

Bertrand Arthur William Russell (1872-1970)

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Biography

Bertrand Russell divided his efforts between philosophy and political advocacy on behalf of a variety of radical causes. He did his most important philosophical work in logic and the philosophy of mathematics between 1900 and 1913, though later he also did important work in epistemology, metaphysics and the philosophy of mind, and continued to contribute to philosophy until the late 1950s. He wrote relatively little on ethics. His political work went on until his death.

In the philosophy of mathematics his position was logicism, the view that all of mathematics can be derived from logical premises, which he attempted to establish in detail by actual derivations, creating in the process what is essentially now the standard formulation of classical logic. Early in this work he discovered the self-referential paradoxes which posed the main difficulty for logicism and which he eventually overcame by the ramified theory of types.

Logic was central to Russell's philosophy from 1900 onwards, and much of his fertility and importance as a philosopher came from his application of the new logic to old problems. Among his most important logical innovations were the modern theory of relations and the theory of descriptions. The latter enabled him to reparse sentences containing the phrase 'the so-and-so' into a form in which the phrase did not appear. The importance of this theory for subsequent philosophy was that it enabled one to recast sentences which apparently committed one to the existence of the so-and-so into sentences in which no such commitment was suggested. This laid the basis for a new method in metaphysics (widely pursued by Russell and others in the first half of the century) in which theories about items of a given kind are reformulated so as to avoid reference to items of that kind.

Logicism itself offers just such a treatment of mathematics and in his later work Russell used the method repeatedly, though the reformulations he suggested were rarely so explicit as the ones he had offered in mathematics. In 1914 he proposed a solution to the problem of the external world by constructing matter out of sensibilia. After 1918 he proposed to construct both mind and matter out of events. After 1940 he treated all particulars as bundles of qualities. In each case his motivation was to avoid postulating anything that could be constructed, thereby eliminating ontological commitments which had no independent evidential support. Outside mathematics, his starting-point was the empirically given and he attempted to make his constructions depend as little as possible upon items not given in experience. He was not, however, a strict empiricist, since he did not think that empirical evidence alone would be sufficient for the constructions and he was always prepared to supplement it in order to obtain them. He wanted to construct, not those items which were empirically warranted, but those which were required by the relevant scientific theories, for he regarded science as the best available, though by no means an infallible, source of truth. The task, in each case, was therefore to reveal the least amount of

apparatus that would have to be assumed in addition to the empirical data in order for the constructions required by science to be possible. This methodology, which he pursued throughout his career, gives an underlying unity to what, more superficially, appears as a series of abrupt changes of position.

1. Life

Russell was born into the Whig aristocracy and inherited many of the values of its most radical wing. His grandfather, a prominent Whig reformer of the 1830s, had twice been prime minister. Russell was orphaned before he was 4 and was brought up by his grandmother who had him educated at home. In 1890 he went to Trinity College, Cambridge, to study mathematics. He gained a first in mathematics in 1893 and then turned to philosophy for his fourth year.

In 1894 he married Alys Pearsall Smith and spent part of his honeymoon in Berlin working on his first book, *German Social Democracy* (1896). The following year he published *An Essay on the Foundations of Geometry*, a revised version of his dissertation for which he had been awarded a six-year fellowship at Trinity. This was the first of a projected series of books on the sciences. After some aborted work on physics, he turned to pure mathematics and logic, producing many of his most important contributions in the period to 1913. While doing so, he lived mainly off unearned income, but in 1910, his capital depleted, he took up a lectureship at Trinity where he taught until 1916.

Though Russell had been born with an interest in politics, it had not hitherto occupied much of his time (despite his study of the German Marxists and vigorous interventions on behalf of free trade and women's suffrage). With the outbreak of war in 1914, however, his interest in philosophy lessened and he threw himself into writing, speaking and organizing on behalf of the pacifists. A conviction under the Defence of the Realm Act led to his dismissal from Trinity in 1916; a further conviction led to his being jailed in 1918. After 1916 he had only relatively short periods of academic employment and was dependent upon writing to make his living – a fact which only partially explains his huge subsequent output.

After the war he visited Russia to see the work of the Bolshevik government, but was disillusioned with its authoritarianism. In 1920–1 he spent a year at the University of Peking (Beijing). Returning to England he married Dora Black in 1921, just in time to legitimize his son; a daughter was born in 1923. Parenthood led him to take an interest in education and with his wife he started an experimental school. It was not, in Russell's eyes, a success, and it was very expensive to run, requiring that he undertake regular lecture tours of the USA to raise money. His involvement with it ended, along with his marriage, in 1932, though Dora continued to run it on her own until 1943.

Through the 1920s and early 1930s Russell wrote prolifically on an astonishing range of topics, producing (among much else) books on Russia, China, relativity, history, education, sexual morality, international relations, religion and the future of society. Though much of this activity was necessary to make ends meet, Russell was a tireless advocate of progressive causes. By the mid-1930s, however, tiring of the precariousness of this way of life and needing now to support two families (he had remarried in 1936 and a third child was born in 1937), he hoped to return to

academic life. This was not easy: positions were scarce and Russell was a controversial figure. In 1938 he lectured at Oxford, followed by visiting appointments at Chicago and UCLA and finally an offer of a permanent position at City College, New York. This last, however, provoked opposition from New York's Catholic community and the appointment was overturned in a celebrated court case.

