



Non-linear Regression - Power Curve

The process of producing a mathematical function that provides the best fit to a series of experimental data points.

The equation for the power curve:

$$y = a \cdot x^b$$

Experimental data

Raw data variables

(Independent variables)

(dependent variables)

<i>data</i> :=	1	1.68
	5	8.39
	10	19.12
	15	33.41
	20	50.98
	25	71.63
	30	95.22
	35	121.64
	40	150.78
	45	182.57
	50	216.93
	55	253.80

$$x := \text{col}(data, 1) = \begin{bmatrix} 1 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ 45 \\ 50 \\ 55 \end{bmatrix}$$

$$y_d := \text{col}(data, 2) = \begin{bmatrix} 1.68 \\ 8.39 \\ 19.12 \\ 33.41 \\ 50.98 \\ 71.63 \\ 95.22 \\ 121.64 \\ 150.78 \\ 182.57 \\ 216.93 \\ 253.8 \end{bmatrix}$$

Total number of data points $n := \text{length}(x) = 12$

finding the logs of the data will allow a linear relationship between x and y to be used

Mean values for x, y, log(x) and log(y)

$$x_{mean} := \sum_{i=1}^n x_i \cdot n^{-1} = 27.5833$$

$$y_{mean} := \sum_{i=1}^n y_{d_i} \cdot n^{-1} = 100.5125$$

$$\ln x_{mean} := \sum_{i=1}^n \ln(x_i) \cdot n^{-1} = 2.9338$$

$$\ln y_{mean} := \sum_{i=1}^n \ln(y_{d_i}) \cdot n^{-1} = 3.9837$$

Least squares fitting - Sum of squares

$$SS_{\ln x} := \sum_{i=1}^n \ln(x_i) = 35.2061$$

$$SS_{\ln y} := \sum_{i=1}^n \ln(y_{d_i}) = 47.8047$$

$$SS_{\ln xy} := \sum_{i=1}^n \ln(x_i) \cdot \ln(y_{d_i}) = 159.3691$$

$$SS_{xsq} := \sum_{i=1}^n \ln(x_i)^2 = 118.2311$$

$$SS_x := \left(\sum_{i=1}^n x_i^2 \right) - n \cdot x_{mean}^2 = 3520.9167$$



Non-linear Regression (linearisation of a non-linear function)

$$b := \frac{n \cdot SS_{lnxy} - SS_{lnx} \cdot SS_{lny}}{n \cdot SS_{xsq} - SS_{lnx}} = 1.2795$$

