

Towards a spectral element framework for tetrahedral meshes using entropy stable two-point flux formulations

Oriol Lehmkuhl* and Jordi Muela*

* Barcelona Supercomputing Center (BSC-CNS)
Plaza Eusebi Güell 1-3, 08034 Barcelona, Spain
e-mail: oriol.lehmkuhl@bsc.es, web page: <http://www.bsc.es>

ABSTRACT

High order spectral element methods based on summation by parts operators and split form discretizations have become a cornerstone for the development of robust and low dissipation solvers for incompressible and compressible flows. The SOD2D framework was originally conceived as a three dimensional spectral element code on unstructured hexahedral meshes, with a strong emphasis on energy consistent formulations, entropy stability and the systematic use of two point numerical fluxes [1].

In the context of the mini symposium on advances in scale-resolving simulation for turbomachinery, this contribution presents the extension of SOD2D towards fully unstructured tetrahedral meshes using a nodal spectral element formulation on simplicial elements. The proposed approach leverages recent developments in tensor product spectral element operators with the summation by parts property on curved triangles and tetrahedra [2], enabling the construction of metric consistent discretizations that preserve the mimetic and stability properties of the original hexahedral framework.

For both incompressible and compressible governing equations, we formulate a unified SEM scheme based on split forms and symmetric two-point fluxes that ensure discrete conservation and entropy stability at the semi discrete level. Attention is devoted to the design of low dissipation stabilisation mechanisms that act selectively on the antisymmetric part of the numerical flux, allowing the method to remain essentially non dissipative in smooth regions while providing sufficient robustness near under resolved gradients and discontinuities.

The resulting tetrahedral spectral element solver is assessed on a series of benchmark problems, including vortex dominated incompressible flows and shock containing compressible configurations. Numerical results demonstrate that the proposed methodology achieves stability and accuracy levels comparable to structured element implementations, while significantly enhancing geometric flexibility. This work contributes to bridging the gap between entropy stable high order formulations and practical simulations on complex industrial geometries within a unified spectral element framework.

REFERENCES

- [1] L. Gasparino, F. Spiga and O. Lehmkuhl. *SOD2D: A GPU-enabled spectral finite elements method for compressible scale-resolving simulations*. Computer Physics Communications, Vol. 297, pp. 109067, 2024.
- [2] T. Montoya and D. W. Zingg. *Efficient tensor-product spectral-element operators with the summation-by-parts property on curved triangles and tetrahedra*. SIAM Journal on Scientific Computing, Vol. 46, No. 4, pp. A2270–A2297, 2024.38, pp. 198-209, 2007.