



Linear Regression - Straight Line

The process of producing a mathematical function that provides the best fit to a series of experimental data points.

Equation of straight line:

$$y = m \cdot x + c$$

Experimental data

Raw data variables

(Independent variables)

(dependent variables)

<i>data</i> :=	1	0.5
	2	1.7
	3	3.4
	4	5.7
	5	8.4
	6	9.8
	7	11.8
	8	13.8
	9	15.8
	10	17.8

$$x := \text{col}(data, 1) =$$

1
2
3
4
5
6
7
8
9
10

$$y_d := \text{col}(data, 2) =$$

0.5
1.7
3.4
5.7
8.4
9.8
11.8
13.8
15.8
17.8

Total number of data points $n := \text{length}(x) = 10$

The sum of squares will allow a linear relationship between x and y to be calculated

Mean values for x, y, log(x) and log(y)

$$x_{mean} := \sum_{i=1}^n x_i \cdot n^{-1} = 5.5$$

$$y_{mean} := \sum_{i=1}^n y_{d_i} \cdot n^{-1} = 8.87$$

Least squares fitting - Sum of squares

$$SS_x := \left(\sum_{i=1}^n (x_i) \right) = 55$$

$$SS_y := \left(\sum_{i=1}^n (y_{d_i}) \right) = 88.7$$

$$SS_{xy} := \left(\sum_{i=1}^n (x_i \cdot y_{d_i}) \right) = 650.9$$

$$SS_{xsq} := \sum_{i=1}^n x_i^2 = 385$$

Linear Regression (Straight line)

$$y = m \cdot x + c$$

Slope of the line:

$$m := \frac{n \cdot SS_{xy} - SS_x \cdot SS_y}{n \cdot SS_{xsq} - (SS_x)^2} = 1.9764$$

Intercept of the line:

$$c := y_{mean} - m \cdot x_{mean} = -2$$



Check of results against the experimental data

Experimental data

Calculated values

$$x = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \end{bmatrix}$$

$$y_d = \begin{bmatrix} 0.5 \\ 1.7 \\ 3.4 \\ 5.7 \\ 8.4 \\ 9.8 \\ 11.8 \\ 13.8 \\ 15.8 \\ 17.8 \end{bmatrix}$$

$$y := m \cdot x + c = \begin{bmatrix} -0.0236 \\ 1.9527 \\ 3.9291 \\ 5.9055 \\ 7.8818 \\ 9.8582 \\ 11.8345 \\ 13.8109 \\ 15.7873 \\ 17.7636 \end{bmatrix}$$

$$f(x) := m \cdot x + c$$



Sum of squares of the residual

$$SS_{res} := \sum_{i=1}^n (y_{d_i} - c - m \cdot x_i)^2$$

Total sum of the squares

$$SS_{tot} := \sum_{i=1}^n (y_i - y_{mean})^2$$

R squared variance

$$R_{sq} := \frac{SS_{tot} - SS_{res}}{SS_{tot}} = 0.9971$$

Coefficient of determination

Result

Formula for straight line graph

$$y := m \cdot x + c$$

$$m = 1.9764$$

$$c = -2$$