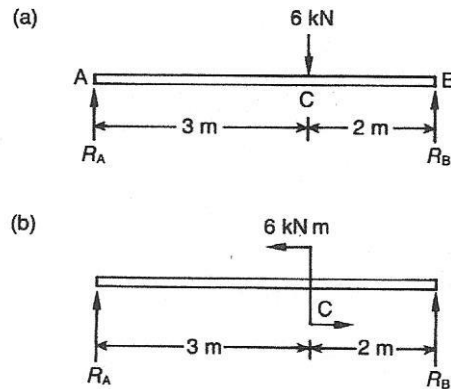


## CHAPTER 3 STATICS

### EXERCISE 20, Page 62

1. Determine the reactions  $R_A$  and  $R_B$  for the simply supported beams shown.



(a) Taking moments about A gives:

$$R_B \times 5 = 6 \times 3$$

from which,  $R_B = \frac{6 \times 3}{5} = 3.6 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 6$$

i.e.  $R_A + 3.6 = 6$

from which,  $R_A = 6 - 3.6 = 2.4 \text{ kN}$

(b) Taking moments about A gives:

$$R_B \times 5 + 6 = 0$$

from which,  $R_B = \frac{-6}{5} = -1.2 \text{ kN}$

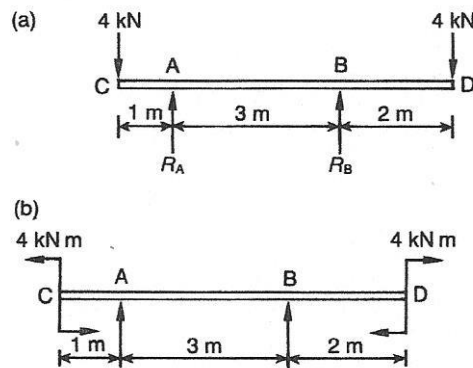
Resolving vertically gives:

$$R_A + R_B = 0$$

i.e.  $R_A - 1.2 = 0$

from which,  $R_A = 1.2 \text{ kN}$

2. Determine the reactions  $R_A$  and  $R_B$  for the simply supported beams shown.



(a) Taking moments about A gives:

$$R_B \times 3 + 4 \times 1 = 4 \times 5$$

i.e.  $R_B = \frac{20 - 4}{3} = 5.333 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 4 + 4$$

from which,  $R_A = 8 - 5.333 = 2.667 \text{ kN}$

(b) Taking moments about A gives:

$$R_B \times 3 = 4 - 4$$

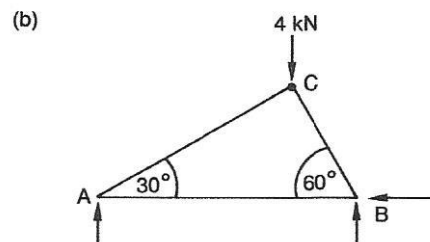
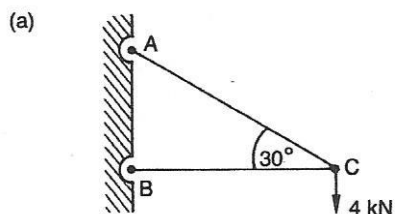
from which,  $R_B = 0$

Resolving vertically gives:

$$R_A + R_B = 0$$

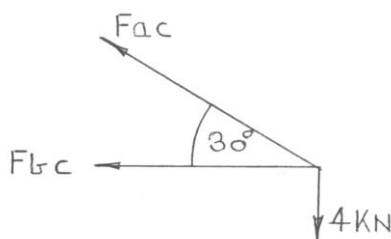
from which,  $R_A = 0$

3. Determine the internal forces in the members of the plane pin-jointed trusses shown below.



(a) Assume unknown internal forces are in tension, and consider the equilibrium of joint C.

Resolving forces vertically gives:



$$F_{ac} \sin 30^\circ = 4$$

from which,  $F_{ac} = \frac{4}{\sin 30^\circ} = 8 \text{ kN (tension)}$

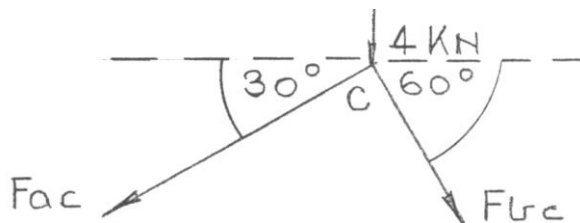
Resolving forces horizontally gives:

$$F_{ac} \cos 30^\circ + F_{bc} = 0$$

i.e.  $8 \cos 30^\circ + F_{bc} = 0$

and  $F_{bc} = -8 \cos 30^\circ = -6.928 \text{ kN (compression)}$

(b) Assume all unknown internal forces are in tension and consider the equilibrium of joint C.



Resolving horizontally gives:

$$F_{ac} \cos 30 = F_{bc} \cos 60$$

from which, 
$$F_{ac} = \frac{\cos 60^\circ}{\cos 30^\circ} = 0.5774 F_{bc} \quad (1)$$

Resolving vertically gives:

$$4 + F_{ac} \sin 30 + F_{bc} \sin 60 = 0$$

or 
$$4 = -0.5 F_{ac} - 0.866 F_{bc} \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$4 = -0.2887 F_{bc} - 0.866 F_{bc}$$

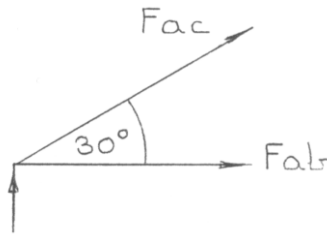
i.e. 
$$4 = -1.1547 F_{bc}$$

and 
$$F_{bc} = \frac{4}{-1.1547} = -\mathbf{3.464 \text{ kN}} \text{ (compression)} \quad (3)$$

Substituting equation (3) into equation (1) gives:

$$F_{ac} = 0.5774 F_{bc} = 0.5774(-3.464) = -\mathbf{2 \text{ kN}} \text{ (compression)} \quad (4)$$

Consider Joint A



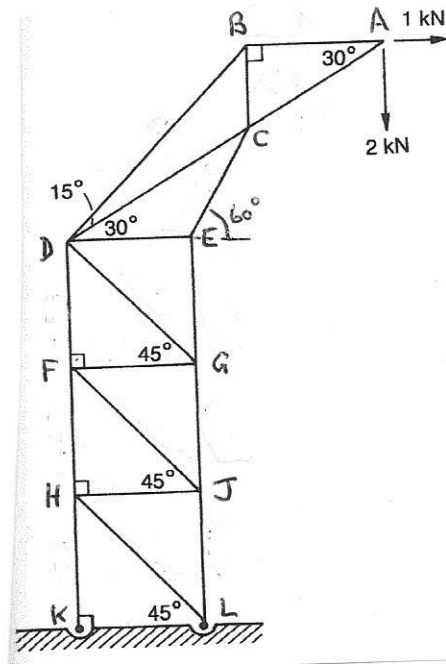
Resolving horizontally gives:

$$F_{ac} \cos 30 + F_{ab} = 0$$

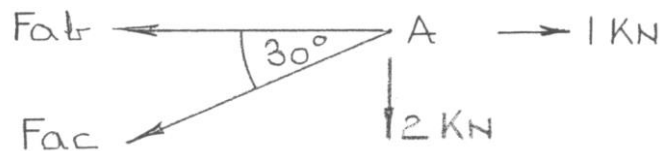
i.e. 
$$F_{ab} = -F_{ac} \cos 30^\circ$$

$$= -(-2) \cos 30^\circ = \mathbf{1.732 \text{ kN}} \text{ (tension)}$$

4. A plane pin-jointed truss is firmly pinned at its base, as shown below. Determine the forces in the members of this truss, stating whether they are in tension or compression.



Assume all unknown forces are in tension, and consider equilibrium of **Joint A**.



Resolving vertically gives:

$$F_{ac} \sin 30^\circ + 2 = 0$$

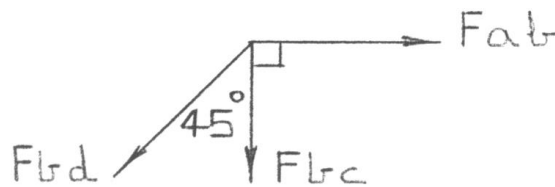
from which,  $F_{ac} = \frac{-2}{\sin 30^\circ} = -4 \text{ kN}$  (compression)

Resolving horizontally gives:

$$F_{ab} + F_{ac} \cos 30^\circ = 1$$

i.e.  $F_{ab} = 1 - 0.866 F_{ac} = 1 - 0.866(-4) = 4.464 \text{ kN}$  (tension)

**Joint B**



Resolving horizontally gives:

$$F_{bd} \cos 45^\circ = F_{ab}$$

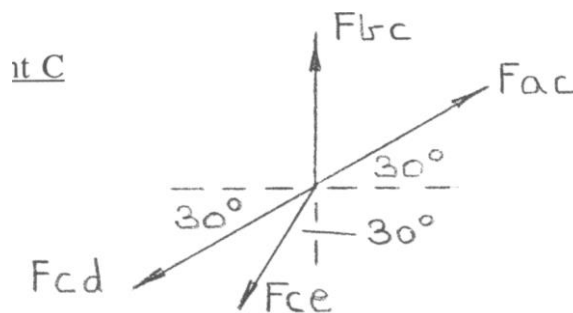
i.e. 
$$F_{bd} = \frac{4.464}{\cos 45^\circ} = \mathbf{6.313 \text{ kN (tension)}}$$

Resolving vertically gives:

$$F_{bc} + F_{bd} \cos 45^\circ = 0$$

i.e. 
$$F_{bc} = -6.313 \cos 45^\circ = \mathbf{-4.464 \text{ kN (compression)}}$$

**Joint C**



Resolving horizontally gives:

$$F_{ac} \cos 30 = F_{cd} \cos 30 + F_{ce} \cos 60$$

i.e. 
$$F_{cd} = F_{ac} - 0.5774 F_{ce}$$

and 
$$F_{cd} = -4 - 0.5774 F_{ce} \quad (1)$$

Resolving vertically gives:

$$F_{bc} + F_{ac} \sin 30 = F_{cd} \sin 30 + F_{ce} \sin 60$$

i.e. 
$$-4.464 - 2 = 0.5 F_{cd} + 0.866 F_{ce} \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$-6.464 = -2 - 0.2887 F_{ce} + 0.866 F_{ce}$$

i.e. 
$$-4.464 = 0.5773 F_{ce}$$

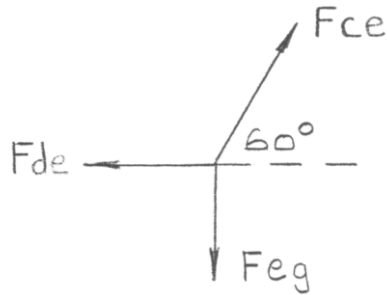
from which, 
$$\mathbf{F_{ce} = \frac{-4.464}{0.5773} = -7.733 \text{ kN (compression)}} \quad (3)$$

Substituting equation (3) into equation (1) gives:

$$\mathbf{F_{cd} = -4 - 0.5774(-7.733)}$$

i.e. 
$$\mathbf{F_{cd} = 0.465 \text{ kN (tension)}}$$

#### Joint E



Resolving horizontally gives:

$$\mathbf{F_{ce} \cos 60 = F_{de}}$$

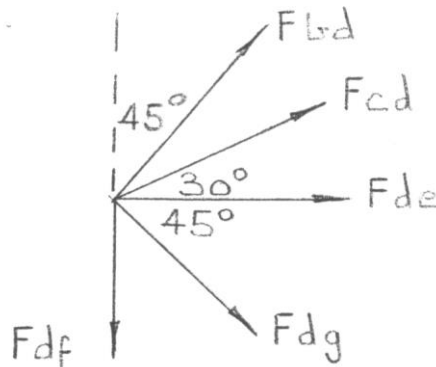
i.e. 
$$\mathbf{F_{de} = -7.733 \cos 60^\circ = -3.867 \text{ kN (compression)}}$$

Resolving vertically gives:

$$\mathbf{F_{ce} \sin 60 = F_{eg}}$$

i.e. 
$$\mathbf{F_{eg} = -7.733 \sin 60^\circ = -6.70 \text{ kN (compression)}}$$

#### Joint D



Resolving horizontally gives:

$$\mathbf{F_{bd} \cos 45^\circ + F_{cd} \cos 30^\circ + F_{de} + F_{dg} \cos 45^\circ = 0}$$

i.e. 
$$\mathbf{6.313 \times 0.707 + 0.465 \times 0.866 - 3.867 + 0.707 F_{dg} = 0}$$

from which, 
$$\mathbf{F_{dg} = \frac{-(6.313 \times 0.707 + 0.465 \times 0.866 - 3.867)}{0.707}}$$

$$= -\mathbf{1.413 \text{ kN (compression)}}$$

Resolving vertically gives:

