

## Abstract objects

Bob Hale

### Philosophical Concept

The central philosophical question about abstract objects is: Are there any? An affirmative answer – given by Platonists or Realists – draws support from the fact that while much of our talk and thought concerns concrete (roughly, spatiotemporally extended) objects, significant parts of it appear to be about objects which lie outside space and time, and are therefore incapable of figuring in causal relationships. The suggestion that there really are such further non-spatial, atemporal and acausal objects as numbers and sets often strikes Nominalist opponents as contrary to common sense. But precisely because our apparent talk and thought of abstracta encompasses much – including virtually the whole of mathematics – that seems indispensable to our best attempts to make scientific sense of the world, it cannot be simply dismissed as confused gibberish. For this reason Nominalists have commonly adopted a programme of reductive paraphrase, aimed at eliminating all apparent reference to and quantification over abstract objects. In spite of impressively ingenious efforts, the programme appears to run into insuperable obstacles.

The simplicity of our initial question is deceptive. Understanding and progress are unlikely without further clarification of the relations between ontological questions and questions about the logical analysis of language, and of the key distinction between abstract and concrete objects. There are both affinities and, more importantly, contrasts between traditional approaches to ontological questions and more recent discussions shaped by ground-breaking work in the philosophy of language initiated by Frege. The importance of Frege's work lies principally in two insights: first, that questions about what kinds of entity there are cannot sensibly be tackled independently of the logical analysis of language; and second, that the question whether or not certain expressions should be taken to have reference cannot properly be separated from the question whether complete sentences in which those expressions occur are true or false.

### 1. Logical and ontological categories

Although what most obviously needs explaining is the abstract–concrete distinction, the relevant notion of *object* also calls for elucidation. There is a familiar, everyday use of the term 'object' in which we may speak of the objects found in the accused's pockets, for example. There are probably no very precise rules governing this use, but it seems clear that being extended in space and time is at least a necessary – but probably not a sufficient – condition for its application. If 'object' is so understood, the term 'abstract object' is straightforwardly self-contradictory. We

should infer, not that Nominalism wins by default, but that some other more general, less restrictive notion of object is in play in philosophical discussions. But if so, how should it be characterized? To avoid begging questions, it might be proposed that anything should be reckoned an object to which we may make reference. Arguably, however, this goes too far the other way – we may as well be said to refer to fiddling as to Nero, when we assert that Nero fiddled, but should be loathe to count fiddling an *object*.

A way forward which preserves this general approach is to take objects to be the referents of expressions of a certain restricted class – what are usually called ‘singular terms’. To take this step is to follow Frege in viewing the ontological categorization of entities as dependent upon a prior logical categorization of expressions. Objects, properties and relations, for example, are essentially the non-linguistic correlates of, respectively, singular terms (for example ‘Nero’, ‘this lake’, ‘the dome of St. Peter’s’ and so on), one-place predicates (‘... fiddles’, ‘... is deep’), and two- or more-place predicates (‘... loves...’, ‘... is taller than...’). An object, on this account, is the referent of an actual or possible singular term.

When *object* is so understood, our opening question is, in an important sense, a distinctively *modern* one. It is not that we can discern no significant common concerns underlying ancient disagreements over the status of Plato’s Forms and the great medieval battle between realists and nominalists over the existence of universals on the one hand, and modern ontological disputes on the other. Traditional and modern discussions share a general concern with the relations between language and the world. At bottom, disagreement over abstract entities is disagreement over whether an adequate account of language–world relations can be provided without reference to any such entities. It remains the case that a fundamental shift has taken place in the way very many philosophers conceive and argue about ontological issues in general, and issues about abstract entities especially. Ancient and medieval disputes focused on the existence of universals as opposed to particulars, with the former thought of as abstract entities which both predicates (‘is red’, ‘is wise’) and corresponding abstract nouns (‘redness’, ‘wisdom’) stand for. But on the Fregean approach, it makes no sense to suppose some one kind of thing to be the common referent of expressions of completely different logical types. This need not mean that there is no significant disagreement between medieval realists and nominalists; but it does mean that they misconceived the issue, or at least ran together questions we should separate. For it is one question whether abstract nouns are to be conceived as genuine singular terms, standing for objects, and a quite distinct question whether the corresponding predicates have reference – if so, then they stand, not for objects, but for properties (concepts, in Frege’s sense) (see Universals).