$$a_0 := lny_{mean} - b \cdot lnx_{mean} = 0.2300$$

Linearisation of
Non-linear function

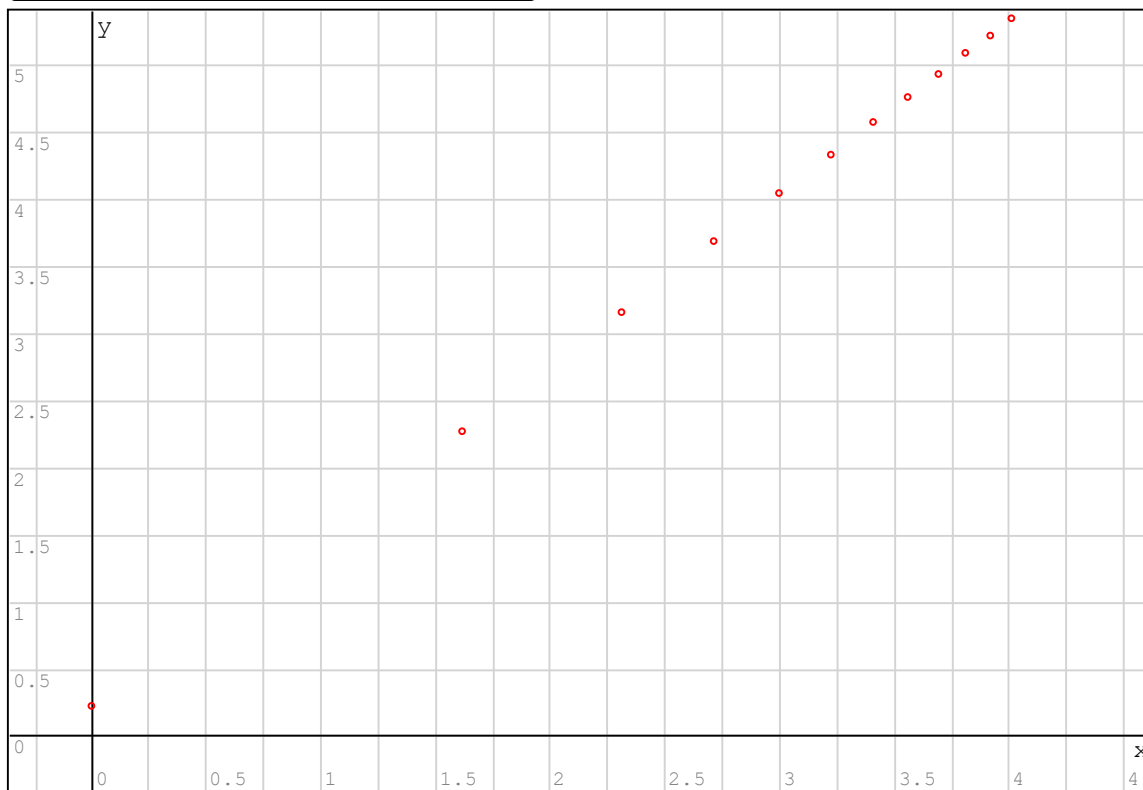
for $i \in [1..n]$
 $x_{ln_i} := \ln(x_i)$

for $i \in [1..n]$
 $y_{ln_i} := a_0 + b \cdot \ln(x_i)$

$$x_{ln} = \begin{bmatrix} 0 \\ 1.6094 \\ 2.3026 \\ 2.7081 \\ 2.9957 \\ 3.2189 \\ 3.4012 \\ 3.5553 \\ 3.6889 \\ 3.8067 \\ 3.912 \\ 4.0073 \end{bmatrix}$$

$$y_{ln} = \begin{bmatrix} 0.23 \\ 2.2892 \\ 3.176 \\ 3.6948 \\ 4.0629 \\ 4.3484 \\ 4.5817 \\ 4.7789 \\ 4.9498 \\ 5.1005 \\ 5.2353 \\ 5.3572 \end{bmatrix}$$

Linearisation of non-linear function
red points show experimental data





Non-linear Regression (nth power curve)

$$y = a \cdot x^b$$

$$a := \frac{\sum_{i=1}^n y_{d_i}}{\sum_{i=1}^n x_i^b} = 1.34025$$

$$b = 1.2795$$

see linearisation above

for $i \in [1..n]$
 $y_i := a \cdot x_i^b$

Check of results against the experimental data

$$x = \begin{bmatrix} 1 \\ 5 \\ 10 \\ 15 \\ 20 \\ 25 \\ 30 \\ 35 \\ 40 \\ \vdots \end{bmatrix}$$

$$y_d = \begin{bmatrix} 1.68 \\ 8.39 \\ 19.12 \\ 33.41 \\ 50.98 \\ 71.63 \\ 95.22 \\ 121.64 \\ 150.78 \\ \vdots \end{bmatrix}$$

Calculated values

$$y = \begin{bmatrix} 1.3402 \\ 10.5074 \\ 25.5067 \\ 42.8507 \\ 61.9175 \\ 82.3772 \\ 104.02 \\ 126.699 \\ 150.3045 \\ \vdots \end{bmatrix}$$

$$f(x) := a \cdot x^b$$



Sum of squares of the residual

$$SS_{res} := \sum_{i=1}^n \left(y_{d_i} - a \cdot (x_i)^b \right)^2$$

Total sum of the squares

$$SS_{tot} := \sum_{i=1}^n \left(y_i - y_{mean} \right)^2$$

R squared variance

$$R_{sq} := \frac{SS_{tot} - SS_{res}}{SS_{tot}} = 0.9746$$

Result for power curve

$$y := a \cdot x^b$$

$$a = 1.3402$$

$$b = 1.2795$$