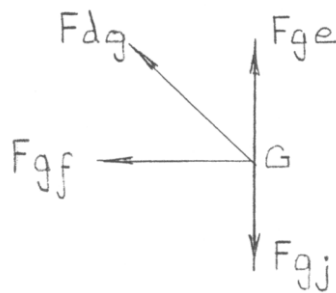
$$F_{bd} \sin 45 + F_{cd} \sin 30 = F_{dg} \sin 45 + F_{df}$$

i.e. 
$$6.313 \times 0.707 + 0.465 \times 0.5 = -1.413 \times 0.707 + F_{df}$$

from which, 
$$F_{df} = 6.313 \times 0.707 + 0.465 \times 0.5 + 1.413 \times 0.707$$

i.e. 
$$\mathbf{F_{df} = 5.695 \text{ kN (tension)}}$$

**Joint G**



Resolving horizontally gives:

$$-F_{dg} \times 0.707 = F_{gf}$$

i.e. 
$$\mathbf{F_{gf} = -(-1.413) \times 0.707 = 1.0 \text{ kN (tension)}}$$

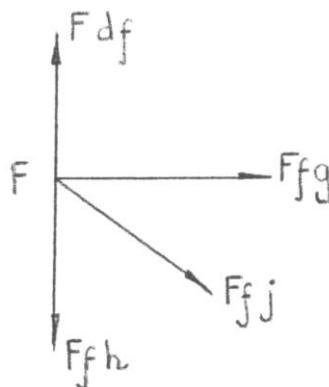
Resolving vertically gives:

$$F_{ge} + F_{dg} \times 0.707 = F_{gj}$$

i.e. 
$$-6.70 - 1.413 \times 0.707 = F_{gj}$$

i.e. 
$$\mathbf{F_{gj} = -7.699 \text{ kN (compression)}}$$

**Joint F**





Resolving horizontally gives:

$$-F_{fj} \times 0.707 = F_{fg}$$

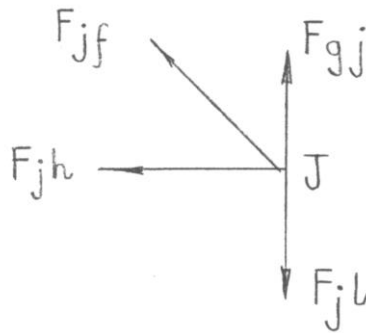
i.e. 
$$F_{fj} = \frac{F_{fg}}{0.707} = -\frac{1.0}{0.707} = -\mathbf{1.414 \text{ kN}} \text{ (compression)}$$

Resolving vertically gives:

$$F_{df} = F_{fj} \times 0.707 + F_{fh}$$

i.e. 
$$F_{fh} = 5.695 - -1.414 \times 0.707 = \mathbf{6.695 \text{ kN}} \text{ (tension)}$$

### Joint J



Resolving horizontally gives:

$$-F_{jf} \times 0.707 = F_{jh}$$

i.e. 
$$F_{jh} = -(-1.414) \times 0.707 = \mathbf{1.0 \text{ kN}} \text{ (tension)}$$

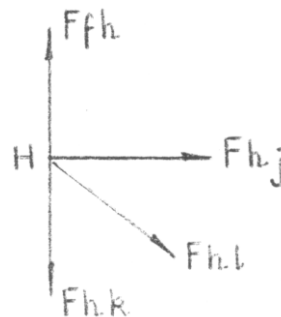
Resolving vertically gives:

$$F_{gj} + F_{jf} \times 0.707 = F_{jl}$$

i.e. 
$$-7.699 - 1.414 \times 0.707 = F_{jl}$$

i.e. 
$$F_{jl} = -\mathbf{8.699 \text{ kN}} \text{ (compression)}$$

### Joint H



Resolving horizontally gives:

$$F_{hl} \cos 45^\circ = -F_{hj}$$

i.e. 
$$F_{hl} = \frac{-1.0}{\cos 45^\circ} = -1.414 \text{ kN (compression)}$$

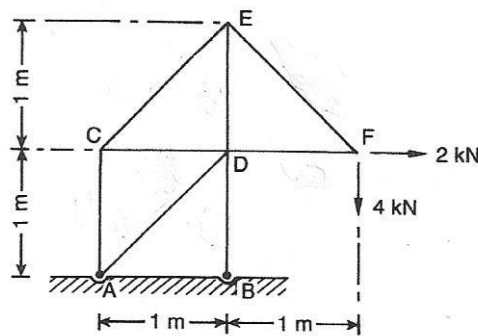
Resolving vertically gives:

$$F_{fh} = F_{hl} \times 0.707 + F_{hk}$$

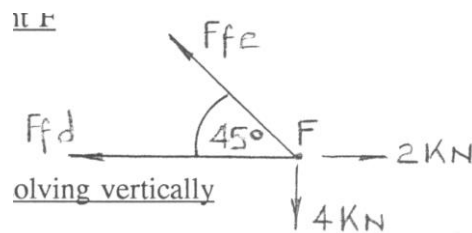
i.e. 
$$F_{hk} = F_{fh} - F_{hl} \times 0.707 = +6.695 - (-1.414) \times 0.707$$
  

$$= 7.695 \text{ kN (tension)}$$

5. The plane pin-jointed truss shown below is firmly pinned at A and B and subjected to two point loads at point F. Determine the forces in the members, stating whether they are tensile or compressive.



**Joint F**



Resolving vertically gives:

$$F_{fe} \times 0.707 = 4$$

i.e. 
$$F_{fe} = \frac{4}{0.707} = 5.658 \text{ kN (tensile)}$$

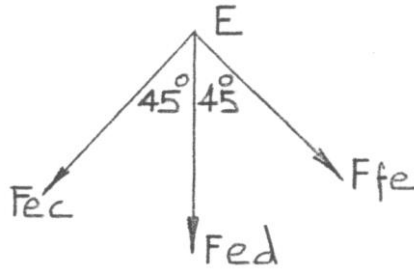
Resolving horizontally gives:

$$F_{fe} \times 0.707 + F_{fd} = 2$$

i.e. 
$$F_{fd} = 2 - F_{fe} \times 0.707 = 2 - 5.658 \times 0.707$$

$$= -2.0 \text{ kN (compressive)}$$

### Joint E



Resolving horizontally gives:

$$F_{ec} = F_{fe} = 5.658 \text{ kN (tensile)}$$

Resolving vertically gives:

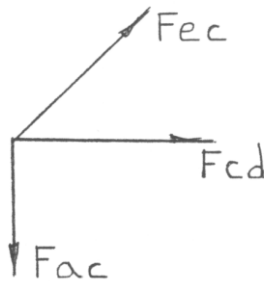
$$F_{ec} \times 0.707 + F_{fe} \times 0.707 + F_{ed} = 0$$

$$\text{i.e. } F_{ed} = -F_{ec} \times 0.707 - F_{fe} \times 0.707$$

$$\text{i.e. } F_{ed} = -5.658 \times 0.707 - 5.658 \times 0.707$$

$$= -8.039 \text{ kN (compressive)}$$

### Joint C



Resolving horizontally gives:

$$F_{ec} \times 0.707 + F_{cd} = 0$$

$$\text{i.e. } F_{cd} = -F_{ec} \times 0.707 = -5.658 \times 0.707$$

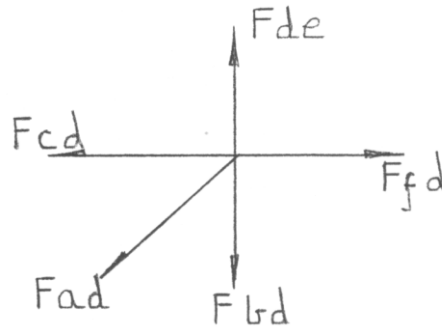
$$= -4.0 \text{ kN (compressive)}$$

Resolving vertically gives:

$$F_{ec} \times 0.707 = F_{ac}$$

$$\text{i.e. } F_{ac} = 5.658 \times 0.707 = 4.0 \text{ kN (tensile)}$$

### Joint D



Resolving horizontally gives:

$$F_{fd} = F_{cd} + F_{ad} \times 0.707$$

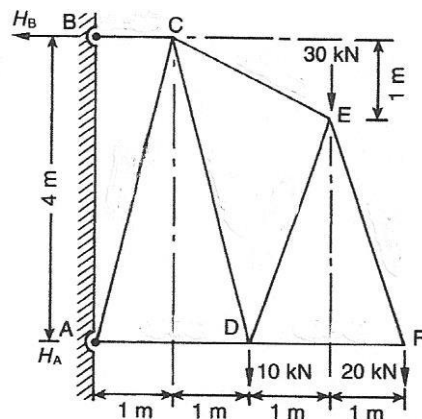
i.e. 
$$F_{ad} = \frac{F_{fd} - F_{cd}}{0.707} = \frac{-2 - -4}{0.707} = \mathbf{2.829 \text{ kN (tensile)}}$$

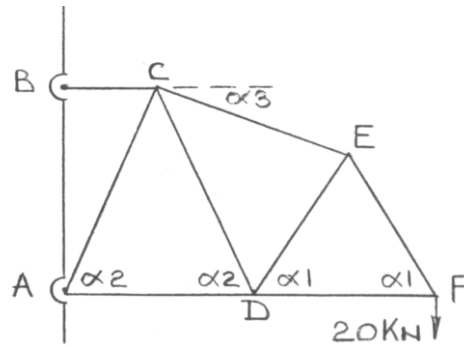
Resolving vertically gives:

$$F_{de} = F_{bd} + F_{ad} \times 0.707$$

i.e. 
$$\begin{aligned} F_{bd} &= F_{de} - F_{ad} \times 0.707 \\ &= -8.039 - 2.829 \times 0.707 \\ &= \mathbf{-10.039 \text{ kN (compressive)}} \end{aligned}$$

**6.** An overhanging pin-jointed roof truss, which may be assumed to be pinned rigidly to the wall at the joints A and B, is subjected to the loading shown below. Determine the forces in the members of the truss, stating whether they are tensile or compressive. Determine, also, the reactions.



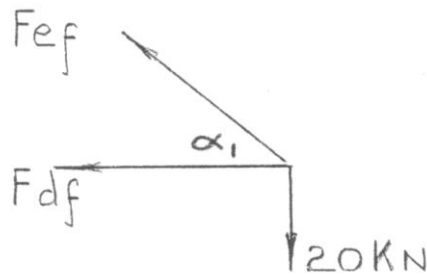


$$\alpha_1 = \tan^{-1} 3 = 71.565^\circ$$

$$\alpha_2 = \tan^{-1} 4 = 75.964^\circ$$

$$\alpha_3 = \tan^{-1} (1/2) = 26.565^\circ$$

### Joint F



Resolving vertically gives;

$$F_{ef} \sin \alpha_1 = 20$$

from which, 
$$F_{ef} = \frac{20}{\sin 71.565^\circ} = \mathbf{21.08 \text{ kN (tensile)}}$$

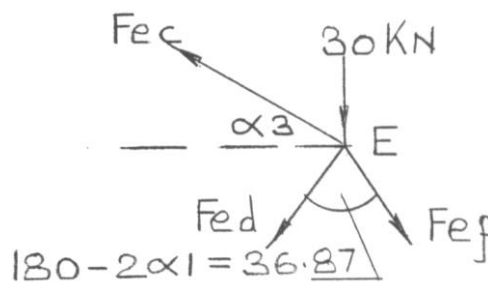
Resolving horizontally gives:

$$F_{df} + F_{ef} \cos \alpha_1 = 0$$

i.e. 
$$F_{df} = -F_{ef} \cos \alpha_1 = -21.08 \times \cos 71.565^\circ$$

$$= -\mathbf{6.666 \text{ kN (compressive)}}$$

### Joint E



Resolving vertically gives:

$$F_{ec} \sin \alpha_3 = 30 + (F_{ed} + F_{ef}) \cos (18.435^\circ)$$

$$\text{i.e.} \quad F_{ec} \sin 26.565^\circ = 30 + (F_{ed} + 21.08) \cos (18.435^\circ)$$

$$\text{i.e.} \quad 0.447 F_{ec} = 30 + 0.949 F_{ed} + 20$$

$$\text{i.e.} \quad F_{ec} = 111.86 + 2.123 F_{ed} \quad (1)$$

Resolving horizontally gives:

$$F_{ec} \cos \alpha_3 + F_{ed} \sin (18.435^\circ) = F_{ef} \sin (18.435^\circ)$$

$$\text{i.e.} \quad 0.894 F_{ec} + 0.316 F_{ed} = 6.666 \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$0.894 (111.86 + 2.123 F_{ed}) + 0.316 F_{ed} = 6.666$$

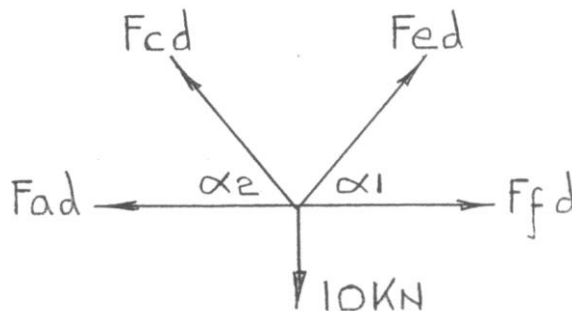
$$\text{i.e.} \quad 100 + 2.214 F_{ed} = 6.666$$

$$\text{i.e.} \quad F_{ed} = -42.16 \text{ kN (compressive)} \quad (3)$$

Substituting equation (3) into equation (1) gives:

$$\begin{aligned} F_{ec} &= 111.86 + 2.123 F_{ed} \\ &= 111.86 + 2.123(-42.16) = 22.35 \text{ kN (tensile)} \end{aligned}$$

### Joint D



Resolving horizontally gives:

$$F_{ad} + F_{cd} \cos \alpha_2 = F_{fd} + F_{ed} \cos \alpha_1$$

$$\text{i.e.} \quad F_{ad} + F_{cd} \cos 75.965^\circ = -6.666 - 42.16 \cos 71.565^\circ$$

$$\text{i.e.} \quad F_{ad} + 0.243 F_{cd} = -6.666 - 42.16 \times 0.316$$

$$\text{i.e.} \quad F_{ad} = -20 - 0.243 F_{cd} \quad (4)$$

Resolving vertically gives:

$$F_{cd} \sin \alpha_2 + F_{ed} \sin \alpha_1 = 10$$

i.e.  $F_{cd} \sin 75.964^\circ + F_{ed} \sin 71.565^\circ = 10$

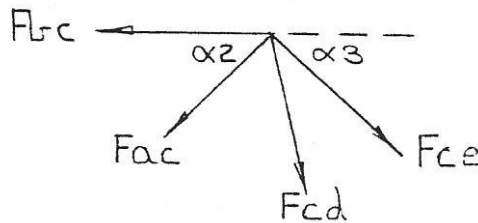
i.e.  $F_{cd} = 10/0.970 - -42.16 \times 0.949/0.970$

i.e.  $F_{cd} = \mathbf{51.557 \text{ kN}}$  (tensile) (5)