In 1940, therefore, Russell found himself unemployed and marooned in America by the war. At this point, the eccentric millionaire Albert Barnes came to his rescue with a five-year appointment to lecture on the history of philosophy at the Barnes Foundation in Philadelphia. Although Barnes fired Russell at the end of 1942, he permanently solved Russell's financial problems, for not only did Russell collect a sizeable sum for breach of contract but the lectures he gave for Barnes became the basis for his hugely successful *History of Western Philosophy* (1945).

Russell returned to England in 1944 to take up a fellowship at Trinity College, where he completed his last great philosophical work, *Human Knowledge* (1948). His return marked not only a mending of his relations with Trinity but also with the British establishment. His continued condemnation of communism in general and the Soviet government in particular was well suited to the beginnings of the cold war and Russell enjoyed a period of unaccustomed respectability. Throughout the 1950s he continued to write prolifically, including a philosophical autobiography, *My Philosophical Development* (1959). He married for a fourth time in 1952.

His respectability was short-lived. After the death of Stalin his hatred of the Soviet government moderated, and the threat of nuclear war came to dominate his thoughts. He wrote extensively on the danger of war during the 1950s but increasingly felt the need for action and in 1958 helped found the Campaign for Nuclear Disarmament, and then the more militant Committee of 100. In 1961 he was jailed once more for inciting demonstrators to civil disobedience. The 1960s were a time of hectic political work for Russell. He lent support to many causes and was especially active in opposing the Vietnam War. His last political statement, on the Middle East, was written two days before his death.

2. Early work

As an undergraduate Russell came under the influence of the neo-Hegelianism which then dominated British philosophy, especially the work of J.M.E. McTaggart, James Ward and F.H. Bradley. His earliest logical views were influenced most by Bradley, especially Bradley's rejection of psychologism. But, like Ward and McTaggart, he rejected Bradley's metaphysical monism in favour of monadism. Even as an idealist, he held that scientific knowledge was the best available and that philosophy should be built around it. Through many subsequent changes, this belief about science and his pluralism remained constant.

In 1895 he conceived the Hegelian plan of writing an encyclopedia of the sciences, together with another series of books on social and political issues, the two to be united in a synthesis of theory and practice. He followed this plan long after its Hegelian inspiration had passed, and in old age he noted that he had written as much of both series as could have been expected – except for the final synthesis, which still eluded him. As a system, the encyclopedia was to be dialectical, with

supercessions between the individual sciences. Within each science, however, transcendental arguments were used to establish its a priori elements. In *An Essay on the Foundations of Geometry*, for example, he attempted to show that projective geometry was a necessary condition for the possibility of external experience. At the same time, he argued that geometry gave rise to contradictions which could only be resolved by a supercession to kinematics. From geometry he turned to arithmetic (which preceded it in the dialectic) and to physics (which followed it), though what he wrote on these subjects remained largely unfinished (see 1983–: vol. 2).

Two main problems caused Russell to abandon this work. First, there was (as G.E. Moore pointed out) a lingering psychologism associated with his use of transcendental arguments. Second, the contradictions which supposedly arose in the special sciences generally were not eliminated by the transition to a new science, but merely reappeared in a new form. They resulted from the use of asymmetrical relations, but only if these relations were treated as internal. Russell resolved both problems in 1898 by abandoning idealism (including internal relations and his Kantian methodology). He called this the one real revolution in his philosophy (see Hegelianism §5; Transcendental arguments).

3. Logicism

The new philosophy was little help with Russell's immediate task – an analysis of the concept of number – until, in 1900, he discovered Peano's symbolic logic. Significantly, Russell's first contribution to Peano's logic was a formal theory of relations on the basis of which he was able to define the cardinal number of a class u as the class of classes coextensive with u and produce a theory of progressions which formed the basis for ordinal arithmetic (1901).

Using the new logic Russell propounded logicism, the view that the whole of pure mathematics can be derived deductively from logical principles, a position he arrived at independently of Frege, who held a similar but more restricted view (and who had anticipated Russell's definition of cardinals in 1884) but whose work Russell discovered only later (see Frege, G. §8). Russell's first full statement of logicism was published in *The Principles of Mathematics* (1903). The interest of this work is mainly philosophical: it contains none of the actual derivations of mathematical results from logical principles. The latter was to be carried out in a projected second volume to be written in collaboration with A.N. Whitehead (see A.N. Whitehead §2). Their work expanded over the next ten years and resulted in the three-volume *Principia Mathematica* (1910–13), in which detailed derivations were given for Cantor's set theory, finite and transfinite arithmetic (including an elegant generalization of ordinal arithmetic called 'relation-arithmetic') and elementary parts of measure theory (see Cantor, G.). A fourth volume, by Whitehead alone, on geometry (including complex number theory) was never completed. As a demonstration of logicism, therefore, *Principia* depends upon much prior arithmetization of mathematics, for example, of analysis, which is not explicitly treated. Even with these allowances much is still left out, such as abstract algebra and statistics (see Numbers §6).

4. Absolute realism

The philosophy which Russell had adopted after abandoning idealism in 1898 and which underlay his early logicism was an extreme realism (to which Moore also subscribed). It was fully stated in *The Principles of Mathematics* and had been anticipated in *A Critical Exposition of the Philosophy of Leibniz* (1900) and in unpublished writings (see 1983–: vol. 2).

In Russell's absolute realism, everything which can be referred to or made the subject of a proposition is a term which has being (though not necessarily existence). Terms are either things, which can occur in a proposition only as its subject(s), or concepts, which may occur as subjects but which may also occur as relations. A proposition is a complex of terms related to each other. All complexes are propositions; all propositions are terms. Terms are neither linguistic nor psychological, but objective constituents of the world. The first task of philosophy is the theoretical analysis of propositions into their constituents. The propositions of logic are unique in that they remain true when any of their terms (apart from logical constants) is replaced by *any* other term.

