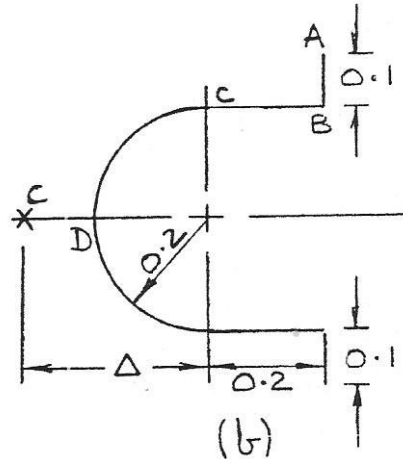
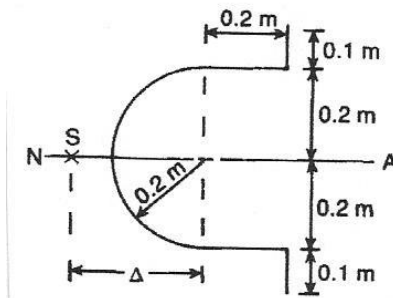
$$F_{AB} = \int_{0.1}^{0.2} \tau t dy = \frac{Ft}{I} \left[\frac{y^3}{6} - 5 \times 10^{-3}y \right]_{0.1}^{0.2}$$

$$= \frac{Ft}{I} \left[(3.333 \times 10^{-4}) - (-3.333 \times 10^{-4}) \right]$$

$$= \frac{6.6667 \times 10^{-4} Ft}{0.0333t} = \mathbf{0.02 F}$$

$$\tau = \frac{F \int_{0.1}^y (dy/t) y}{tI} = \frac{F}{I} \left(\frac{y^2}{2} - 5 \times 10^{-3} \right)$$

BC



$$I_{NA} = 0.016t + 1.667 \times 10^{-4}t + 0.1t \times 0.25^2 \times 2 + 0.01257t$$

i.e. $I_{NA} = 0.0412t$

AB

$$\int y dA = \int_{0.2}^y t dy y$$

$$= t \left[\frac{y^2}{2} \right]_{0.2}^y = \frac{t}{2} [y^2 - 0.09]$$

$$\tau = \frac{F \int y dA}{tI} = \frac{0.5 F t}{tI} (y^2 - 0.09)$$

$$F_{AB} = \int \tau t dy = \frac{0.5 F t}{I} \left[\frac{y^3}{3} - 0.09y \right]_{0.2}^{0.25}$$

$$= \frac{0.5 F t}{I} [(-1.53 \times 10^{-2}) - (-1.8 \times 10^{-2})]$$

i.e. $F_{AB} = \frac{1.35 \times 10^{-3} F t}{I}$

At B, $\int y dA = 0.1t \times 0.25 = 0.025t$

At C, $\int y dA = 0.025t + 0.04t = \mathbf{0.065t}$

$$\tau_B = \mathbf{0.025F/I}$$

$$\tau_C = \mathbf{0.065F/I}$$

$$F_{BC} = \mathbf{9 \times 10^{-3} Ft/I}$$

CB

$$\int y dA = 0.065t + 0.04t \sin \phi$$

$$\tau_\phi = \frac{F}{I} (0.065 + 0.04 \sin \phi)$$

$$F_{CD} = \int_0^\pi \tau_\phi 0.2 d\phi t$$

$$= \frac{0.2Ft}{I} [0.065\phi - 0.04 \cos \phi]_0^\pi$$

$$= \frac{0.2Ft}{I} [(0.065\pi + 0.04) - (0 - 0.04)]$$

i.e. $F_{CD} = \frac{\mathbf{0.0568Ft}}{I}$

Taking moments about 0 gives:

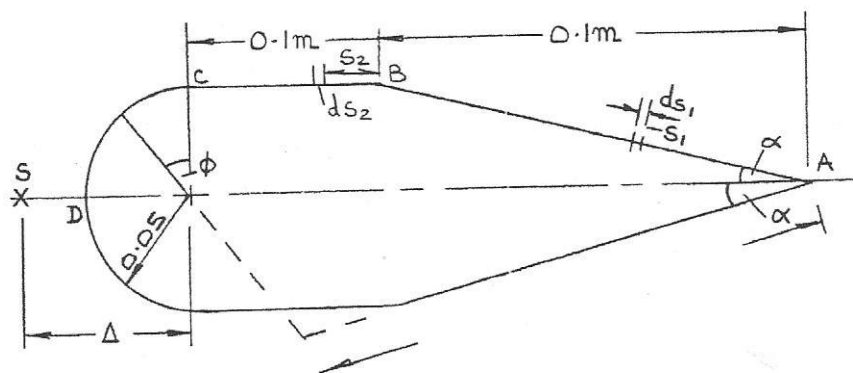
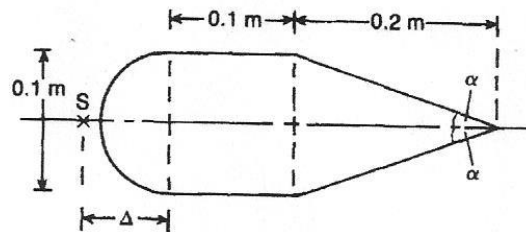
$$F\Delta = \frac{Ft}{I} [-1.35 \times 10^{-3} \times 0.2 \times 2 + 9 \times 10^{-3} \times 0.2 \times 2 + 0.0568 \times 2]$$

$$= \frac{Ft}{0.0412t} [-5.4 \times 10^{-4} + 3.6 \times 10^{-3} + 0.1136]$$

from which, **shear centre position, $\Delta = 2.83$ m**

EXERCISE 62, Page 359

1. Determine the shear centre position for the thin-walled closed tube shown which is of uniform thickness.



$$\alpha = \tan^{-1}(0.05/0.2) = 14.04^\circ$$

$$I = 2 \times \int_0^{0.206} t ds_1 (s_1 \sin \alpha)^2 + 0.1 \times t \times 0.05^2 \times 2 + \int_0^\pi (t R d\phi) (R \cos \phi)^2$$

$$= 0.1176t \left[\frac{s_1^3}{3} \right]_0^{0.206} + 5 \times 10^{-4} t + t R^3 \int_0^\pi \cos^2 \phi d\phi$$

$$= \left[3.427 \times 10^{-4} + 5 \times 10^{-4} + R^3 \int_0^\pi \left(\frac{1 + \cos 2\phi}{2} d\phi \right) \right] t$$

$$= \left[8.427 \times 10^{-4} + \frac{0.05^3}{2} \left(\phi + \frac{\sin 2\phi}{2} \right) \right] t$$

i.e. $I = 1.039 \times 10^{-3} t$

AB

$$\tau_{s_1} = \frac{F}{tI} \int (t ds_1) s_1 \sin \alpha$$

$$= \frac{F}{1.039 \times 10^{-3} t} \times 0.2426 \left[\frac{s_1^2}{2} \right]$$

$$= \mathbf{116.75 F s_1^2 t}$$

$$\tau_B = \frac{4.95 F}{t}$$

BC

As 't' is uniform,

$$\tau_{s_2} = \frac{4.95 F}{t} + \frac{F}{t I} \int_0^{s_2} (t ds_2) \times 0.05$$

$$= \frac{4.95 F}{t} + \frac{48.12 F s_2}{t}$$

$$\tau_c = \frac{9.76 F}{t}$$

CD

As 't' is uniform,

$$\tau_\phi = \frac{9.76 F}{t} + \frac{F}{t I} \int_0^\phi (t R d\phi) R \cos \phi$$

$$= \frac{9.76 F}{t} + \frac{F R^2}{1.04 \times 10^{-3} t} [\sin \phi]_0^\pi$$

i.e.