Substituting equation (5) into equation (4) gives:

$$\begin{aligned} F_{ad} &= -20 - 0.243 F_{cd} \\ &= -20 - 0.243(51.557) = -\mathbf{32.528 \text{ kN}} \text{ (compressive)} \end{aligned}$$

### Joint C



Resolving vertically gives:

$$F_{ac} \sin \alpha_2 + F_{cd} \sin \alpha_2 + F_{ce} \sin \alpha_3 = 0$$

i.e.  $F_{ac} \sin 75.964^\circ + F_{cd} \sin 75.964^\circ + F_{ce} \sin 26.565^\circ = 0$

i.e.  $0.970 F_{ac} + 51.557 \times 0.970 + 22.35 \times 0.447 = 0$

and  $F_{ac} = \frac{-51.557 \times 0.970 - 22.35 \times 0.447}{0.970} = -\mathbf{61.856 \text{ kN}}$  (compressive)

Resolving horizontally gives:

$$F_{bc} + F_{ac} \cos \alpha_2 = F_{cd} \cos \alpha_2 + F_{ce} \cos \alpha_3$$

i.e.  $F_{bc} - 61.856 \cos 75.964^\circ = 51.557 \cos 75.964^\circ + 22.35 \cos 26.565^\circ$

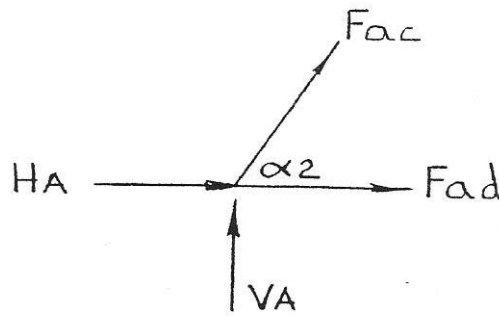
$$F_{bc} = +61.856 \times 0.243 + 51.557 \times 0.243 + 22.35 \times 0.894$$

$$F_{bc} = \mathbf{47.540 \text{ kN}}$$
 (tensile)

### Reactions

#### Joint B

$$H_B = F_{bc} = \mathbf{47.540 \text{ kN}}$$



### Joint A

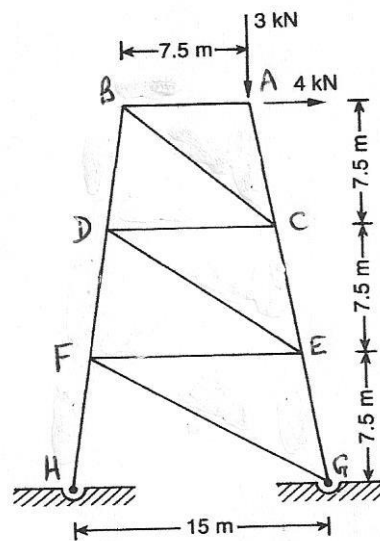
Resolving vertically gives:

$$V_A = -F_{ac} \sin \alpha_2 = -(-61.856) \sin 75.964^\circ = \mathbf{60.01 \text{ kN}}$$

Resolving horizontally gives:

$$H_A = -F_{ad} - F_{ac} \cos \alpha_2 = -32.528 + 61.856 \times 0.243 = \mathbf{47.559 \text{ kN}}$$

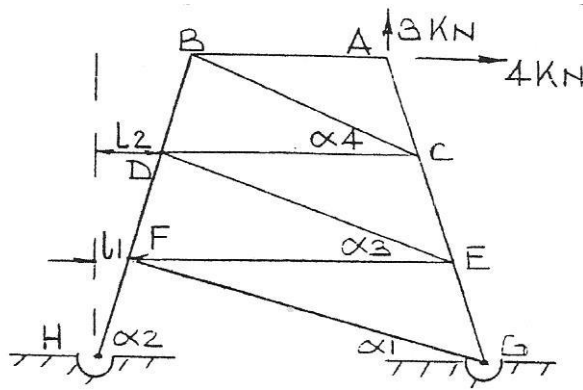
7. Determine the forces in the symmetrical pin-jointed truss shown, stating whether they are tensile or compressive.



$$\alpha_2 = \tan^{-1} \left( \frac{7.5 \times 3}{3.75} \right) = 80.538^\circ$$

$$l_1 = \frac{7.5}{\tan \alpha_2} = 1.25 \text{ m}$$





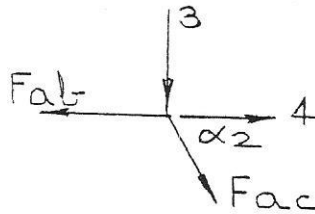
$$l_2 = 2.5 \text{ m}$$

$$\alpha_1 = \tan^{-1} \frac{7.5}{(15 - 1.25)} = 28.61^\circ$$

$$\alpha_3 = \tan^{-1} \frac{7.5}{(15 - 3.75)} = 33.69^\circ$$

$$\alpha_4 = \tan^{-1} \frac{7.5}{(15 - 3.75 - 2.5)} = 40.60^\circ$$

**Joint A**



Resolving vertically gives:

$$3 + F_{ac} \sin \alpha_2 = 0$$

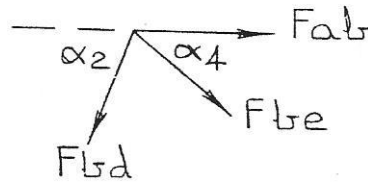
$$\begin{aligned} \text{i.e.} \quad F_{ac} &= -\frac{3}{\sin 80.538^\circ} = -\frac{3}{0.986} \\ &= -3.043 \text{ kN (compressive)} \end{aligned}$$

Resolving horizontally gives:

$$F_{ab} = F_{ac} \cos \alpha_2 + 4$$

$$\begin{aligned} \text{i.e.} \quad F_{ab} &= -3.043 \cos 80.538^\circ + 4 \\ &= +3.50 \text{ kN (tensile)} \end{aligned}$$

### Joint B



Resolving vertically gives:

$$F_{bd} \sin \alpha_2 + F_{bc} \sin \alpha_4 = 0$$

i.e.  $F_{bd} \sin 80.538^\circ + F_{bc} \sin 40.60^\circ = 0$

i.e.  $0.986 F_{bd} = -0.651 F_{bc}$

i.e.  $F_{bd} = -0.660 F_{bc}$  (1)

Resolving horizontally gives:

$$F_{bd} \cos \alpha_2 = F_{ab} + F_{bc} \cos \alpha_4$$

i.e.  $F_{bd} \cos 80.538^\circ = F_{ab} + F_{bc} \cos 40.60^\circ$

i.e.  $0.164 F_{bd} = 3.50 + 0.759 F_{bc}$  (2)

Substituting equation (1) into equation (2) gives:

i.e.  $0.164(-0.660 F_{bc}) = 3.50 + 0.759 F_{bc}$

and  $(0.759 + 0.108) F_{bc} = -3.50$

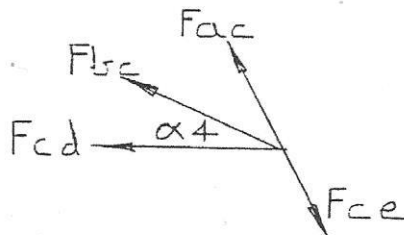
i.e.  $F_{bc} = -4.037 \text{ kN}$  (compressive) (3)

Substituting equation (3) into equation (1) gives:

$$F_{bd} = -0.660 F_{bc} = -0.660 \times -4.037$$

i.e.  $F_{bd} = 2.664 \text{ kN}$  (tensile)

### Joint C



Resolving horizontally gives:

$$F_{cd} + F_{bc} \cos \alpha_4 + F_{ac} \cos \alpha_2 = F_{ce} \cos \alpha_2$$

i.e.  $F_{cd} = 4.037 \times 0.759 + 3.043 \times 0.164 + F_{ce} \times 0.164$

i.e.  $F_{cd} = 3.563 + 0.164 F_{ce}$  (4)

Resolving vertically gives:

$$F_{ac} \sin \alpha_2 + F_{bc} \sin \alpha_4 = F_{ce} \sin \alpha_2$$

i.e.  $0.986 F_{ac} + 0.651 F_{bc} = 0.986 F_{ce}$

i.e.  $0.986(-3.043) + 0.651(-4.037) = 0.986 F_{ce}$

i.e.  $-5.6285 = 0.986 F_{ce}$

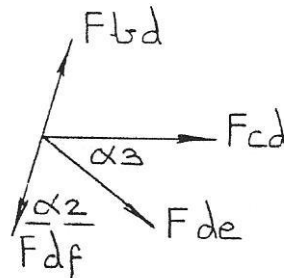
and  $F_{ce} = -5.6285/0.986 = -5.708 \text{ kN (compressive)}$  (5)

Substituting equation (5) into equation (4) gives:

$$F_{cd} = 3.563 + 0.164(-5.708)$$

i.e.  $F_{cd} = 2.627 \text{ kN (tensile)}$

### Joint D



Resolving vertically gives:

$$F_{bd} \sin \alpha_2 = F_{de} \sin \alpha_3 + F_{df} \sin \alpha_2$$

i.e.  $2.664 \times 0.986 = F_{de} \times 0.555 + 0.986 F_{df}$

and  $F_{df} = 2.664 - 0.563 F_{de}$  (6)

Resolving horizontally gives:

$$F_{bd} \cos \alpha_2 + F_{cd} + F_{de} \cos \alpha_3 = F_{df} \cos \alpha_2$$

i.e.  $0.164 \times 2.664 + 2.627 + 0.832 F_{de} = 0.164 F_{df}$  (7)

Substituting equation (5) into equation (6) gives:

$$0.164 \times 2.664 + 2.627 + 0.832 F_{de} = 0.164(2.664 - 0.563 F_{de})$$

i.e.  $0.437 + 2.627 = F_{de}(-0.832 - 0.0923) + 0.437$

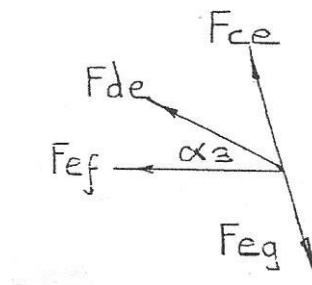
from which,  $F_{de} = -2.842 \text{ kN}$  (compressive) (8)

Substituting equation (8) into equation (6) gives:

$$F_{df} = 2.664 - 0.563(-2.842)$$

$$F_{df} = 4.264 \text{ kN (tensile)}$$

### Joint E



Resolving vertically gives:

$$F_{de} \sin \alpha_3 + F_{ce} \sin \alpha_2 = F_{eg} \sin \alpha_2$$

i.e.  $-2.842 \times 0.555 - 5.708 \times 0.986 = 0.986 F_{eg}$

i.e.  $F_{eg} = -7.2054/0.986 = -7.308 \text{ kN}$  (compressive)

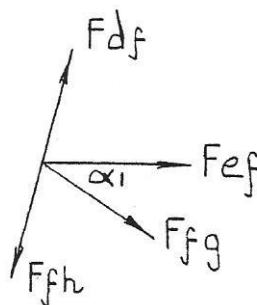
Resolving horizontally gives:

$$F_{ef} + F_{de} \cos \alpha_3 + F_{ce} \cos \alpha_2 = F_{eg} \cos \alpha_2$$

i.e.  $F_{ef} = -0.164 \times 7.308 + 0.832 \times 2.842 + 0.164 \times 5.708$

and  $F_{ef} = 2.102 \text{ kN}$  (tensile)

### Joint F



Resolving vertically gives:

$$F_{df} \sin \alpha_2 = F_{fg} \sin \alpha_1 + F_{fh} \sin \alpha_2$$

$$\text{i.e.} \quad 4.264 \times 0.986 = F_{fg} \times 0.479 + F_{fh} \times 0.986$$

$$\text{i.e.} \quad F_{fh} = 4.264 - 0.486 F_{fg} \quad (9)$$

Resolving horizontally gives:

$$F_{df} \cos \alpha_2 + F_{ef} + F_{fg} \cos \alpha_1 = F_{fh} \cos \alpha_2$$

$$\text{i.e.} \quad 4.264 \times 0.164 + 2.102 = 0.164 \times F_{fh} - 0.878 F_{fg}$$

$$\text{i.e.} \quad 2.801 = 0.164 \times F_{fh} - 0.878 F_{fg}$$

$$\text{i.e.} \quad 0.164 \times F_{fh} = 2.801 + 0.878 F_{fg}$$

$$\text{and} \quad F_{fh} = 2.801/0.164 + 0.878/0.164 F_{fg}$$

$$\text{i.e.} \quad F_{fh} = 17.079 + 5.354 F_{fg} \quad (10)$$

Substituting equation (10) into equation (9) gives:

$$17.079 + 5.354 F_{fg} = 4.264 - 0.486 F_{fg}$$

$$5.840 F_{fg} = -12.815$$

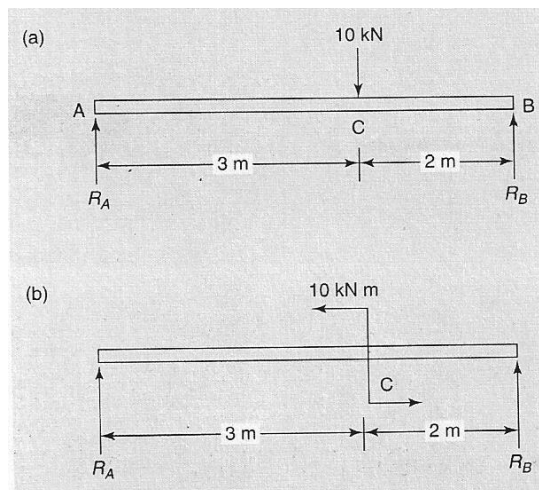
$$\text{i.e.} \quad F_{fg} = -12.815/5.840 = -2.194 \text{ kN (compressive)} \quad (11)$$