In 1901 Russell discovered that this position fell prey to self-referential paradoxes. For example, if the combination of any number of terms is a new term, the combination of *all* terms is a term distinct from any term. This paradox is related to the greatest cardinal paradox which arose in Cantor's transfinite arithmetic. Cantor had shown that the power-set, $\wp(S)$, of any set S has a higher cardinality than S . But if S is the set of all cardinals, the cardinality of $\wp(S)$ will then be a cardinal greater than any in S . While thinking about this problem Russell came upon an even more troubling contradiction, now known as Russell's paradox, which affected the foundations of set theory itself. It seems natural to suppose that some sets are, but most sets are not, members of themselves. If we consider, however, the set of all sets that are not members of themselves and ask whether this set is a member of itself we arrive at Russell's paradox. For supposing that it is a member of itself, then it lacks the defining property of the set and so is not a member of itself. But if it is not a member of itself then it has the defining property of the set and so is a member of itself. The paradox is simply proven given the unrestricted comprehension axiom of naive set theory, that for any property (or propositional function) ϕ there is a set ϕ of just those things that satisfy ϕ ; that is to say, ϕ . Let ϕx be $\sim(x \in x)$ and let a be $\{x: \sim(x \in x)\}$, then substituting in the comprehension axiom gives:

$$\{x : \sim(x \in x)\} \in \{x: \sim(x \in x)\} \equiv \sim(\{x : \sim(x \in x)\} \in \{x: \sim(x \in x)\})$$

These paradoxes in set theory are related to other self-referential paradoxes. The best-known is the liar paradox, generated by the person who says 'I am lying': if what they say is true, it is a lie; if it is a lie then they speak the truth. The long delay in the publication of *Principia* was largely due to the difficulty of devising a logic which was both paradox-free and strong enough to support pure mathematics (see Paradoxes of set and property §4; Cantor's theorem; Continuum hypothesis).

5. Solutions to the paradoxes: the zig-zag theory

In *The Principles of Mathematics* Russell offered no firm solution to the paradoxes, but in the next few years he tried a variety of approaches. The first was what he called the 'zig-zag theory'. The intuitive idea behind it was simple: only 'fairly simple' propositional functions determine

classes; ‘complicated and recondite’ functions do not (1906a: 145–6). The problem was to specify the degree of complexity which marked the divide. Russell’s only published remarks on the theory were in a survey article written after he had abandoned the theory and are vague on this point (1906a). Unpublished writings reveal that he tried a number of formulations (1983–: vol. 4). The chief difficulty is to find an appropriately restricted comprehension axiom: versions with weak constraints permit new contradictions to be derived, but if the constraints are too strong, logicism (for example, Russell’s definition of cardinal number) is undermined. Russell found no way of showing that a comprehension axiom strong enough to support logicism would be paradox-free.

The zig-zag approach has some affinity to Quine’s ‘New Foundations’ theory (Quine 1937) and some more recent type-free theories (Feferman 1984), but the former imposes quite different constraints on comprehension, while the latter is inconsistent with logicism.

6. Solutions to the paradoxes: the substitutional theory

Russell’s next attempt went considerably further. It occupied him from 1905 to 1906 and started from the theory of incomplete symbols he had devised in 1905 (see §9 below). Under the substitutional theory Russell eliminated classes, relations and predicates as terms over which variables ranged. Variables now range over entities, which are either simple individuals or complexes (including propositions). The fundamental notion of the theory is that of substituting one entity for another in a complex to yield a new complex. This is written $p^b_a!q$ and read ‘ p with b substituted for a yields q ’, where p and q need not be propositions, though in the interesting cases they will be. $p^b_a!q$ will always be true or false whatever p , a and b are. If a does not occur in p then $p^b_a!q$. Propositional functions and classes of individuals are expressed by matrices, for example, ‘ p / a ’, in which p is called the prototype and a the argument. b is a member of the class represented by ‘ p / a ’, if $p^b_a!q$ and q is true. Propositional functions of propositional functions (or classes of classes) are expressed by the matrix ‘ $q / (p / a)$ ’. Russell hoped to avoid the paradoxes because the application of a matrix to itself, which generates many of them, is impossible in the system. For example, $p^{p/a}_a$

is ill-formed, because ‘ p / a ’ is an incomplete expression, not the name of an entity, and thus there is nothing which is substituted for a . Thus one can express with matrices everything that could previously be expressed by means of classes or functions except, as Russell puts it, for ‘certain limiting cases...[which] are precisely those that lead to contradictions’ (1906b: 171–2).

The ability to capture an iterative concept of classes by substitutional means preserved Russell’s definition of cardinal number and thus permitted the logicist programme to proceed. Russell spent much of 1906 developing logicism on a substitutional basis (see 1983–: vol. 5). The above account illustrates the lengths he was prepared to go to in order to preserve the unrestricted variables of the Principles. However, the fact that the theory allowed propositions as substituends permitted a propositional paradox to be derived. After several attempts to eliminate it and finding

that, each time, it returned in more complicated forms, Russell reluctantly abandoned the substitutional theory.

7. Solutions to the paradoxes: the theory of types

In 1906 Russell adopted the solution which appears in *Principia*, namely the theory of types which banned self-reference by stratifying terms and expressions into complex hierarchies of disjoint sub-classes. The expression ‘all terms’, for example, is then meaningless unless restricted to terms of specified type(s), and a combination of terms of a given type is a term of different type; functions are of a different type to their arguments; classes of a different type to their members.