$$\tau_\phi = \frac{9.76 F}{t} + 2.404 \sin \phi \frac{F}{t}$$

$$\tau_0 = \frac{\oint \tau ds}{\oint ds}$$

However,

$$\oint ds = 0.206 \times 2 + 0.1 \times 0.2 + \pi \times 0.05 = \mathbf{0.769 \text{ m}}$$

and

$$\begin{aligned} \oint \tau ds &= 2 \times \int_0^{0.206} \frac{116.8 F s_1^2}{t} ds_1 + 2 \times \int_0^{0.1} \left(\frac{4.9 F}{t} + \frac{48.12 F s_2}{t} \right) ds_2 \\ &\quad + \int_0^\pi \left(\frac{9.76 F}{t} + 2.4 \sin \phi \frac{F}{t} \right) 0.05 d\phi \end{aligned}$$

$$= \frac{233.6F}{t} \left[\frac{s_1^3}{3} \right]_0^{0.206} + \left[9.9F \frac{s_2}{t} + \frac{48.12Fs_2^2}{t} \right]_0^{0.1} + \left[\frac{0.488F\phi}{t} - \frac{0.12F \cos \phi}{t} \right]_0^\pi$$

$$= \frac{F}{t} (0.681 + 0.09 + 0.481 + 1.533 + 0.24)$$

$$= \frac{3.925F}{t}$$

$$\tau_0 = \frac{-3.925F}{t} \times \frac{1}{0.769}$$

i.e. $\tau_0 = \frac{-5.104F}{t}$

Taking moments about '0' gives:

$$F(\Delta + R) = 2 \times \int_0^{0.206} \left(\tau_0 + \frac{116.8Fs_1^2}{t} \right) \times t ds_1 \times 0.3 \cos \alpha \times \tan \alpha$$

+

$$2 \int_0^{0.1} \left[\tau_0 + \left(\frac{4.95F}{t} + \frac{48.12Fs_2}{t} \right) \right] t ds_2 \times 0.05$$

$$= \int_0^\pi \left[\tau_0 + \left(\frac{9.76F}{t} + 2.4 \frac{F}{t} \sin \phi \right) \right] 0.05^2 d\phi t$$

or
$$F(\Delta) = 0.6F \left[-5.104s_1 + \frac{116.8s_1^3}{3} \right]_0^{0.206} \times 0.243$$

$$+ 0.1F \left[-5.104s_2 + 4.95s_2 + 48.12 \frac{s_2^2}{2} \right]_0^{0.1}$$

Hence,
$$\Delta = 0.05^2 F [-5.1041 + 9.76\phi - 2.4 \cos \phi]_0^\pi$$

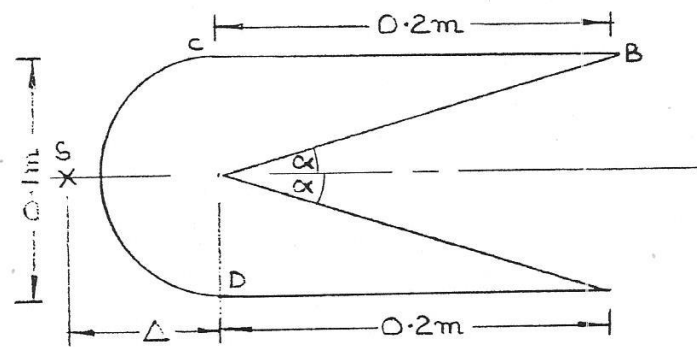
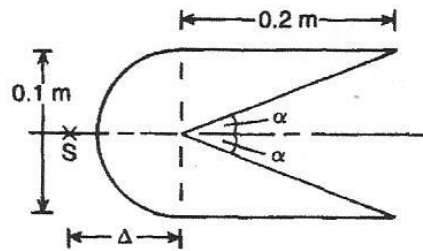
$$= 0.146(-1.051 + 0.34) + 0.1(-0.51 + 0.495 + 0.241)$$

$$+ 0.05^2 (-16.03 + 30.66 + 4.8)$$

$$= -0.104 + 0.023 + 0.049 = -0.032 \text{ m}$$

i.e. the shear centre position, $\Delta = -0.032 \text{ m}$

2. Determine the shear centre position for the thin-walled closed tube shown, which is of uniform thickness.



OB

$$\alpha = \tan^{-1}(0.05 / 0.2) = 14.04^\circ$$

$$I = 2 \int_0^{0.206} (t ds_1) (s_1 \sin \alpha)^2 ds_1 + 2 \times 0.2 \times t \times 0.05^2 + \int_0^\pi t R d\phi (R \cos \phi)^2$$

$$= \frac{2}{3} \times 0.05886 \left[\frac{s_1^3}{3} \right]_0^{0.206} t + 1.963 \times 10^{-4} t$$

$$= 3.43 \times 10^{-4} t + 1 \times 10^{-3} t + 1.963 \times 10^{-4} t$$

i.e.