Substituting equation (11) into equation (9) gives:

$$F_{fh} = 4.264 - 0.486(-2.194)$$

$$\text{i.e.} \quad F_{fh} = 5.330 \text{ kN (tensile)}$$

**8. Determine the reactions  $R_A$  and  $R_B$  for the simply supported beams shown below.**



(a) Taking moments about A gives:

$$R_B \times 5 = 10 \times 3$$

from which,  $R_B = \frac{10 \times 3}{5} = 6 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 10$$

i.e.  $R_A + 6 = 10$

from which,  $R_A = 10 - 6 = 4 \text{ kN}$

(b) Taking moments about A gives:

$$R_B \times 5 + 10 = 0$$

from which,  $R_B = \frac{-10}{5} = -2 \text{ kN}$

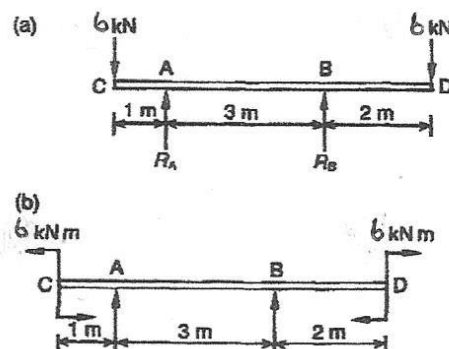
Resolving vertically gives:

$$R_A + R_B = 0$$

i.e.  $R_A - 2 = 0$

from which,  $R_A = 2 \text{ kN}$

9. Determine the reactions  $R_A$  and  $R_B$  for the simply supported beams shown below.



(a) Taking moments about A gives:

$$R_B \times 3 + 6 \times 1 = 6 \times 5$$

i.e.  $R_B = \frac{30-6}{3} = 8 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 6 + 6$$

from which,  $R_A = 12 - 8 = 4 \text{ kN}$

(b) Taking moments about A gives:

$$R_B \times 3 = 6 - 6$$

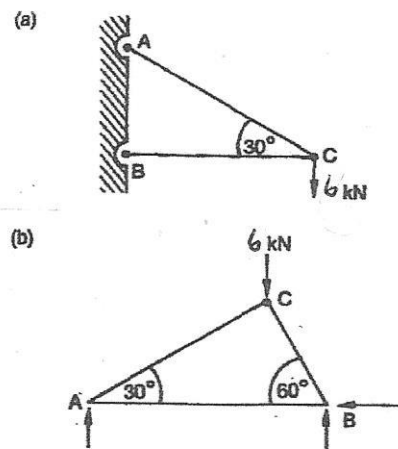
from which,  $R_B = 0$

Resolving vertically gives:

$$R_A + R_B = 0$$

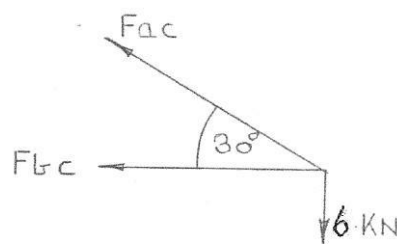
from which,  $R_A = 0$

**10.** Determine the internal forces in the members of the plane pin-jointed trusses shown below.



(a) Assume unknown internal forces are in tension, and consider the equilibrium of joint C.

Resolving forces vertically gives:



$$F_{ac} \sin 30^\circ = 6$$

from which,  $F_{ac} = \frac{6}{\sin 30^\circ} = \mathbf{12 \text{ kN}}$  (tension)

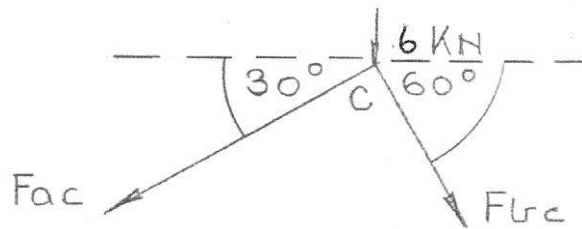
Resolving forces horizontally gives:

$$F_{ac} \cos 30^\circ + F_{bc} = 0$$

i.e.  $12 \cos 30^\circ + F_{bc} = 0$

and  $F_{bc} = -12 \cos 30^\circ = \mathbf{-10.392 \text{ kN}}$  (compression)

(b) Assume all unknown internal forces are in tension and consider the equilibrium of joint C.



Resolving horizontally gives:

$$F_{ac} \cos 30 = F_{bc} \cos 60$$

from which,  $F_{ac} = \frac{\cos 60^\circ}{\cos 30^\circ} F_{bc} = 0.5774 F_{bc}$  (1)

Resolving vertically gives:

$$6 + F_{ac} \sin 30 + F_{bc} \sin 60 = 0$$

or  $6 = -0.5 F_{ac} - 0.866 F_{bc}$  (2)

Substituting equation (1) into equation (2) gives:

$$6 = -0.2887 F_{bc} - 0.866 F_{bc}$$

i.e.  $6 = -1.1547 F_{bc}$

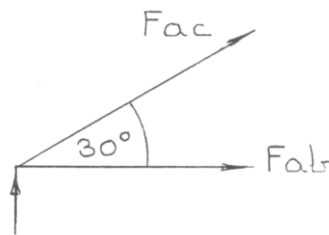
and  $F_{bc} = \frac{6}{-1.1547} = \mathbf{-5.196 \text{ kN}}$  (compression) (3)

Substituting equation (3) into equation (1) gives:

$$F_{ac} = 0.5774 F_{bc} = 0.5774(-5.196) = \mathbf{-3 \text{ kN}}$$
 (compression) (4)

Consider Joint A





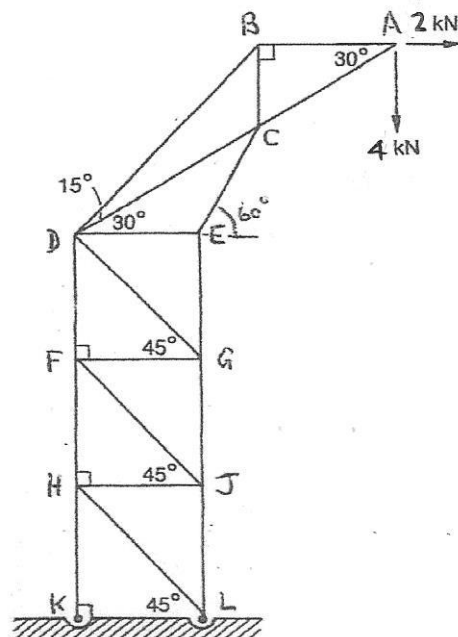
Resolving horizontally gives:

$$F_{ac} \cos 30 + F_{ab} = 0$$

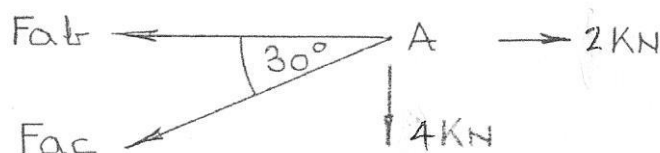
i.e.  $F_{ab} = -F_{ac} \cos 30^\circ$

$$= -(-3) \cos 30^\circ = \mathbf{2.598 \text{ kN (tension)}}$$

**11.** A plane pin-jointed truss is firmly pinned at its base, as shown. Determine the forces in the members of this truss, stating whether they are in tension or compression.



Assume all unknown forces are in tension, and consider equilibrium of **Joint A**.



Resolving vertically gives:

$$F_{ac} \sin 30^\circ + 4 = 0$$

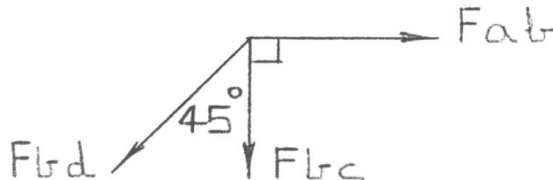
from which,  $F_{ac} = \frac{-4}{\sin 30^\circ} = -8 \text{ kN (compression)}$

Resolving horizontally gives:

$$F_{ab} + F_{ac} \cos 30^\circ = 2$$

i.e.  $F_{ab} = 2 - 0.866 F_{ac} = 2 - 0.866(-8) = 8.928 \text{ kN (tension)}$

### Joint B



Resolving horizontally gives:

$$F_{bd} \cos 45^\circ = F_{ab}$$

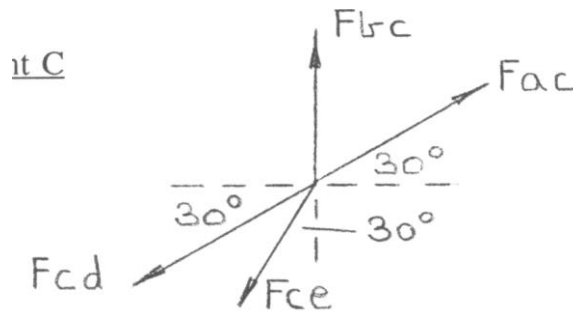
i.e.  $F_{bd} = \frac{8.928}{\cos 45^\circ} = 12.626 \text{ kN (tension)}$

Resolving vertically gives:

$$F_{bc} + F_{bd} \cos 45^\circ = 0$$

i.e.  $F_{bc} = -12.626 \cos 45^\circ = -8.928 \text{ kN (compression)}$

### Joint C



Resolving horizontally gives:

$$F_{ac} \cos 30 = F_{cd} \cos 30 + F_{ce} \cos 60$$

i.e.  $F_{cd} = F_{ac} - 0.5774 F_{ce}$

and  $F_{cd} = -8 - 0.5774 F_{ce} \quad (1)$

Resolving vertically gives:

$$F_{bc} + F_{ac} \sin 30 = F_{cd} \sin 30 + F_{ce} \sin 60$$

$$\text{i.e.} \quad -8.928 - 4 = 0.5 F_{cd} + 0.866 F_{ce} \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$-12.928 = -4 - 0.2887 F_{ce} + 0.866 F_{ce}$$

$$\text{i.e.} \quad -8.928 = 0.5774 F_{ce}$$

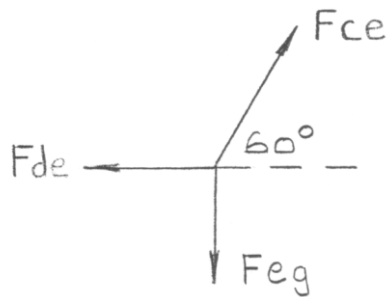
$$\text{from which,} \quad F_{ce} = \frac{-8.928}{0.5774} = -15.462 \text{ kN (compression)} \quad (3)$$

Substituting equation (3) into equation (1) gives:

$$F_{cd} = -8 - 0.5774(-15.462)$$

$$\text{i.e.} \quad F_{cd} = 0.928 \text{ kN (tension)}$$

**Joint E**



Resolving horizontally gives:

$$F_{ce} \cos 60 = F_{de}$$

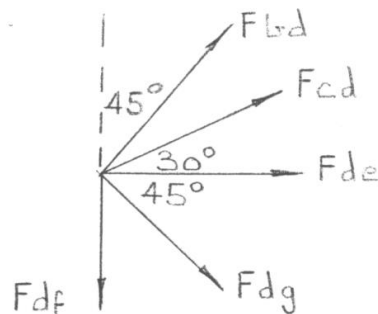
$$\text{i.e.} \quad F_{de} = -15.462 \cos 60^\circ = -7.731 \text{ kN (compression)}$$

Resolving vertically gives:

$$F_{ce} \sin 60 = F_{eg}$$

$$\text{i.e.} \quad F_{eg} = -15.462 \sin 60^\circ = -13.390 \text{ kN (compression)}$$

**Joint D**



Resolving horizontally gives:

$$F_{bd} \cos 45^\circ + F_{cd} \cos 30^\circ + F_{de} + F_{dg} \cos 45^\circ = 0$$

i.e.  $12.626 \times 0.707 + 0.928 \times 0.866 - 7.731 + 0.707 F_{dg} = 0$

from which,  $F_{dg} = \frac{-(12.626 \times 0.707 + 0.928 \times 0.866 - 7.731)}{0.707}$

$$= -2.828 \text{ kN (compression)}$$

Resolving vertically gives:

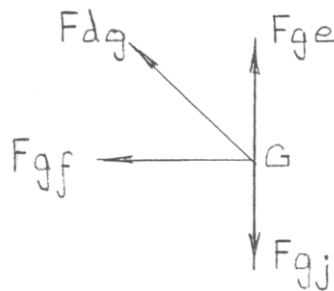
$$F_{bd} \sin 45^\circ + F_{cd} \sin 30^\circ = F_{dg} \sin 45^\circ + F_{df}$$

i.e.  $12.626 \times 0.707 + 0.928 \times 0.5 = -2.828 \times 0.707 + F_{df}$

from which,  $F_{df} = 12.626 \times 0.707 + 0.928 \times 0.5 + 2.828 \times 0.707$

i.e.  $F_{df} = 11.390 \text{ kN (tension)}$

### Joint G



Resolving horizontally gives:

$$-F_{dg} \times 0.707 = F_{gf}$$

i.e.  $F_{gf} = -(-2.828) \times 0.707 = 2.0 \text{ kN (tension)}$

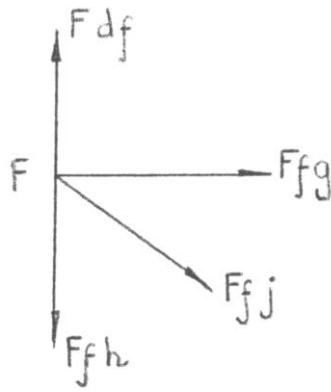
Resolving vertically gives:

$$F_{ge} + F_{dg} \times 0.707 = F_{gj}$$

i.e.  $-13.390 - 2.828 \times 0.707 = F_{gj}$

i.e.  $F_{gj} = -15.390 \text{ kN (compression)}$

### Joint F



Resolving horizontally gives:

$$-F_{fj} \times 0.707 = F_{fg}$$

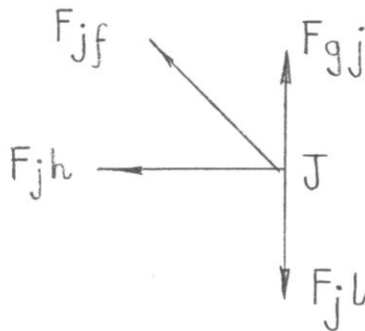
i.e. 
$$F_{fj} = \frac{F_{fg}}{0.707} = -\frac{2.0}{0.707} = -\mathbf{2.828 \text{ kN}}$$
 (compression)