Ironically, a simple version of the theory had appeared in *The Principles of Mathematics* (appendix B). There, propositional functions are assigned a range of significance, ‘a range within which x must lie if $\phi(x)$ is to be a proposition at all’ (1903: 525). Significance ranges form types in that ‘if x belongs to the range of significance of $\phi(x)$, there is a class of objects, the *type* of x , all of which must belong to the range of significance of $\phi(x)$ ’ (1903: 525). Every significance range either is a type or is the union of several types. Types are hierarchically arranged: individuals constitute the lowest type, classes of individuals the next, classes of classes of individuals the next after that, and so on. Two-place relations (in extension) form a different hierarchy, three-place relations yet another, and so on. The theory bears resemblance to Chwistek’s simple type theory (Chwistek 1921), though there are important differences, notably that in Russell’s theory all ranges form a type, and propositions and numbers form distinct types outside the hierarchy. Since propositions are not stratified into types, Russell’s theory yields a propositional version of the contradiction (1903: 527).

Russell anticipated that the type theory of the *Principles* would have to be ‘transformed into some subtler shape before it can answer all difficulties’ (1903: 523). One reason he did not pursue this immediately was due to his reluctance to abandon the unrestricted variables of the *Principles* upon which his account of logic depended (see Urquhart 1989). Only when other avenues had been explored did he return to type theory. In the type theory of the *Principles* the type of a function had been determined exclusively by the type(s) of its argument(s). The ‘subtler shape’ of the theory Russell developed after 1906 – the ‘ramified theory of types’ – was obtained by adding order distinctions within each type based upon the form of the function. In a controversy with Poincaré, Russell had acknowledged that all the paradoxes involved violations of the ‘vicious circle principle’: No totality can contain members defined in terms of itself; or, in the formulation Russell preferred, ‘whatever involves an apparent [bound] variable must not be among the possible values of that variable’ (1906c: 198). The ramified theory of types was designed to build order-restrictions into the bound variables themselves in such a way that violations of the vicious circle principle would be prevented by the symbolism itself.

Functions are significant only when their arguments for each argument-place are drawn from a specified range of items appropriate to that place. Russell expresses this by saying that a function presupposes a totality for each of its argument-places; such a totality is the significance range for that argument-place. As before, individuals constitute the lowest such totality. First-order

functions take only individuals as arguments; second-order functions take either first-order functions or individuals as arguments; third-order functions take either second-order functions, first-order functions or individuals as arguments; and so on. All the arguments of any function must be of a lower order than that of the function. If an n th-order function has an argument of order $n - 1$, the function is said to be *predicative*. Non-predicative functions all involve quantification over functions of lower order. (For example, if Russell is an individual, ‘Russell is clever’ employs a first-order predicative function of individuals, whereas ‘Russell has all the (first-order) qualities of a great philosopher’ employs a non-predicative function of second-order involving the quantification $(\phi)(x \text{ is a great philosopher} \supset \phi x)$, where the variable ϕ ranges over the totality of first-order functions. All bound variables range over only one such totality.)

Propositions form a similar hierarchy: *elementary* propositions have only individuals as terms; first-order propositions are either elementary or involve quantification over individuals; second-order propositions involve quantification over first-order propositions; and so on. First-order propositions presuppose no totality except that of individuals and, in general, propositions of n th-order presuppose totalities only of propositions of order $n - 1$. The hierarchy of propositions can be derived from the hierarchy of functions since a proposition involving a totality of n th-order propositions can be reduced to one involving a totality of n th-order functions. Observing the restrictions imposed by these hierarchies ensures that no violations of the vicious circle principle can occur. The liar paradox is avoided, for instance, because what the liar asserts is ‘ $(p)(I \text{ affirm } p \text{ and } p \text{ is false})$ ’. For this to be significant the bound variable ‘ p ’ must range over propositions of some specified order, while the proposition the liar asserts will be of a higher order and thus not among the propositions the liar asserts to be false.

In 1908, Russell sketched the development of logic and set theory up to the definition of cardinal and ordinal numbers on the basis of the ramified theory, a process that is completed in *Principia*. There are some differences between the 1908 presentation followed above and that in *Principia*: for instance, in *Principia* elementary propositions are not included among first-order propositions and propositional quantification is avoided. This is related to a more important change, namely Russell’s treatment of propositions in *Principia* as incomplete symbols (see §10) which requires a radical distinction between elementary propositions and those involving first-order quantification and is the basis for his claim that the propositional hierarchy is derived from the functional one. Some elements of the substitutional theory survive into ramified type theory, indeed Landini (1993) goes so far as to call the 1908 version a ‘modification’ of the substitutional theory. As in the substitutional theory, classes remain incomplete symbols, and Russell presents the hierarchy of functions in explicitly substitutional terms (see *Theory of types*).

8. Problems with ramified types

Russell’s reluctance to embrace the ramified theory of types was well-motivated, for it raised serious obstacles to logicism. In the first place, Russell’s definition of a cardinal number as the class of all similar classes is no longer admissible. There is no longer a class of all similar classes, but only a class of all similar first-order classes, a class of all similar second-order classes, and so on. (Strictly, of course, the definition will proceed in terms of the hierarchy of functions, but the class terminology is more familiar and admissible as a *façon de parler*.) There

then ceases to be a unique cardinal 2, for example, but a different 2 at each level of the hierarchy; ordinary talk about 2 is typically ambiguous between these different cardinals. Similar problems affect the statement of logical principles which, strictly, would need to be stated for each order. In *Principia* this is avoided by the use of free variables which are typically ambiguous. Thus $\vdash \cdot \forall x \cdot \neg \phi x$ asserts the law of excluded middle for monadic propositions of undetermined order; whereas $\vdash \cdot (\phi) \cdot \forall x \cdot \neg \phi x$ asserts it for those of some particular order. These results are untoward but not intolerable.