## **2. The abstract–concrete distinction**

Abstract objects can be neither seen nor heard, nor can they be tasted, felt or smelled. But for several reasons it would be unsatisfactory to take inaccessibility to sense-perception as the basis

of our distinction. Besides importing an unwanted relativity to human sensory faculties, it would fail to draw the distinction clearly, there being room for dispute over what should count as perceiving something. If the range of sense-perception is taken as including only what can be discerned with the naked organ, as it were, the condition for being concrete is clearly too restrictive. The range might be extended to allow for detection via more or less remote effects, but once the criterion is loosened in this way, the proposal slides into taking capacity for involvement in causal interactions as the mark of the concrete. This suggestion avoids the difficulties with a sensory-access criterion but, even if extensionally correct, does not go to the heart of the matter. We expect capacities in general to have some categorical basis. Why are concrete objects capable of causal interaction but abstract objects not? The answer, it would seem, should yield a more illuminating account of the distinction. Partly for this reason, a more promising account of the distinction sees lack of location in space or time as distinctive of the abstract – what cannot be anywhere, anywhen, cannot be a factor in the causal nexus.

Although it is widely endorsed and gives intuitively correct results in the cases to which philosophers have attended, this account is nevertheless flawed. This is because there are candidates for abstract status which, though plainly lacking spatial properties, are not wholly atemporal. In the sense in which two pairs of players at different chessboards may be said to be engaged in one and the same game, the game of chess is plausibly taken to be an abstract object; but while it is not located anywhere, it has not always existed, but was devised at a certain time. Other examples are natural languages, many if not all works of art, and words and letters in the type- as opposed to token-sense (roughly, the sense in which there are just six, not eight, distinct letters in the word ‘abstract’) (see Type/token distinction). Thus while the abstract–concrete distinction undoubtedly has much to do with spatiality and temporality, it does not seem straightforwardly identifiable with the distinction between what has spatial or temporal position and what has neither. An alternative proposal of considerable interest is that concrete objects are those which are, in principle, capable of being picked out ostensively, while abstract objects are those to which we can refer only by means of some functional expression (Dummett 1973: ch. 14). Thus we may pick out a particular tree by the words ‘That beech’, perhaps accompanying our utterance with a pointing gesture; but we cannot, for example, literally point to a certain shape or number – rather, we must refer to them as, say, the shape of such and such a vase or the number of eggs in the carton (Noonan 1976; Hale 1987: ch. 3).

### **3. Grounds for belief in abstract objects**

Many philosophers, appealing to Ockham’s Razor – the principle that entities should not be multiplied beyond necessity – deem it mortally sinful to believe in abstract objects unless such belief is unavoidable, but disagree about whether it is actually avoidable. Orthodox nominalists hope to avoid it by carrying through a programme of reductive paraphrase. However, in view of the resistance of various kinds of apparent reference to/quantification over abstract objects to elimination by reductive paraphrase or re-interpretation in concrete terms, this does not appear

feasible as a completely general means of escaping commitment to abstract objects (see Ontological commitment). This has led some philosophers to conclude that reference to and quantification over domains including abstract objects is indispensable to a fully adequate account of the world. There is a strong appearance that this is the case with reference to mathematical entities – numbers of various kinds, functions and more generally, sets. On the face of it, the natural sciences, and physics especially, require substantial use of arithmetic and analysis, and the latter in turn draws fairly heavily on set theory. This argument – known as the Quine–Putnam Indispensability Argument – provides, if accepted, a strong *indirect* reason for believing in numbers and sets at least: scientific theories require acceptance of mathematical theories, so that whatever reasons we have to believe that our best scientific theories are true is reason to accept mathematical theories, and so to believe in the abstract objects of which they speak.