$$I = 1.539 \times 10^{-3} t$$

$$\tau_{s_1} = \frac{F}{tI} \int_0^{s_1} (s_1 \sin \alpha) t ds_1 = \frac{F}{I} \left[\frac{s_1^2}{2} \right]_0^{s_1} \sin \alpha$$

i.e.

$$\tau_{s_1} = \frac{78.82 F s_1^2}{t}$$

and

$$\tau_B = \frac{3.345 F}{t}$$

BC

As 't' is constant,

$$\begin{aligned}\tau_{s_2} &= \frac{3.345F}{t} + \frac{F}{tI} \int_0^{s_2} t ds_2 \times 0.05 \\ &= \frac{3.345F}{t} + \frac{32.49Fs_2}{t} \\ \tau_c &= \frac{9.84F}{t}\end{aligned}$$

CD

$$\begin{aligned}\tau_\phi &= \frac{9.84F}{t} + \frac{F}{tI} \int_0^\phi (R d\phi t) R \cos \phi \\ &= \frac{9.84F}{t} + \frac{1.62F \sin \phi}{t}\end{aligned}$$

$$\tau = \frac{-\int \tau ds}{\int ds}$$

$$\begin{aligned}\int ds &= 0.206 \times 2 + 0.2 \times 2 + \pi \times 0.05 \\ &= 0.969 \text{ m}\end{aligned}$$

$$\begin{aligned}\int \tau_s ds &= 2 \int_0^{0.206} \frac{78.82Fs_1^2}{t} ds_1 + \frac{2F}{t} \int_0^{0.2} (3.345 + 32.49s_2) ds_2 \\ &\quad + \frac{F}{t} \int_0^\pi (9.84 + 1.62 \sin \phi) \times 0.05 d\phi \\ &= \frac{157.64F}{t} [s_1^3]_0^{0.206} + \frac{2F}{t} \left[3.345s_2 + \frac{32.49s_2^2}{2} \right]_0^{0.2} \\ &\quad + \frac{0.05F}{t} [9.84\phi - 1.62 \cos \phi]_0^\pi \\ &= \frac{F}{t} (0.46 + 2.636 + 1.71) = \frac{4.8F}{t} \\ \tau_0 &= -\frac{4.8F}{t} \times \frac{1}{0.969}\end{aligned}$$

i.e. $\tau_0 = -\frac{4.96F}{t}$

Taking moments about 0 gives:

$$F(\Delta + 0.05) = 2F \int_0^{0.2} (-4.96 + 3.35 + 32.49s_2) ds_2 \times 0.05$$

+

$$F \int_0^\pi (-4.96 + 9.84 + 1.62 \sin \phi) \times 0.05^2 d\phi$$

or $(\Delta + 0.05) = 0.1 \left[-1.61s_2 + 16.2s_2^2 \right]_0^{0.2} + \left[0.0122\phi - 4.05 \times 10^{-3} \cos \phi \right]_0^\pi$

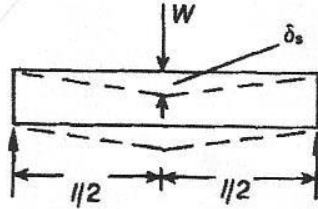
$$= 0.1(-0.322 + 0.65) + 0.0383 - 8.1 \times 10^{-3}$$

$$= 0.0328 + 0.0464$$

from which, **the shear centre position, $\Delta = 0.0792$ m**

EXERCISE 63, Page 361

1. Determine the maximum deflection due to shear for the simply supported beam shown. It may be assumed that the beam cross-section is rectangular, of constant width b and of constant depth d .