A similar, but more troubling difficulty threatened Russell's treatment of the continuum, for on the ramified theory the least upper bound of a bounded set of real numbers is a number of a higher level. The most serious difficulty, however, concerns the principle of mathematical induction: namely, that any function satisfied by 0, and satisfied by $x + 1$ if satisfied by x , is satisfied by all natural numbers. These two difficulties threaten the logicist programme for transfinite and finite arithmetic respectively. Russell surmounted them by making the assumption that for every nonpredicative function there is an equivalent predicative function in the same variables. This is his 'axiom of reducibility': $(\exists f) (\phi x \equiv_x f ! x)$ (1910–13: 12.1).

It has often been objected that the axiom of reducibility undoes all the good that ramifying type theory achieved, on the grounds that it renders the ramified theory equivalent to the simple theory, thereby readmitting all the paradoxes ramification was designed to prevent (Ramsey 1925; Copi 1950). Myhill (1979) shows that this objection is ill-founded.

Frank Ramsey (1925) proposed that the paradoxes should be divided into those which belong to logic and set theory and those which are semantical or linguistic and that logic need solve only the former. Since the simple theory of types solves the logical paradoxes, the ramified theory is not needed in logic. This suggestion has been very influential. However, it still leaves the semantic paradoxes to be dealt with and Ramsey's demarcation between the two kinds is hardly adequate. His suggestion that the semantic paradoxes are in some way empirical is neither clear nor plausible. Moreover, the fact that both types of paradox arise from self-reference suggests a uniform treatment. Above all, Ramsey's distinction does little justice to Russell's appealing intuition that the same underlying logic should be applied universally. It is clear now that semantics is capable of just as rigorous, formal treatment as set theory and the suggestion that the proper formal treatment of each requires different logical principles lacks *prima facie* plausibility (see Semantic paradoxes and theories of truth §2).

Under the influence of Wittgenstein, Russell came to accept the view that logical propositions were all tautologies. From this point of view the axiom of reducibility is clearly unsatisfactory (Wittgenstein 1922). From much the same point of view, Ramsey argued that it was not only contingent but a contingent falsehood – though it is not clear that his counterexample is a good one (Ramsey 1925). In the second edition of *Principia*, therefore, Russell tried to dispense with the axiom. He conceded that much of transfinite arithmetic would be lost as a result, but hoped none the less to save the theory of natural numbers by proving a version of the principle of mathematical induction within the constraints of the ramified theory and without the axiom of reducibility. Gödel (1944) noted the flaw in the proof and Myhill (1974) has since shown it to be

irremediable. It follows, therefore, that even finite arithmetic cannot be logicized within the constraints of the ramified theory without the axiom of reducibility.

9. The theory of incomplete symbols

Both the substitutional theory and ramified type theory were closely linked to one of Russell's most important contributions to logic: his theory of definite descriptions and the more general theory of incomplete symbols it presaged. The theory of descriptions arose out of problems in Russell's theory of denoting in *The Principles of Mathematics*. It is plain that 'I met Quine' and 'I met the author of *Quiddities*' are different propositions, even though Quine is the author of *Quiddities*. In the *Principles*, the first proposition contains Quine, while the second contains a 'denoting concept' (expressed by the definite description 'the author of *Quiddities*') which denotes Quine. When Quine occurs in a proposition the proposition is about Quine, but when a denoting concept occurs the proposition is not about the concept but what the concept denotes. This theory, if it is to be storable, requires that there be some way in which a denoting concept, rather than its denotation, can be denoted. After much effort, Russell in 'On Denoting' (1905) concluded that this was impossible and eliminated denoting concepts as intermediaries between denoting phrases and their denotations by means of his theory of descriptions.

Russell showed in a broad (though not comprehensive) range of cases how denoting phrases could be eliminated in favour of predicates and quantified variables by providing an analysis of the sentence in which they occurred. This advance was made possible by Russell's study of Frege, whose theory of quantification replaced the intricate but unavailing theory of the *Principles*. Russell analysed sentences containing definite descriptions, such as, 'The successor of 1 is even', thus:

1. $(x) (x \text{ succeeds } 1 \ \& \ (y) (y \text{ succeeds } 1 \supset y = x) \ \& \ x \text{ is even})$

The three conjuncts in this analysis ensure, respectively, that a successor of 1 exists, that there is at most one successor of 1 and that whatever succeeds 1 is even.

In cases where the definite description does not denote, the sentence gets the value *false* on account of the falsity of the first conjunct of its analysis. This enabled Russell to treat as meaningful sentences containing non-denoting definite descriptions without having to suppose either that nonexistent objects were denoted (as in Meinong's theory of objects) or that there were such things as denoting concepts (as in Russell's 1903 theory) or Fregean senses. The subsequent literature has emphasized the first point, but it was the second which motivated Russell.

Some counterintuitive consequences of Russell's analysis were removed by his introduction of scope distinctions among the occurrences of descriptions within sentences. Not all problems are thus removed, however, and there have been many subsequent attempts to do better either through improvements to Russell's theory, or through 'free logics', or by means of Meinongian theories which permit quantification over nonexistent objects.