This argument has been vigorously contested, particularly by Field (1980), who argues – in support of a new and highly unorthodox brand of nominalism – that there is, contrary to appearances, no need for mathematical theories to be true for their use in science to be justified. It is enough that such theories should have a certain strong kind of consistency property, which he calls ‘conservativeness’. Since a nominalist can accept mathematical theories as having this property without believing them to be true, they have no need to engage in any kind of reductive translation programme of the sort previously mentioned – they can simply use mathematical theories while denying that they are literally true, thereby avoiding commitment to the abstract objects their truth requires. Among the difficulties confronting this approach, one important assumption Field makes is worth highlighting. Field takes the Quine–Putnam argument to offer the only ground worth taking seriously for holding mathematical theories to be true, so that if he is able to undermine it, there remains no pressure to take on the ontological commitments they import. If Field’s assessment were correct, the best grounds we could have for believing maths and so on, would be indirect and a posteriori. But this assessment rests upon the challengeable assumption that the only statements we may justifiably accept on other-than-indirect a posteriori grounds are those directly ascertainable as true by observation. Perhaps we should take seriously, as he does not, the possibility that belief in the truth of mathematical statements and acceptance of their ontology may be warranted a priori.

#### **4. Grounds for disbelief**

Unquestionably the most important arguments against abstract objects are epistemological. One is that – in view of the presumed causal inertia of abstract objects – to construe statements of some given kind as having their truth-conditions constituted by states of affairs essentially involving such objects, puts those statements irretrievably beyond the reach of our knowledge. Crudely, if mathematical statements have Platonistic truth-conditions, we could not possibly know them to be true; since we do have mathematical knowledge, Platonism is false. In its simplest and earliest versions, this argument relies upon a very exacting form of causal theory of

knowledge, which takes it to be an invariably necessary condition for a thinker  $X$  to know that  $p$ , that  $X$ 's true belief that  $p$  should itself be caused by, or otherwise suitably causally related to, the fact that  $p$  (see Knowledge, causal theory of). A problem with this argument is that while such a strong condition (just how strong depends on how precisely the vague phrase 'suitable causal relation' is understood) may be satisfied in standard cases of perceptual and memory knowledge, it is very hard to see how it could be quite generally met, even when restricted in scope to ordinary empirical knowledge concerning perfectly concrete matters. Our inductively grounded belief that all aardvarks have bugs is, we may suppose, causally induced by inspection of a large and suitably varied contingent of bug-infested aardvarks – but there is no sort of causal relation, however complicated or attenuated, of which it may with any plausibility be claimed both that it holds between our general belief and the fact that all past, present or future aardvarks have bugs and that its holding is epistemically significant. If knowledge does not demand a suitable causal link in every case, the argument against Platonism collapses, at least in its present form.

A related argument alleges that no satisfactory sense can be made of the idea that we are capable of identifying reference to or thought about abstract objects. And once again, the argument in its simplest form rests upon an eminently challengeable assumption – in this case, that identifying reference or thought about a particular object *always* requires a suitable causal link between the speaker/thinker (or their utterance/thought) and the object in question (see Reference).

Opponents of Platonism may hope to fashion more sophisticated causal analyses of knowledge and reference which are strong enough to sustain versions of these objections without being so strong as to be independently objectionable, but none has yet come forth.