Resolving vertically gives:

$$F_{df} = F_{fj} \times 0.707 + F_{fh}$$

i.e. 
$$F_{fh} = 11.390 - 2.828 \times 0.707 = \mathbf{13.390 \text{ kN}}$$
 (tension)

#### Joint J



Resolving horizontally gives:

$$-F_{jf} \times 0.707 = F_{jh}$$

i.e. 
$$F_{jh} = -(-2.828) \times 0.707 = \mathbf{2.0 \text{ kN}}$$
 (tension)

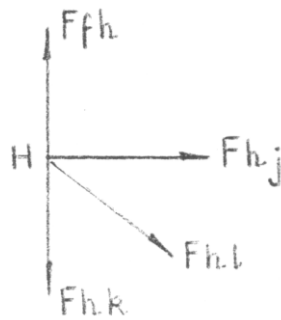
Resolving vertically gives:

$$F_{gj} + F_{jf} \times 0.707 = F_{jl}$$

i.e. 
$$-15.390 - 2.828 \times 0.707 = F_{jl}$$

i.e. 
$$F_{jl} = -\mathbf{17.390 \text{ kN}}$$
 (compression)

#### Joint H



Resolving horizontally gives:

$$F_{hl} \cos 45^\circ = -F_{hj}$$

i.e. 
$$F_{hl} = \frac{-2.0}{\cos 45^\circ} = -2.828 \text{ kN (compression)}$$

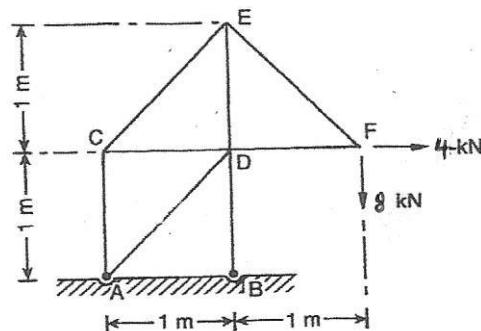
Resolving vertically gives:

$$F_{fh} = F_{hl} \times 0.707 + F_{hk}$$

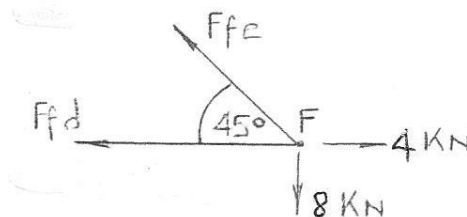
i.e. 
$$F_{hk} = F_{fh} - F_{hl} \times 0.707 = +13.390 - (-2.828) \times 0.707$$
  

$$= 15.390 \text{ kN (tension)}$$

**12.** The plane pin-jointed truss shown is firmly pinned at A and B and subjected to two point loads at point F. Determine the forces in the members, stating whether they are tensile or compressive.



**Joint F**



Resolving vertically gives:

$$F_{fe} \times 0.707 = 8$$

i.e. 
$$F_{fe} = \frac{8}{0.707} = \mathbf{11.315 \text{ kN (tensile)}}$$

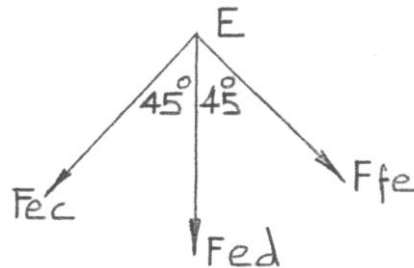
Resolving horizontally gives:

$$F_{fe} \times 0.707 + F_{fd} = 4$$

i.e. 
$$F_{fd} = 4 - F_{fe} \times 0.707 = 4 - 11.315 \times 0.707$$
  

$$= -\mathbf{4.0 \text{ kN (compressive)}}$$

### Joint E



Resolving horizontally gives:

$$F_{ec} = F_{fe} = \mathbf{11.315 \text{ kN (tensile)}}$$

Resolving vertically gives:

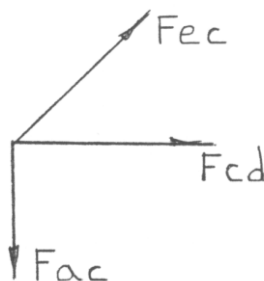
$$F_{ec} \times 0.707 + F_{fe} \times 0.707 + F_{ed} = 0$$

i.e. 
$$F_{ed} = -F_{ec} \times 0.707 - F_{fe} \times 0.707$$

i.e. 
$$F_{ed} = -11.315 \times 0.707 - 11.315 \times 0.707$$
  

$$= -\mathbf{16.0 \text{ kN (compressive)}}$$

### Joint C



Resolving horizontally gives:

$$F_{ec} \times 0.707 + F_{cd} = 0$$

i.e. 
$$F_{cd} = -F_{ec} \times 0.707 = -11.315 \times 0.707$$

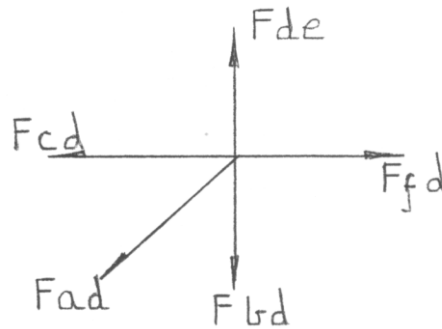
$$= -8.0 \text{ kN (compressive)}$$

Resolving vertically gives:

$$F_{ec} \times 0.707 = F_{ac}$$

i.e.  $F_{ac} = 11.315 \times 0.707 = 8.0 \text{ kN (tensile)}$

**Joint D**



Resolving horizontally gives:

$$F_{fd} = F_{cd} + F_{ad} \times 0.707$$

i.e.  $F_{ad} = \frac{F_{fd} - F_{cd}}{0.707} = \frac{-4 - -8}{0.707} = 5.658 \text{ kN (tensile)}$

Resolving vertically gives:

$$F_{de} = F_{bd} + F_{ad} \times 0.707$$

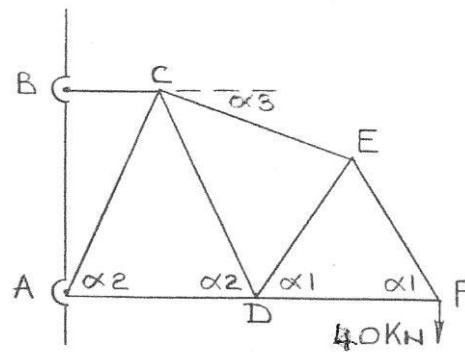
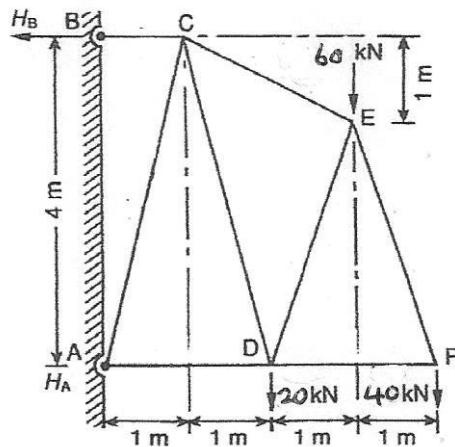
i.e.  $F_{bd} = F_{de} - F_{ad} \times 0.707$

$$= -16.0 - 5.658 \times 0.707$$

$$= -20.0 \text{ kN (compressive)}$$

**13.** An overhanging pin-jointed roof truss, which may be assumed to be pinned rigidly to the wall at the joints A and B, is subjected to the loading shown. Determine the forces in the members of the truss, stating whether they are tensile or compressive. Determine also the reactions.



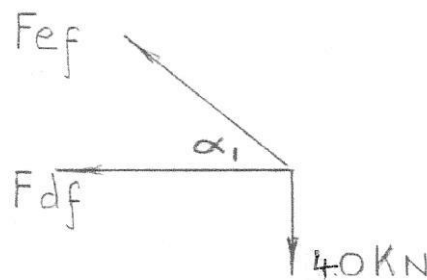


$$\alpha_1 = \tan^{-1} 3 = 71.565^\circ$$

$$\alpha_2 = \tan^{-1} 4 = 75.964^\circ$$

$$\alpha_3 = \tan^{-1} (1/2) = 26.565^\circ$$

**Joint F**



Resolving vertically gives;

$$F_{ef} \sin \alpha_1 = 40$$

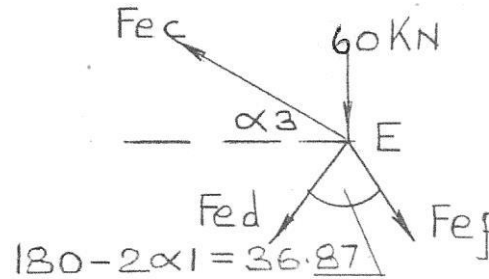
from which, 
$$F_{ef} = \frac{40}{\sin 71.565^\circ} = 42.16 \text{ kN (tensile)}$$

Resolving horizontally gives:

$$F_{df} + F_{ef} \cos \alpha_1 = 0$$

$$\begin{aligned}
 \text{i.e.} \quad F_{df} &= -F_{ef} \cos \alpha_1 = -42.16 \times \cos 71.565^\circ \\
 &= -13.332 \text{ kN (compressive)}
 \end{aligned}$$

### Joint E



Resolving vertically gives:

$$F_{ec} \sin \alpha_3 = 60 + (F_{ed} + F_{ef}) \cos (18.435^\circ)$$

$$\text{i.e.} \quad F_{ec} \sin 26.565^\circ = 60 + (F_{ed} + 42.16) \cos (18.435^\circ)$$

$$\text{i.e.} \quad 0.447 F_{ec} = 60 + 0.949 F_{ed} + 40$$

$$\text{i.e.} \quad F_{ec} = 233.71 + 2.123 F_{ed} \quad (1)$$

Resolving horizontally gives:

$$F_{ec} \cos \alpha_3 + F_{ed} \sin (18.435^\circ) = F_{ef} \sin (18.435^\circ)$$

$$\text{i.e.} \quad 0.894 F_{ec} + 0.316 F_{ed} = 13.332 \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$0.894 (233.71 + 2.123 F_{ed}) + 0.316 F_{ed} = 13.332$$

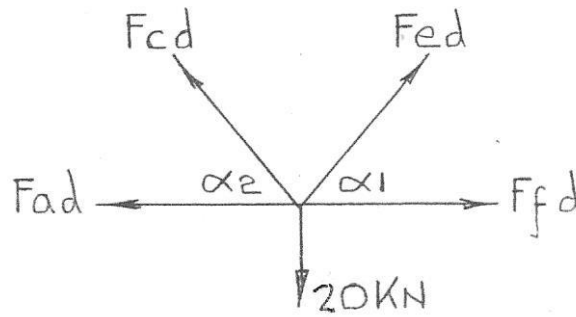
$$\text{i.e.} \quad 208.94 + 2.214 F_{ed} = 13.332$$

$$\text{i.e.} \quad F_{ed} = -88.35 \text{ kN (compressive)} \quad (3)$$

Substituting equation (3) into equation (1) gives:

$$\begin{aligned}
 F_{ec} &= 233.71 + 2.123 F_{ed} \\
 &= 233.71 + 2.123(-88.35) = 46.14 \text{ kN (tensile)}
 \end{aligned}$$

### Joint D



Resolving horizontally gives:

$$F_{ad} + F_{cd} \cos \alpha_2 = F_{fd} + F_{ed} \cos \alpha_1$$

i.e.  $F_{ad} + F_{cd} \cos 75.965^\circ = -13.332 - 88.35 \cos 71.565^\circ$

i.e.  $F_{ad} + 0.243 F_{cd} = -13.332 - 88.35 \times 0.316$

i.e.  $F_{ad} = -41.25 - 0.243 F_{cd}$  (4)

Resolving vertically gives:

$$F_{cd} \sin \alpha_2 + F_{ed} \sin \alpha_1 = 20$$

i.e.  $F_{cd} \sin 75.964^\circ + F_{ed} \sin 71.565^\circ = 20$

i.e.  $F_{cd} = 20/0.970 - 88.35 \times 0.949/0.970$

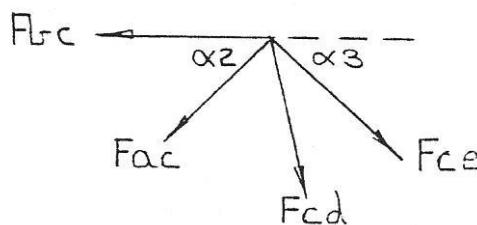
i.e.  $F_{cd} = 107.06 \text{ kN (tensile)}$  (5)

Substituting equation (5) into equation (4) gives:

$$F_{ad} = -41.25 - 0.243 F_{cd}$$

$$= -41.25 - 0.243(107.06) = -67.27 \text{ kN (compressive)}$$

**Joint C**



Resolving vertically gives:

$$F_{ac} \sin \alpha_2 + F_{cd} \sin \alpha_2 + F_{ce} \sin \alpha_3 = 0$$

i.e.  $F_{ac} \sin 75.964^\circ + F_{cd} \sin 75.964^\circ + F_{ce} \sin 26.565^\circ = 0$

i.e.  $0.970 F_{ac} + 107.06 \times 0.970 + 46.14 \times 0.447 = 0$

and  $F_{ac} = \frac{-107.06 \times 0.970 - 46.14 \times 0.447}{0.970} = -128.32 \text{ kN (compressive)}$

Resolving horizontally gives:

$$F_{bc} + F_{ac} \cos \alpha_2 = F_{cd} \cos \alpha_2 + F_{ce} \cos \alpha_3$$

i.e.  $F_{bc} - 197.67 \cos 75.964^\circ = 107.06 \cos 75.964^\circ + 46.14 \cos 26.565^\circ$