Obviously the theory will not permit the use of definite descriptions as substitution- values for the variables in (1) and this leads to a sharp distinction in Russell's philosophy of language

between definite descriptions and names. Since Russell thought that most ordinary names were in fact disguised definite descriptions, he distinguished between descriptions and ‘logically proper names’ which alone could be substituted for variables. (The latter were names of objects of acquaintance – represented in ordinary language by ‘this’ and ‘that’.) By treating ordinary names as descriptions he could use the theory of descriptions to account for the informative nature of identity statements such as ‘Hesperus = Phosphorus’.

The principle of contextual definition employed in the theory of descriptions, which allowed sentences containing a description to be rephrased to eliminate the description, led Russell, after 1905, to make a sharp distinction between the grammatical and the logical form of a proposition. He applied similar techniques to other kinds of expression (for instance, to class names and to propositions). Russell held that the expressions thus eliminated, which he called ‘incomplete symbols’, had meaning only in the context of a sentence and were meaningless in isolation. The various ways in which they were eliminated constituted his ‘theory of incomplete symbols’ (see Descriptions; Existence §2; Free logics §§1, 3; Proper names §4; Sense and reference; Strawson, P.F. §2).

10. Theories of truth and judgment

The theory of incomplete symbols made possible Russell’s substitutional theory, in which class names and relations were treated as incomplete symbols. The appearance of a propositional paradox in the substitutional theory convinced Russell that propositions should also be treated as incomplete symbols. This was accomplished by his multiple relation theory of judgment which became part of the philosophical underpinnings of ramified type theory. In Russell’s early realism, true and false propositions alike were treated as subsistent complexes and belief as a relation between a mind and a proposition. This leaves it obscure why we prefer to believe true propositions. In the multiple relation theory, belief and other ‘propositional’ attitudes were treated as many-place (‘multiple’) relations between a mind and the individual constituents of the erstwhile proposition. Thus ‘*m* believes that *aRb*’ has the form ‘*B*(*m*, *a*, *R*, *b*)’, not ‘*B*(*m*, *p*)’. Apparent references to propositions are eliminated by subsuming their constituents within an actual psychological complex including a mind and related by some ‘propositional’ attitude. Propositions thus become fabrications of the mind. The belief represented by ‘*B*(*m*, *a*, *R*, *b*)’ will be a true belief just in case there is a complex *a-R-b*.

Russell had considered such a theory as early as 1906, but put it aside while he worked on the substitutional theory in which propositions were needed as entities. The theory was taken up again and developed in writings from 1910 to 1913. The final development of the theory, in *Theory of Knowledge* (1913), was left unpublished by Russell because of criticisms from Wittgenstein, then his student at Cambridge. Wittgenstein’s criticisms are perhaps most simply expressed as a dilemma. Either the constituents of a belief (*a*, *R* and *b* in the example above) are assigned to types or they are not. If they are not, then the ‘propositions’ fabricated by thought will not be subject to the ramified type hierarchy (it will be possible to judge nonsense, as Wittgenstein puts it) and the paradoxes will reappear. If they are, then they must be assigned to types by some prior judgment, to which the same considerations apply, and an infinite regress results (Sommerville 1981; Griffin 1985).

11. Logical atomism

Russell used the name ‘logical atomism’ for all his philosophical work after 1898, though now it is used mainly to describe his position from 1905 to 1919. The philosophy was atomistic because it took items in certain categories to be basic; it was logical because it sought to construct items in other categories by rigorous logical means. This involved a two-fold process: an analysis of concepts in ordinary or scientific use to discover which items were basic, followed by a logical reconstruction of the science from the basic items up. Although Russell’s early logicist constructions were clearly along these lines, this analytic methodology came fully into its own only with the advent of the theory of incomplete symbols in 1905.

One important consequence of the theory of incomplete symbols was that the ontological commitments of a theory could be reduced by reformulating the theory to remove expressions which apparently denoted problematic entities. It was unnecessary to posit the existence of items which could be logically constructed. This formed the basis for what Russell called the ‘supreme maxim in scientific philosophizing’, his version of Ockham’s razor: ‘Wherever possible, logical constructions are to be substituted for inferred entities’ (1983–: vol. 8, 11).

Parallel results held in epistemology. The theory of incomplete symbols showed how knowledge of a wide range of items could be achieved by knowledge (by acquaintance) of a much narrower range. Acquaintance was a direct, two-place cognitive relation, and objects of acquaintance were epistemically basic. Knowledge of other items was obtained by description, which always depended upon knowledge by acquaintance (1911). The distinction between knowledge by acquaintance and knowledge by description first appeared in ‘On Denoting’, where Russell first formulated his ‘principle of acquaintance’: ‘in every proposition that we can apprehend...all the constituents are really entities with which we have immediate acquaintance’ (1983–: vol. 4, 427). All other apparent constituents were to be eliminated by analysis. These innovations in 1905 marked the end of Russell’s extreme realism, though he remained a Platonist in that he included universals among the objects of acquaintance (see Knowledge by acquaintance and description; Logical atomism §2; Ontological commitment).

12. Our knowledge of the external world

Russell has traditionally been regarded as the direct heir of the British empiricists, yet his work prior to 1910 belongs more naturally to the continental rationalist tradition. None of the thinkers who influenced him during this period – Kant, Bradley, Cantor, Peano, Frege – were empiricists, and it was not until 1912, when he was commissioned to write a short introduction to philosophy, *The Problems of Philosophy*, that he undertook a serious study of the British empiricists, especially Berkeley and Hume. Hume’s influence appears directly in the chapter on induction (and in a paper of 1912, ‘On the Notion of Cause’); Berkeley’s was more indirect (since Russell rejected his idealism) and appears in Russell’s treatment of the external world.