A more powerful epistemological objection appeals to the thought that, even if knowledge is not to be analysed in specifically causal terms, we should expect to be able to provide a naturalistic explanation of our tendency to get things right significantly more often than not, in any area where we are disposed to credit ourselves with a capacity for knowledge (see Reliabilism). In the absence of causal or other natural relations between ourselves and abstract objects, it is hard to see how any such credible explanation might run for any region of discourse whose statements are supposed to carry Platonistic truth-conditions. The argument relies on the assumption that ontological views are tenable only to the extent that they leave space for a credible epistemology. The arguments reviewed here confront Platonism with a strong challenge, even if they could not, by their very nature, tell decisively against it.

## References and further reading

- Dummett, M. (1973) *Frege: Philosophy of Language*, London: Duckworth.  
(Hard going, but the best introduction to Frege's approach to the analysis of language. Chapters 2, 4 and 14 are especially relevant.)
- Field, H. (1980) *Science without Numbers*, Oxford: Blackwell.

(Introduction and chapter 1 give a clear outline of the approach discussed in §3 above.)

Field, H. (1989) *Realism, Mathematics and Modality*, Oxford: Blackwell.

(Occasionally technically difficult, but generally very readable. Chapters 1 and 2 provide an excellent account of Field's overall position; chapter 7.2 develops the epistemological argument against Platonism mentioned in §4 above.)

Frege, G. (1884) *The Foundations of Arithmetic*, trans. J.L. Austin, Oxford: Blackwell, 1953.

(Essential reading for anyone wishing to pursue the present subject in depth. See especially §§60–8.)

Frege, G. (1892) 'On concept and object' in P. Geach and M. Black (eds) *Translations from the Philosophical Writings of Gottlob Frege*, Oxford: Blackwell, 1970.

(Especially relevant to §1 above. Provides a very readable informal elucidation of Frege's contrasted notions of object and concept, indicating their connection with his fundamental distinction, at the level of language, between proper names and predicates.)

Goodman, N. and Quine, W.V.O. (1947) 'Steps towards a constructive nominalism', *Journal of Symbolic Logic* 12: 105–122; repr. in N. Goodman *Problems and Projects*, Indianapolis, In and New York: Bobbs-Merrill, 1972, 173–198.

(Classic nominalist manifesto. Technically quite difficult after opening sections.)

Hale, R. (1987) *Abstract Objects*, Oxford: Blackwell.

(Along with Noonan (1976), chapter 3 develops a proposal to base the abstract–concrete distinction on the different kinds of identity criterion appropriate to the different kinds of object. Chapters 1–3 are relevant to §§1–2 above, chapter 5 to §3, and chapters 4, and 7 to §4.)

Hale, R. (1994) 'Is Platonism epistemologically bankrupt?', *Philosophical Review* 103 (2): 299–325.

(Defends Platonism against arguments discussed in §4 above.)

Hale, R. and Wright, C. (1993) 'Nominalism & the Contingency of Abstract Objects' *Journal of Philosophy* 89 (3): 111–135.

(Criticizes Field's version of nominalism.)

Lewis, D. (1987) *On the Plurality of Worlds*, Oxford: Blackwell.

(Very readable. Chapter 2 discusses problems with the abstract–concrete distinction.)

Noonan, H. (1976) 'Dummett on abstract objects', *Analysis* 36 (2): 49–54.

(Difficult but rewarding discussion, relevant to §2 above.)

Putnam, H. (1971) 'Philosophy of logic', in *Mathematics, Matter and Method: Philosophical Papers*, vol. 1, Cambridge: Cambridge University Press, 1979, 2nd edn.

(Technically difficult in places, but contains important material relevant to §3 above, including presentation of indispensability arguments.)

Quine, W.V.O. (1948) 'On what there is', in *From a Logical Point of View*, New York: Harper Torchbooks, 1961, 1–19.

(Classic paper on ontology, with strong nominalist sympathies.)

Wright, C. (1983) *Frege's Conception of Numbers as Objects*, Aberdeen: Aberdeen University Press.

(Excellent exposition and vigorous defence of Frege's position. Chapters 1 and 2 are especially relevant.)

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