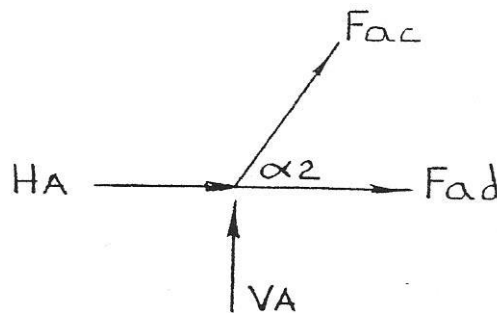
$$F_{bc} = +197.67 \times 0.243 + 107.06 \times 0.243 + 46.14 \times 0.894$$

$$F_{bc} = 98.45 \text{ kN (tensile)}$$

## Reactions

### Joint B

$$H_B = F_{bc} = 98.45 \text{ kN}$$



### Joint A

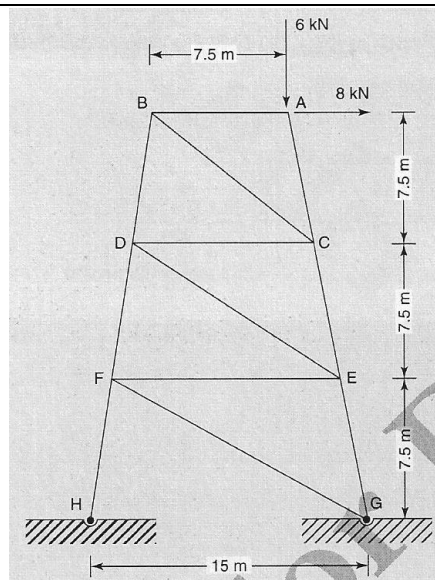
Resolving vertically gives:

$$V_A = -F_{ac} \sin \alpha_2 = -(-128.322) \sin 75.964^\circ = 124.49 \text{ kN}$$

Resolving horizontally gives:

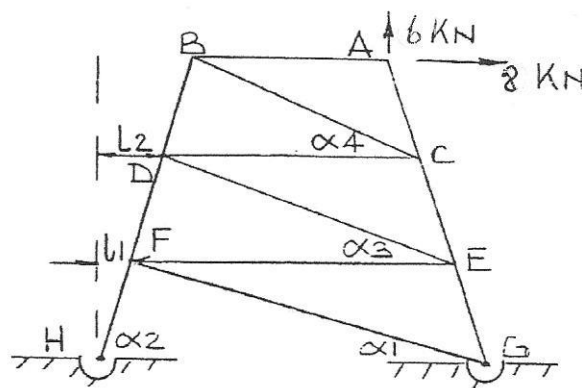
$$H_A = -F_{ad} - F_{ac} \cos \alpha_2 = -67.27 + 128.322 \times 0.243 = 98.45 \text{ kN}$$

**14.** Determine the forces in the symmetrical pin-jointed truss shown stating whether they are tensile or compressive.



$$\alpha_2 = \tan^{-1} \left( \frac{7.5 \times 3}{3.75} \right) = 80.538^\circ$$

$$l_1 = \frac{7.5}{\tan \alpha_2} = 1.25 \text{ m}$$



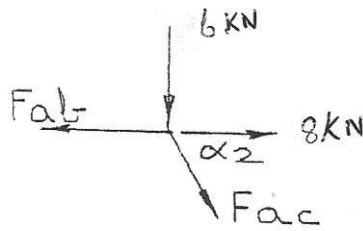
$$l_2 = 2.5 \text{ m}$$

$$\alpha_1 = \tan^{-1} \frac{7.5}{(15 - 1.25)} = 28.61^\circ$$

$$\alpha_3 = \tan^{-1} \frac{7.5}{(15 - 3.75)} = 33.69^\circ$$

$$\alpha_4 = \tan^{-1} \frac{7.5}{(15 - 3.75 - 2.5)} = 40.60^\circ$$

**Joint A**



Resolving vertically gives:

$$6 + F_{ac} \sin \alpha_2 = 0$$

i.e. 
$$F_{ac} = -\frac{6}{\sin 80.538^\circ} = -\frac{6}{0.986}$$

$$= -6.085 \text{ kN (compressive)}$$

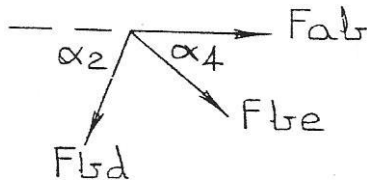
Resolving horizontally gives:

$$F_{ab} = F_{ac} \cos \alpha_2 + 8$$

i.e. 
$$F_{ab} = -6.085 \cos 80.538^\circ + 8$$

$$= +7.0 \text{ kN (tensile)}$$

### Joint B



Resolving vertically gives:

$$F_{bd} \sin \alpha_2 + F_{bc} \sin \alpha_4 = 0$$

i.e. 
$$F_{bd} \sin 80.538^\circ + F_{bc} \sin 40.60^\circ = 0$$

i.e. 
$$0.986 F_{bd} = -0.651 F_{bc}$$

i.e. 
$$F_{bd} = -0.660 F_{bc} \quad (1)$$

Resolving horizontally gives:

$$F_{bd} \cos \alpha_2 = F_{ab} + F_{bc} \cos \alpha_4$$

i.e. 
$$F_{bd} \cos 80.538^\circ = 7.0 + F_{bc} \cos 40.60^\circ$$

i.e. 
$$0.164 F_{bd} = 7.0 + 0.759 F_{bc} \quad (2)$$

Substituting equation (1) into equation (2) gives:

i.e.  $0.164(-0.660 F_{bc}) = 7.0 + 0.759 F_{bc}$

and  $(0.759 + 0.108) F_{bc} = -7.0$

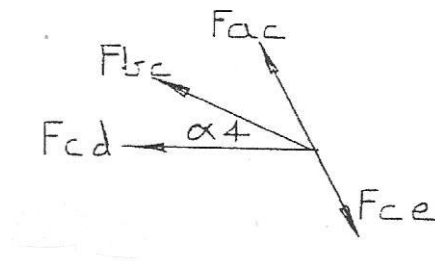
i.e.  $F_{bc} = -8.074 \text{ kN (compressive)}$  (3)

Substituting equation (3) into equation (1) gives:

$$F_{bd} = -0.660 F_{bc} = -0.660 \times -8.074$$

i.e.  $F_{bd} = 5.329 \text{ kN (tensile)}$

### Joint C



Resolving horizontally gives:

$$F_{cd} + F_{bc} \cos \alpha_4 + F_{ac} \cos \alpha_2 = F_{ce} \cos \alpha_2$$

i.e.  $F_{cd} = 8.074 \times 0.759 + 6.085 \times 0.164 + F_{ce} \times 0.164$

i.e.  $F_{cd} = 7.126 + 0.164 F_{ce}$  (4)

Resolving vertically gives:

$$F_{ac} \sin \alpha_2 + F_{bc} \sin \alpha_4 = F_{ce} \sin \alpha_2$$

i.e.  $0.986 F_{ac} + 0.651 F_{bc} = 0.986 F_{ce}$

i.e.  $0.986(-6.085) + 0.651(-8.074) = 0.986 F_{ce}$

i.e.  $-11.256 = 0.986 F_{ce}$

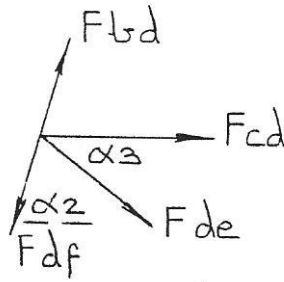
and  $F_{ce} = -11.256/0.986 = -11.416 \text{ kN (compressive)}$  (5)

Substituting equation (5) into equation (4) gives:

$$F_{cd} = 7.126 + 0.164(-11.416)$$

i.e.  $F_{cd} = 5.254 \text{ kN (tensile)}$

### Joint D



Resolving vertically gives:

$$F_{bd} \sin \alpha_2 = F_{de} \sin \alpha_3 + F_{df} \sin \alpha_2$$

i.e.  $5.329 \times 0.986 = F_{de} \times 0.555 + 0.986 F_{df}$

and  $F_{df} = 5.329 - 0.563 F_{de}$  (6)

Resolving horizontally gives:

$$F_{bd} \cos \alpha_2 + F_{cd} + F_{de} \cos \alpha_3 = F_{df} \cos \alpha_2$$

i.e.  $5.329 \times 0.164 + 5.254 + 0.832 F_{de} = 0.164 F_{df}$  (7)

Substituting equation (6) into equation (7) gives:

$$5.329 \times 0.164 + 5.254 + 0.832 F_{de} = 0.164(5.329 - 0.563 F_{de})$$

i.e.  $0.874 + 5.254 = F_{de} (-0.832 - 0.0923) + 0.874$

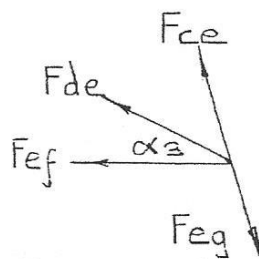
from which,  $F_{de} = -5.684 \text{ kN}$  (compressive) (8)

Substituting equation (8) into equation (6) gives:

$$F_{df} = 5.329 - 0.563(-5.684)$$

$$F_{df} = 8.529 \text{ kN (tensile)}$$

### Joint E



Resolving vertically gives:

$$F_{de} \sin \alpha_3 + F_{ce} \sin \alpha_2 = F_{eg} \sin \alpha_2$$

i.e.  $-5.684 \times 0.555 - 11.416 \times 0.986 = 0.986 F_{eg}$



i.e.  $F_{eg} = -14.411/0.986 = -14.615 \text{ kN}$  (compressive)

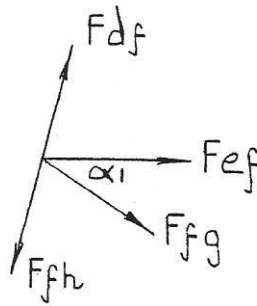
Resolving horizontally gives:

$$F_{ef} + F_{de} \cos \alpha_3 + F_{ce} \cos \alpha_2 = F_{eg} \cos \alpha_2$$

i.e.  $F_{ef} = 0.164 \times -14.615 + 0.832 \times 5.684 + 0.164 \times 11.416$

and  $F_{ef} = 4.204 \text{ kN}$  (tensile)

### Joint F



Resolving vertically gives:

$$F_{df} \sin \alpha_2 = F_{fg} \sin \alpha_1 + F_{fh} \sin \alpha_2$$

i.e.  $8.529 \times 0.986 = F_{fg} \times 0.479 + F_{fh} \times 0.986$

i.e.  $F_{fh} = 8.529 - 0.486 F_{fg}$  (9)

Resolving horizontally gives:

$$F_{df} \cos \alpha_2 + F_{ef} + F_{fg} \cos \alpha_1 = F_{fh} \cos \alpha_2$$

i.e.  $8.529 \times 0.164 + 4.204 = 0.164 \times F_{fh} - 0.878 F_{fg}$

i.e.  $5.603 = 0.164 \times F_{fh} - 0.878 F_{fg}$

i.e.  $0.164 \times F_{fh} = 5.603 + 0.878 F_{fg}$

and  $F_{fh} = 5.603/0.164 + 0.878/0.164 F_{fg}$

i.e.  $F_{fh} = 34.165 + 5.354 F_{fg}$  (10)

Substituting equation (10) into equation (9) gives:

$$34.165 + 5.354 F_{fg} = 8.529 - 0.486 F_{fg}$$

$$5.840 F_{fg} = -25.636$$

i.e.  $F_{fg} = -25.636/5.840 = -4.390 \text{ kN}$  (compressive) (11)

Substituting equation (11) into equation (9) gives:

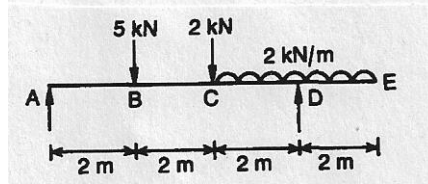
$$F_{th} = 8.529 - 0.486(-4.390)$$

i.e.  **$F_{th} = 10.663 \text{ kN}$**  (tensile)

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## EXERCISE 21, Page 76

1. Determine the bending moments and shearing forces at the points A, B, C, D and E for the simply supported beam shown. Determine, also, the position of the point of contraflexure.