In *Problems* Russell held that empirical knowledge is based on direct acquaintance with sense-data and that matter itself, of which we have only knowledge by description, is postulated as the best explanation of sense-data. He soon became dissatisfied with this idea and, inspired by his logicist constructions of mathematical concepts, proposed instead that matter be logically

constructed out of sense-data, thereby obviating dubious inferences to material objects as the causes of sensations. The actual data of sense, however, are too fragmentary for the construction of items with the expected properties of matter (such as permanence). To solve this problem Russell was led to postulate unsensed sensibilia in addition to sense-data. It is important to realize that for Russell, unlike most sense-datum theorists, sense-data were physical, located in physical space at varying distances from the place at which common sense located the material object. It is thus logically, though not practically, possible for more than one mind to be acquainted with the same sense-datum at the same time. Sense-data are merely sensibilia with which a mind happens to be acquainted, so properly speaking matter is constructed out of sensibilia rather than sense-data. In addition to matter, Russell also sought to construct space and time. All the sense-data with which a single mind is acquainted are located within a perspective, a private space peculiar to that mind. Similar spaces exist at every point at which a mind could be located. These private spaces Russell calls perspectives. Physical space is the set of all such perspectives. The arrangement of perspectives in physical space is achieved by the variation in the qualities of the sensibilia (such as their size and shape) as they appear in the perspectives: for instance, if the sensibilia in perspectives P_1 and P_2 are more similar than those in perspectives P_3 and P_4 , P_1 and P_2 will be located closer together in physical space than P_3 and P_4 .

This theory received its fullest expression in *Our Knowledge of the External World* (1914), though it remained in programmatic form; detailed constructions along similar lines were undertaken by Carnap (1928) (see Carnap, R. §1). Although Russell hoped that eventually a correlation between sense-data and the differential equations of physics would be forthcoming, it is difficult to see how this would ever be possible. Many of the standard objections to sense-data do not apply to Russell's theory, but it is still difficult to see how the gap between experiential evidence and physical theory could be bridged in the deductively rigorous way Russell wanted. For example, it is difficult to see how perspectives could be uniquely arranged in physical space when no sensible occurs in more than one perspective. Nor is it clear how sensibilia could be grouped together to form the material objects required by either common sense or physics. Moreover, since sensibilia are physical and objective it would seem that they are still subject to misapprehension by the mind, thereby reintroducing the difficulties with illusion which had allegedly made material objects unviable as objects of acquaintance (see *Perception*, epistemic issues in).

13. Neutral monism

Before 1919 Russell wrote little about the philosophy of mind, though his dualism was evident in his analyses of judgment and acquaintance. By 1919, however, he came to think that mind, like matter, should be constructed out of more primitive material, a view akin to James's neutral monism (see James, W.). This change is somewhat puzzling, since Russell had criticized James's theory in some detail in *Theory of Knowledge* (1913) and, more tentatively, in 'The Philosophy of Logical Atomism' (1918). He was influenced partly by Hume's arguments for the unintrospectability of the self, but most importantly by Wittgenstein's treatment of belief, which 'shows...that there is no such thing as the soul' (Wittgenstein 1922: 5.5421).

Russell's neutral monism began with a new theory of belief in 'On Propositions' (1919a). He identified the contents of beliefs with images ('image-propositions') and words ('word-propositions'), understood as certain sorts of events – now the basic elements of his construction – which are neutral in the sense of being neither physical nor mental. Truth was analysed in terms of resemblance and causal relations. Wider issues are treated in *The Analysis of Mind* (1921), where the influence of contemporary trends in psychology, especially behaviourism, is evident. He never fully embraced behaviourism (for example, he did not attempt a behaviourist elimination of images), but he was prepared to push it as far as he thought it would go. He offered, for example, a behaviourist account of desire, about which he subsequently had doubts.

On Russell's neutral monism, minds and matter are constructed out of events and differ only in that they are organized by different causal laws: material objects by those of physics, minds by 'mnemic' causal laws, in which a past state of the mind is among the proximate causes of its current state. The theory is a precursor of later mind–brain identity theories.

In 1927, Russell returned to the problem of matter – this time, paying more attention to contemporary physics. Part I of *The Analysis of Matter* is devoted to relativity and quantum theory, and Part III deals with the causal and spatiotemporal order of events. In Part II, starting from a causal theory of perception, Russell asks what can be inferred from percepts (considered as events occurring within one's head) about the nature of their supposed external causes. Using the dictum that like causes have like effects, Russell answers that we can infer only the structure of the external world from the structure of our percepts – a doctrine known as structural realism.

Russell quickly abandoned structural realism when M.H.A. Newman (1928) showed that any set with the right cardinality could be arranged so as to have the same structure as the world – a result analogous to that claimed in Putnam's model-theoretic argument against realist theories of reference (Demopoulos and Friedman 1989). Russell, however, never abandoned neutral monism, though he left the theory incomplete. Constructions were suggested rather than provided (the fullest sketches offered were the treatment of belief in the second edition of *Principia* (1925) and a 1936 paper on temporal order). Moreover, while Russell had much to say about consciousness, he said very little about intentionality and (more surprisingly) personal identity. On the physics side, the breakdown of causality in quantum theory was a serious problem, given the importance Russell attached to causal relations in his constructions, but in 1927 it was perhaps not yet so well-established a fact as to demand explanation (see *Neutral monism*).