$$F = \frac{W}{2}$$

and

$$\tau = \frac{3W}{bd^3} \left(\frac{d^2}{4} - y^2 \right)$$

$$\text{SSE} = \int \frac{\tau^2}{2G} d(\text{vol})$$

$$= \frac{2}{2} \int \frac{9W^2}{Gb^2d^6} \left(\frac{d^2}{4} - y^2 \right)^2 l b dy$$

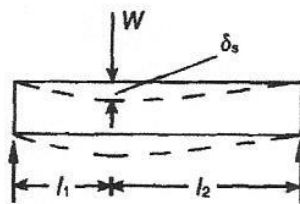
i.e.

$$\text{SSE} = \frac{3W^2l}{20Gbd}$$

$$\text{WD} = \frac{1}{2} W \delta_s$$

from which, **maximum deflection**, $\delta_s = \frac{3Wl}{10Gbd}$

2. Determine the maximum deflection due to shear for the simply supported beam shown. It may be assumed that the beam cross-section is rectangular, of constant width b and of constant depth d .



$$F_1 = \frac{W l_2}{l} \quad \text{and} \quad F_2 = \frac{W l_1}{l}$$

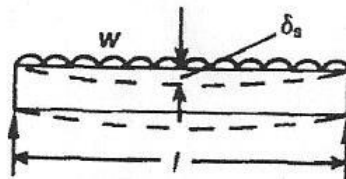
$$\begin{aligned} \text{SSE} &= \frac{2}{2} \int_0^{d/2} \frac{36W^2 l_1^2}{Gb^2 d^6 l^2} \left(\frac{d^2}{4} - y^2 \right)^2 b l_1 dy \\ &= 36 \left(\frac{W^2}{Gb^2 d^6 l^2} \right) \times b (l_2^2 l_1 + l_1^2 l_2) \left[\frac{d^4 y}{16} - \frac{d^2 y^3}{6} + \frac{y^5}{5} \right]_0^{d/2} \\ &= 36 \left(\frac{W^2 (l_2^2 l_1 + l_1^2 l_2)}{60 b d l^2} \right) = 36 \left(\frac{W^2 (l_1 + l_2) l_1 l_2}{60 b d l^2} \right) \end{aligned}$$

i.e. $\text{SSE} = \frac{3W^2 l_1 l_2}{5Gbdl}$ since $l_1 + l_2 = l$

$$\text{WD} = \frac{1}{2} W \delta_s$$

from which, **maximum deflection**, $\delta_s = \frac{6Wl_1 l_2}{5Gbdl}$

3. Determine the maximum deflection due to shear for the simply supported beam shown. It may be assumed that the beam cross-section is rectangular, of constant width b and of constant depth d .



At x , $F = \frac{wl}{2} - wx = w \left(\frac{l}{2} - x \right)$

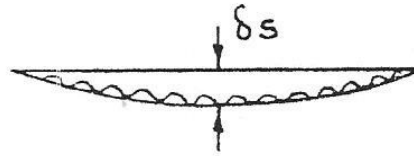
$$\tau = \frac{6F}{bd^3} \left(\frac{d^2}{4} - y^2 \right)$$

$$\text{SSE} = \frac{2 \times 2}{2} \int_0^{l/2} \int_0^y \frac{36F^2}{Gb^2 d^6} \left(\frac{d^2}{4} - y^2 \right) b dx dy$$

$$\begin{aligned}
 &= \frac{72bd^5}{Gb^2d^660} w^2 \int_0^{l/2} \left(\frac{l}{2} - x \right)^2 dx \\
 &= \frac{6w^2}{5Gb d} \int_0^{l/2} \left(\frac{l^2}{4} - lx + x^2 \right) dx \\
 &= \frac{6w^2}{5Gb d} \left[\frac{l^2 x}{4} - \frac{l x^2}{2} + \frac{x^3}{3} \right]_0^{l/2}
 \end{aligned}$$

i.e.
$$SSE = \frac{6w^2 l^3}{120Gb d}$$

$$WD = \frac{1}{2} \times wl \times \delta_s \times \frac{2}{3}$$



and
$$\delta_s = \frac{3 \times WD}{wl}$$

i.e. the maximum deflection,
$$\delta_s = \frac{3wl^2}{20Gb d}$$