Taking moments about D gives:

$$\begin{aligned} R_A \times 6 &= 5 \times 4 + 2 \times 2 \\ &= 24 \end{aligned}$$

from which,  $R_A = 4 \text{ kN}$

Resolving vertically gives:

$$R_A + R_D = 5 + 2 + 2 \times 4$$

i.e.  $R_D = 15 - R_A = 15 - 4$

from which,  $R_D = 11 \text{ kN}$

**Bending moments (M)**

$$M_A = 0$$

$$M_B = R_A \times 2 = 8 \text{ kN m}$$

$$M_C = R_A \times 4 - 5 \times 2 = 6 \text{ kN m}$$

$$M_D = R_A \times 6 - 5 \times 4 - 2 \times 2 - 2 \times 2 \times 1 = -4 \text{ kN m}$$

$$M_E = 0$$

**Shearing forces (F)**

$$F_A = 4 \text{ kN}$$

$$F_{B-} = 4 \text{ kN}; F_{B+} = 4 - 5 = -1 \text{ kN}$$

$$F_{C-} = 4 - 5 = -1 \text{ kN}; F_{C+} = 4 - 5 - 2 = -3 \text{ kN}$$

$$F_{D-} = 4 - 5 - 2 - 2 \times 2 = -7 \text{ kN}; F_{D+} = 4 - 5 - 2 - 2 \times 2 + R_D = 4 \text{ kN}$$

$$F_E = 0$$

The point of contraflexure lies in span CD, and let this be at distance  $x$  from A

$$M = R_A \times x - 5(x-2) - 2(x-4) - 2(x-4) \times \frac{(x-4)}{2} = 0$$

$$\text{i.e.} \quad 0 = 4x - 5x + 10 - 2x + 8 - (x^2 - 8x + 16)$$

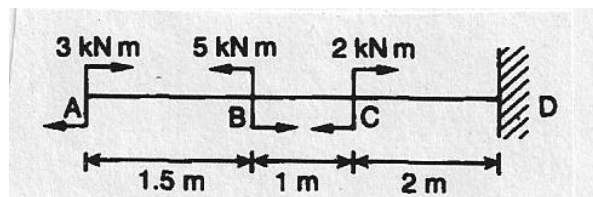
$$\text{i.e.} \quad 0 = -3x + 18 - x^2 + 8x - 16$$

$$\text{i.e.} \quad 0 = -x^2 + 5x + 2 \quad \text{which is a quadratic equation}$$

By quadratic formula or by calculator,  $x = 5.37 \text{ m}$

Hence, **the point of contraflexure,  $x = 5.37 \text{ m}$  from A**

**2.** Determine the bending moments and shearing forces at the points A, B, C and D on the cantilever shown.



**Bending moments (M)**

$$M_A = 3 \text{ kN m}$$

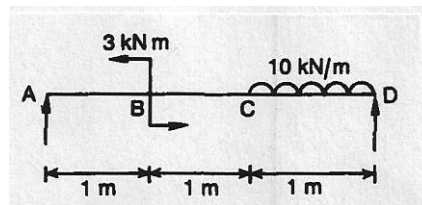
$$M_{B-} = 3 \text{ kN m}; \quad M_{B+} = 3 - 5 = -2 \text{ kN m}$$

$$M_{C-} = 3 - 5 = -2 \text{ kN m}; \quad M_{C+} = 3 - 5 + 2 = 0$$

**Shearing Forces (F)**

$$F_A = F_B = F_C = F_D = 0$$

**3.** Determine the bending moments and shearing forces at the points A, B, C and D on the beam shown.



Taking moments about D gives:

$$R_A \times 3 = 3 + 10 \times 1 \times 0.5$$

from which,  $R_A = 8/3 = 2.667 \text{ kN}$

Resolving vertically gives:

$$R_A + R_D = 10 \times 1$$

from which,  $R_D = 10 - R_A = 10 - 2.667 = 7.333 \text{ kN}$

**Bending moments (M)**

$$M_A = 0$$

$$M_{B-} = R_A \times 1 = 2.667 \text{ kN m}$$

$$M_{B+} = 2.667 - 3 = -0.333 \text{ kN m}$$

$$M_C = R_A \times 2 - 3 = 2.33 \text{ kN m}$$

$$M_D = 0$$

**Shearing Forces (F)**

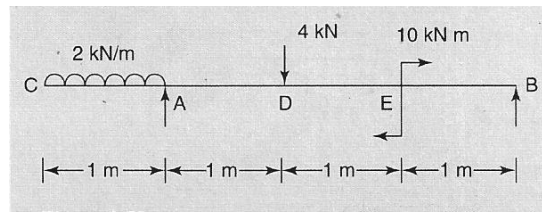
$$F_A = R_A = 2.667 \text{ kN}$$

$$F_B = 2.667 \text{ kN}$$

$$F_C = 2.667 \text{ kN}$$

$$F_D = -R_D = -7.333 \text{ kN}$$

4. A uniform-section beam is simply supported at A and B, as shown. Determine the bending moments and shearing forces at the points C, A, D, E and B.



Taking moments about B gives:

$$R_A \times 3 + 10 = 2 \times 1 \times 3.5 + 4 \times 2$$

i.e.  $R_A \times 3 = 15 - 10$

from which,  $R_A = 1.667 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 2 \times 1 + 4$$

from which,  $R_B = 6 - 1.667 = 4.333 \text{ kN}$

**Bending moments (M)**

$$M_C = 0$$

$$M_A = -2 \times 1 \times 0.5 = -1 \text{ kN m}$$

$$M_D = -2 \times 1 \times 1.5 + R_A \times 1$$

i.e.  $M_D = -3 + 1.667 = -1.333 \text{ kN m}$

$$M_{E-} = -2 \times 1 \times 2.5 + R_A \times 2 - 4 \times 1 = -5.667 \text{ kN m}$$

$$M_{E+} = -5.667 + 10 = 4.333 \text{ kN m}$$

$$M_B = 0$$

**Shearing Forces (F)**

$$F_C = 0$$

$$F_{A-} = -2 \times 1 = -2 \text{ kN}$$

$$F_{A+} = -2 + R_A = -0.333 \text{ kN}$$

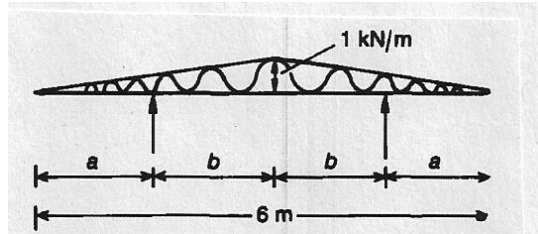
$$F_{D-} = -0.333 \text{ kN}$$

$$F_{D+} = -0.333 - 4 = -4.333 \text{ kN}$$

$$F_E = -4.333 \text{ kN}$$

$$F_B = -R_B = -4.333 \text{ kN}$$

5. A simply supported beam supports a distributed load, as shown below. Obtain an expression for the value of  $a$ , so that the bending moment at the support will be of the same magnitude as that at mid-span.



$$2a + 2b = 6$$

$$a + b = 3 \quad (1)$$

$$w_a = \frac{a}{a+b} = \frac{a}{3} \times 1$$

$$R = \frac{a+b}{2} = 1.512$$

$$M_A = a \times \frac{a}{6} \times \frac{a}{3} = \frac{a^3}{18} \quad 13$$

$$M_{\text{central}} = 2 \times 3 \times 1 \times 1 - 1.5 \times b = 1.5 - 1.5(3-a)$$

$$M_A = M_{\text{central}}$$

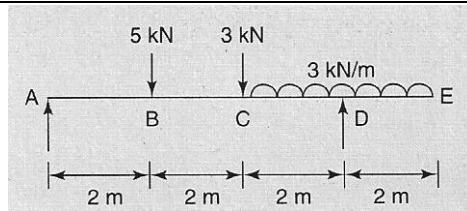
$$\frac{a^3}{18} = -\frac{3}{2} + \frac{3}{2}(3-a)$$

$$\frac{a^3}{18} = -1.5 + 1.5 \times 3 - 1.5a \quad 14$$

i.e. 
$$\frac{a^3}{18} + 1.5a = 3$$

from which,  $a = 1.788 \text{ m}$  (by calculator or the Newton-Raphson method)

6. Determine the bending moments and shearing forces at the points A, B, C, D and E for the simply supported beam shown. Determine also the position of the point of contraflexure.



Taking moments about D gives:

$$R_A \times 6 = 5 \times 4 + 3 \times 2$$

$$= 26$$

from which,  $R_A = 26/6 = 4.333 \text{ kN}$

Resolving vertically gives:

$$R_A + R_D = 5 + 3 + 3 \times 4$$

i.e.  $R_D = 20 - R_A = 20 - 4.333$

from which,  $R_D = 15.667 \text{ kN}$

**Bending moments (M)**

$$M_A = 0$$

$$M_B = R_A \times 2 = 8.666 \text{ kN m}$$

$$M_C = R_A \times 4 - 5 \times 2 = 7.332 \text{ kN m}$$

$$M_D = R_A \times 6 - 5 \times 4 - 3 \times 2 - 3 \times 2 \times 1 = -6.0 \text{ kN m}$$

$$M_E = 0$$

**Shearing forces (F)**

$$F_A = 4.333 \text{ kN}$$

$$F_{B-} = 4.333 \text{ kN}; F_{B+} = 4.333 - 5 = -0.667 \text{ kN}$$

$$F_{C-} = 4.333 - 5 = -0.667 \text{ kN}; F_{C+} = 4.333 - 5 - 3 = -3.667 \text{ kN}$$

$$F_{D-} = 4.333 - 5 - 3 - 3 \times 2 = -9.667 \text{ kN}; F_{D+} = 4.333 - 5 - 3 - 3 \times 2 + R_D = 6.0 \text{ kN}$$

$$F_E = 0$$

The point of contraflexure lies in span CD, and let this be at distance  $x$  from A

$$M = R_A \times x - 5(x-2) - 3(x-4) - 3(x-4) \times \frac{(x-4)}{2} = 0$$



i.e.  $0 = 4.333x - 5x + 10 - 3x + 12 - 1.5(x^2 - 8x + 16)$

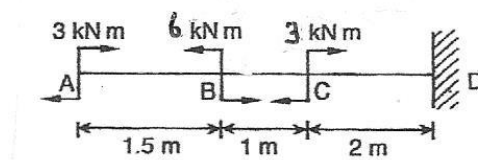
i.e.  $0 = 4.333x - 5x + 10 - 3x + 12 - 1.5x^2 + 12x - 24$

i.e.  $0 = -1.5x^2 + 8.333x - 2$  which is a quadratic equation

By quadratic formula or by calculator,  $x = 5.30 \text{ m}$

Hence, **the point of contraflexure,  $x = 5.30 \text{ m}$  from A**

**7.** Determine the bending moments and shearing forces at the points A, B, C and D on the cantilever shown.



**Bending moments (M)**

$$M_A = 3 \text{ kNm}$$

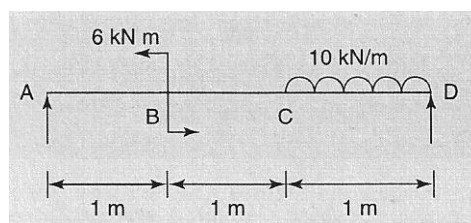
$$M_{B-} = 3 \text{ kNm}; \quad M_{B+} = 3 - 6 = -3 \text{ kNm}$$

$$M_{C-} = 3 - 6 = -3 \text{ kNm}; \quad M_{C+} = 3 - 6 + 3 = 0$$

**Shearing Forces (F)**

$$F_A = F_B = F_C = F_D = 0$$

**8.** Determine the bending moments and shearing forces at the points A, B, C and D on the beam shown below.



Taking moments about D gives:

$$R_A \times 3 = 6 + 10 \times 1 \times 0.5$$

from which,  $R_A = 11/3 = 3.667 \text{ kN}$

Resolving vertically gives:

$$R_A + R_D = 10 \times 1$$

from which,  $R_D = 10 - R_A = 10 - 3.667 = 6.333 \text{ kN}$

**Bending moments (M)**

$$M_A = 0$$

$$M_{B-} = R_A \times 1 = 3.667 \text{ kN m}$$

$$M_{B+} = 3.667 - 6 = -2.333 \text{ kN m}$$

$$M_C = R_A \times 2 - 6 = 1.333 \text{ kN m}$$

$$M_D = 0$$

**Shearing Forces (F)**

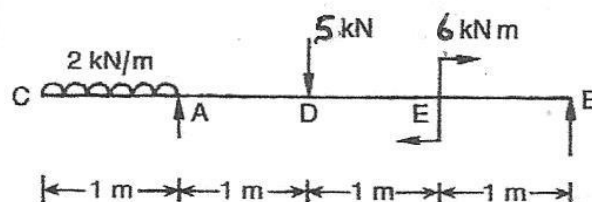
$$F_A = R_A = 3.667 \text{ kN}$$

$$F_B = 3.667 \text{ kN}$$

$$F_C = 3.667 \text{ kN}$$

$$F_D = -R_D = -6.333 \text{ kN}$$

9. A uniform-section beam is simply supported at A and B, as shown. Determine the bending moments and shearing forces at points C, A, D, E and B.



Taking moments about B gives:

$$R_A \times 3 + 6 = 2 \times 1 \times 3.5 + 5 \times 2$$

i.e.  $R_A \times 3 = 17 - 6$

from which,  $R_A = 11/3 = 3.667 \text{ kN}$

Resolving vertically gives:

$$R_A + R_B = 2 \times 1 + 5$$

from which,

$$R_B = 7 - 3.667 = \mathbf{3.333 \text{ kN}}$$

**Bending moments (M)**

$$M_C = 0$$

$$M_A = -2 \times 1 \times 0.5 = -\mathbf{1 \text{ kN m}}$$

$$M_D = -2 \times 1 \times 1.5 + R_A \times 1$$

i.e.

$$M_D = -3 + 3.667 = -\mathbf{0.667 \text{ kN m}}$$

$$M_{E-} = -2 \times 1 \times 2.5 + R_A \times 2 - 5 \times 1 = -\mathbf{2.666 \text{ kN m}}$$

$$M_{E+} = -2.667 + 6 = \mathbf{3.333 \text{ kN m}}$$

$$M_B = 0$$

**Shearing Forces (F)**

$$F_C = 0$$

$$F_{A-} = -2 \times 1 = -\mathbf{2 \text{ kN}}$$

$$F_{A+} = -2 + R_A = -\mathbf{1.667 \text{ kN}}$$

$$F_{D-} = -\mathbf{1.667 \text{ kN}}$$

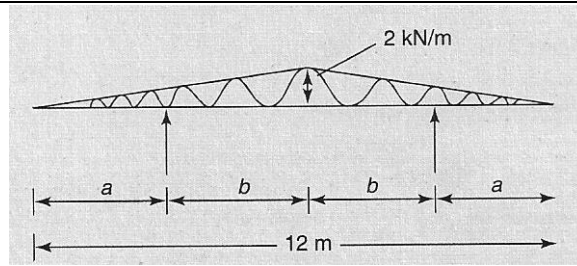
$$F_{D+} = -1.667 - 5 = -\mathbf{6.667 \text{ kN}}$$

$$F_E = -\mathbf{6.667 \text{ kN}}$$

$$F_B = -R_B = -\mathbf{3.333 \text{ kN}}$$

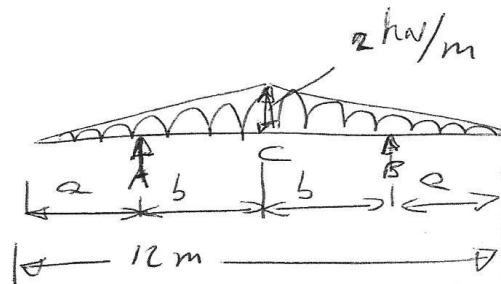
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**10.** A simply supported beam supports a distributed load, as shown below. Obtain an expression for the value of  $a$ , so that the bending moment at the support will be of the same magnitude as that at mid-span.