14. Later epistemology

Russell's penchant for reductive analysis was taken further in *An Inquiry into Meaning and Truth* (1940) where the terms of atomic facts, hitherto sense-data or events, are construed as bundles of compresent qualities. These qualities are the meanings of object-words, and bundles of them are denoted by logically proper names. This position was retained in *Human Knowledge* (1948); it marks the culmination of Russell's long battle with substance/attribute metaphysics. Despite its title, the *Inquiry* was more concerned with metaphysics and

epistemology than with semantics. The meaning theory proposed there was causal and (in inspiration) behaviourist, though once again Russell did not expect a purely behaviourist analysis to be adequate.

Russell's allegiance to empiricism, like behaviourism, was never very strong and by 1936 he was writing on 'The Limits of Empiricism'. Even in *The Analysis of Matter* there are clear indications that he thought pure empiricism would render science impossible. In the *Analysis* he thought that Berkeleyan scepticism about matter could be avoided by construction, but that Hume's scepticism about induction was so far untouched. He tackled induction most thoroughly in *Human Knowledge* (1948), which contains his only extended treatment of probability. The problem for this book was set in 'The Limits of Empiricism', where he asked what non-empirical principles must be assumed in order for science to be possible. Originally, he thought that induction itself would be sufficient. In *Human Knowledge*, however, he showed that inductive inference was not always truth-preserving (for reasons later popularized by Nelson Goodman as 'the new riddle of induction') (see *Induction, epistemic issues* in §1).

In place of induction, he proposed five 'postulates of non-demonstrative inference' as the minimum extra-logical, non-empirical apparatus required for science. His methodology here returned to that which had guided his work on *Principia* where he had sought the minimum logical apparatus necessary to support mathematics. It even had some resemblance to his earlier Kantian methodology, though Russell pointed out that he proposed his postulates as scientific hypotheses, not as synthetic a priori truths. The postulates, unfortunately, were vaguely stated in *Human Knowledge*, and Russell left more detailed work on them unpublished, probably on account of the frosty reception the book received.

By the time *Human Knowledge* was published linguistic philosophy was at its height and Russell's work had become very unfashionable. Russell had no time for Wittgenstein's later philosophy, nor for the ordinary language philosophy it inspired. He continued to think that understanding the world, rather than language, was the chief task of philosophy and his last philosophical writings, in the 1950s, were unavailing attempts to stem the tide of linguistic philosophy (see *Ordinary language philosophy, school of §§1–3*).

List of works

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(The formalization of Russell's theory of relations in Peano's logical notation; an early statement of logicism which includes the Frege–Russell definition of cardinal number.)

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(A defence of logicism against criticisms by Poincaré. In it Russell recognizes violations of the vicious circle principle as the source of the paradoxes.)

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(A popular introduction to philosophy, with the emphasis on epistemology.)

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Russell, B.A.W. (1913) *Theory of Knowledge*, in Russell (1983–), vol. 8.

(A major unfinished work on epistemology. Extant parts give detailed discussion of acquaintance and belief.)

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(A summary of Russell's philosophical development to date with a survey of outstanding problems.)

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(A formal construction of temporal order based on Russell's event ontology.)

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(As much metaphysics and epistemology as semantics. An important statement of Russell's later position.)

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(A popular, entertaining and brilliantly witty history. Scholars may sneer, but it's well worth reading.)

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(A collection of important essays, edited by Marsh in collaboration with Russell.)

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(Russell's philosophical autobiography.)

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(The full autobiography of Russell's life and times, including his political activism.)

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(Further essays, with more of a bias towards logic. Contains a very useful three-part bibliography.)

References and further reading

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(The only serious attempt actually to furnish the logical constructions proposed in Russell (1914). Technical.)

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(Still one of the best things written on Russell's logic.)

Grattan-Guinness, I. (1977) *Dear Russell – Dear Jourdain. A commentary on Russell's logic based on his correspondence with Philip Jourdain*, London: Duckworth.

(Difficult and technical, but an important and fascinating source on Russell's logic.)

Grayling, A.C. (1996) *Russell*, Past Masters series, Oxford: Oxford University Press.

(A very concise popular introduction to Russell's thought, including his politics.)

Griffin, N. (1985) 'Russell's Multiple Relation Theory of Judgment', *Philosophical Studies* 47: 213–247.

(An account of the development of the multiple relation theory and its collapse under Wittgenstein's criticisms.)

Griffin, N. (1991) *Russell's Idealist Apprenticeship*, Oxford: Clarendon Press.

(A detailed survey of Russell's neo-Hegelian work and the reasons that led him to abandon it.)

Jager, R. (1972) *The Development of Bertrand Russell's Philosophy*, London: Allen & Unwin.

(The most comprehensive book on Russell – including his social and political thought. Sometimes superficial, but a useful survey for the beginner.)

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(The definitive refutation of the structural realism of Russell (1927).)

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(Though dated, the essays are generally of high quality. Contains an exceptionally useful, though now somewhat dated, bibliography.)

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(Includes a proposal, like that in Russell's zig-zag theory, to avoid the paradoxes by restricting comprehension – though Quine's restriction differs from any proposed by Russell.)

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(Contains the influential proposal to distinguish logical from semantic paradoxes and to solve only the former within logic, thereby obviating the need for ramified, as distinct from simple, type theory. An important original contribution to logic in the Russellian tradition.)

Ryan, A. (1988) *Bertrand Russell: A Political Life*, London: Allen Lane.

(A survey of Russell's political engagements; more narrative than analysis.)

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(Several useful papers on Russell's early logic and his later epistemology.)

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(An excellent, concise account of Russell's attempts to solve the paradoxes between 1903 and 1908 and why he was so reluctant to embrace ramified type theory. Not difficult.)

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Taylor & Francis
Not for distribution