In the diagram below,  $R_A = R_B$  and  $a + b = 6$  m

and  $R_A = R_B = (a + b) \times \frac{2}{2} = 6$  kN



The bending moment at A,  $BM_A = \frac{2}{a+b} \times \frac{a}{2} \times \frac{a}{3} = \frac{2}{6} \times \frac{a^2}{6}$

i.e.  $BM_A = \frac{a^2}{18}$  (1)

The bending moment at C,  $BM_C = R_A \times b - (a+b) \times \frac{2}{2} \times \frac{(a+b)}{3}$

$$= 6b - 6 \times \frac{2}{2} \times \frac{6}{3}$$

i.e.  $BM_C = 6b - 12$  (2)

However, equation (1) = equation (2),

i.e.  $\frac{a^2}{18} = 6b - 12$  (3)

But,  $a + b = 6$  or  $b = 6 - a$  (4)

Substituting equation (4) into equation (3) gives:

$$\frac{a^2}{18} = 6(6 - a) - 12$$

i.e.  $\frac{a^2}{18} = 36 - 6a - 12 = 24 - 6a$

i.e.  $a^2 = 18(24 - 6a)$

i.e.  $a^2 = 432 - 108a$

or  $a^2 + 108a - 432 = 0$

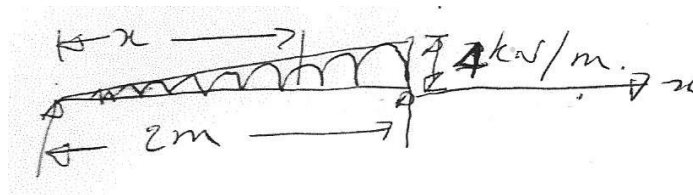
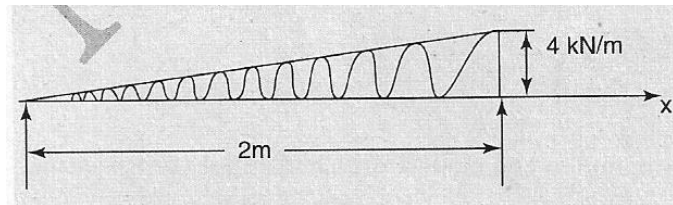
Using the quadratic formula or by calculator,  $a = 3.862$  m (or  $-111.86$  m, which is not possible)

Hence, **the value of  $a$ , so that the bending moment at the support will be of the same magnitude as that at mid-span is:  $a = 3.862$  m**

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## EXERCISE 22, Page 81

1. Determine expressions for the shearing force and bending moment distributions for the hydrostatically loaded beam shown, which is simply supported at its ends. Find also the position and value of the maximum bending moment.



At  $x$ ,  $w = -2x$

$$\frac{dF}{dx} = w = -2x$$

hence, 
$$F = \int_0^x -2x \, dx = -\frac{2x^2}{2} + A$$

i.e. 
$$F = -x^2 + A \quad (1)$$

$$M = \int_0^x F \, dx = \int_0^x (-x^2 + A) \, dx$$

$$M = -\frac{x^3}{3} + Ax + B \quad (2)$$

At  $x = 0$ ,  $M = 0$

Hence,  $B = 0$

At  $x = 2$ ,  $M = 0$

Hence, 
$$0 = -\frac{2^3}{3} + 2A \quad (3)$$

From equation (3),  $A = \frac{2^3}{3(2)} = 1.333$  (4)

Hence,  $F = -x^2 + 1.333$

From equation (2),  $M = -\frac{x^3}{3} + 1.333x$  or  $M = -0.333x^3 + 1.333x$  (5)

For maximum bending moment,

$$\frac{dM}{dx} = 0 = -x^2 + 1.333$$

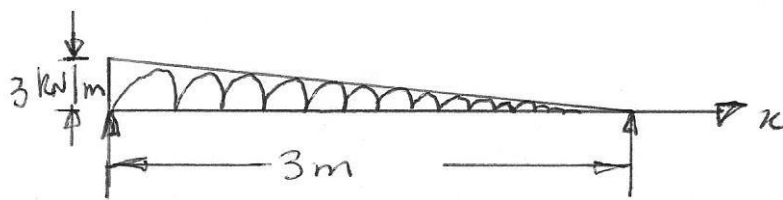
from which,  $x = \sqrt{1.333} = 1.155 \text{ m}$  (6)

Hence, **maximum bending moment,**

$$M = -\frac{x^3}{3} + 1.333x = -\frac{(1.155)^3}{3} + 1.333 \times 1.155$$

i.e.  $M = 1.026 \text{ kN m}$

2. Determine expressions for the shearing force and bending moment diagrams for the hydrostatically load beam shown, which is simply supported at its ends. Hence, or otherwise, determine the position and value of the maximum bending moment.

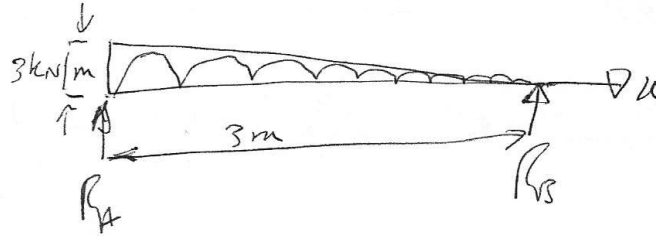


By inspection, at  $x$ ,  $w = -3 + x$

$$\frac{dF}{dx} = w = -3 + x$$
 (1)

hence,  $F = \int (-3 + x) dx$

i.e.  $F = -3x + \frac{x^2}{2} + A$  (2)



$$\frac{dM}{dx} = F$$

hence, 
$$M = \int_0^x F dx = \int_0^x \left( -3x + \frac{x^2}{2} + A \right) dx$$

i.e. 
$$M = -\frac{3x^2}{2} + \frac{x^3}{6} + Ax + B \quad (3)$$

At  $x = 0$ ,  $M = 0$

hence, 
$$B = 0 \quad (4)$$

At  $x = 3$ ,  $M = 0$

hence, 
$$0 = -\frac{3(3)^2}{2} + \frac{(3)^3}{6} + A(3)$$

i.e. 
$$0 = -13.5 + 4.5 + 3A$$

i.e. 
$$A = \frac{9}{3} = 3 \quad (5)$$

Hence, from equation (2),

$$F = -3x + 0.5x^2 + 3 \quad \text{or} \quad F = 3 - 3x + 0.5x^2$$

From equation (3), 
$$M = -1.5x^2 + \frac{x^3}{6} + 3x \quad \text{or} \quad M = 3x - 1.5x^2 + \frac{x^3}{6} \quad (6)$$

For maximum bending moment,

$$\frac{dM}{dx} = 0 = -3x + \frac{x^2}{2} + 3$$

i.e. 
$$0.5x^2 - 3x + 3 = 0$$



from which,

$$x = \frac{- -3 \pm \sqrt{(-3)^2 - 4(0.5)(3)}}{2(0.5)} = \frac{3 \pm \sqrt{3}}{1}$$

= **1.268 m** or 4.73 m which is ignored

Hence, **maximum bending moment,**

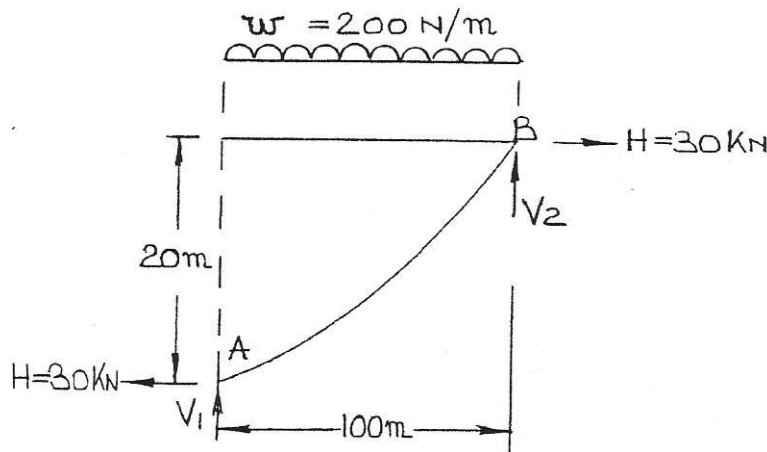
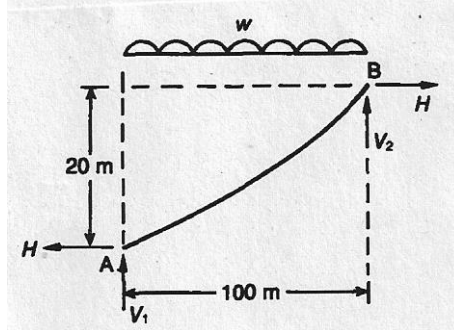
$$M = -1.5(1.268)^2 + \frac{(1.268)^3}{6} + 3(1.268)$$

i.e.  **$M = 1.732 \text{ kN m}$**

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## EXERCISE 23, Page 86

1. Determine the maximum tensile force in the cable shown, and the vertical reactions at its ends, given the following:  $w = 200 \text{ N/m}$  and  $H = 30 \text{ kN}$



Taking moments about B gives:

$$H \times 20 + V_1 \times 100 = 200 \times 100 \times 50$$

i.e.  $30000 \times 20 + V_1 \times 100 = 1000000$

and  $V_1 = \frac{10^6 - 600000}{100} = 15$

i.e.  $V_1 = 4 \text{ kN}$

Resolving vertically gives:

$$V_1 + V_2 = 200 \times 100$$

i.e.  $V_2 = 20000 - 4000 = 16 \text{ kN}$

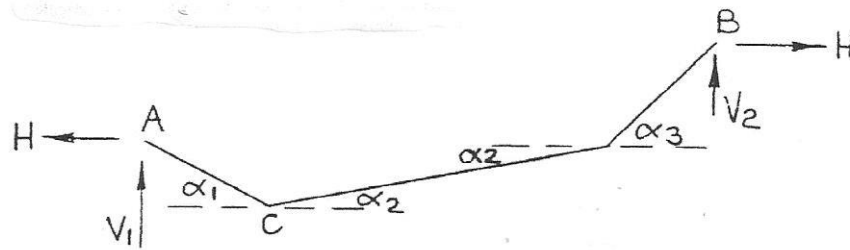
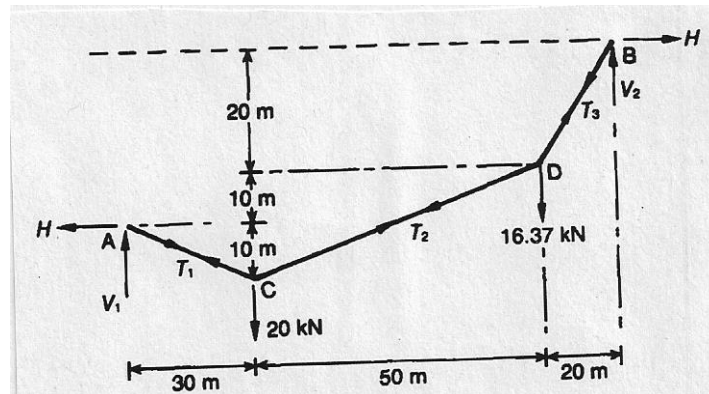
To find maximum tension:

$$T_A = \sqrt{[4^2 + 30^2]} = 30.27 \text{ kN}$$

and

$$T_B = \sqrt{[16^2 + 30^2]} = \mathbf{34.0 \text{ kN}} = \text{maximum tension}$$

2. Determine the tensile forces in the cable shown, together with the end reactions.



$$\alpha_1 = \tan^{-1} (1/3) = 18.43^\circ$$

$$\alpha_2 = \tan^{-1} (20/50) = 21.80^\circ$$

$$\alpha_3 = \tan^{-1} (1) = 45^\circ$$

**Joint C**

Resolving horizontally gives:

$$T_1 / \cos \alpha_1 = T_2 \cos \alpha_2$$

and

$$T_1 = (\cos \alpha_2 / \cos \alpha_1) T_2 = 0.979 T_2 \quad (1)$$

Resolving vertically gives:

$$T_1 \sin \alpha_1 + T_2 \sin \alpha_2 = 20$$

i.e.

$$0.316 T_1 + 0.371 T_2 = 20 \quad (2)$$

Substituting equation (1) into equation (2) gives:

$$0.316(0.979 T_2) + 0.371 T_2 = 20$$

i.e.  $0.680 T_2 = 20$

from which,  $T_2 = 20/0.680 = \mathbf{29.41 \text{ kN}}$  (3)

Substituting equation (3) into equation (1) gives:

$$T_1 = 0.979(29.41)$$

i.e.  $T_1 = \mathbf{28.79 \text{ kN}}$

### **Joint A**

Resolving horizontally gives:

$$\begin{aligned} H &= T_1 \cos \alpha_1 \\ &= (28.79)(\cos 18.43^\circ) \end{aligned}$$

i.e.  $H = \mathbf{27.31 \text{ kN}}$  (4)

Resolving vertically gives:

$$V_1 = T_1 \sin \alpha_1$$

i.e.  $V_1 = 28.79 \sin 18.43^\circ = \mathbf{9.10 \text{ kN}}$

### **Joint D**

Resolving horizontally gives:

$$T_3 \cos \alpha_3 = T_2 \cos \alpha_2$$

i.e.  $T_3 \cos 45^\circ = 29.41 \cos 21.80^\circ$

from which,  $T_3 = 27.307/\cos 45^\circ = \mathbf{38.62 \text{ kN}}$

### **Joint B**

$$V_2 = T_3 \sin \alpha_3$$

i.e.  $V_2 = 38.62 \sin 45^\circ = \mathbf{27.31 \text{ kN}}$

and  $H_2 = T_3 \cos \alpha_3$

$$= 38.62 \cos 45^\circ$$

i.e.

$$H_2 = 27.31 \text{ kN}$$

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