

GCSE Maths

GRADE BOOSTER HIGHER
Revision Workbook

ANSWERS

tutor2u 

1.1 Multiples, Factors, LCM, HCF and Prime Factors

A) Key Words Crossword

Across: 2) Highest; 3) Lowest; 4) Overlap; 7) Union

Down: 1) Factors; 5) Venn; 6) Prime

B) Questions

1) 2, 7, 19, 37, 61, 89

2. a) Factors of 15 = 1, 3, 5, 15

Multiples of 15 = 15, 30, 45, 60, 75

Factors of 18 = 1, 2, 3, 6, 9, 18

Multiples of 18 = 18, 36, 54, 72, 90

2. b) LCM = 90

HCF = 3

3.) 180

2

2

3

45

3

5

$2^3 \times 3^2 \times 5$

C) Using Set Diagrams to find the LCM and HCF

1) $a = 2, b = 2, c = 1$

2) LCM = 630, HCF = 21

D) EXAM QUESTIONS

1.) a) 84

b) 6

2.) 1.00 PM

1.2 Calculations with Fractions

A) Methods

Instructions

| | | |
|---|---------------------------|--|
| Find the lowest common denominator | Multiply the numerators | Keep the first fraction the same |
| Find the equivalent fractions | Multiply the denominators | Flip the second fraction |
| Calculate (do not add or subtract the denominators) | Simplify | Change the problem to a multiplication, then calculate |
| Simplify | | Simplify |

B) Mixed Numbers & Improper Fractions

Correct answers (circled):

Corrections:

$$2\frac{3}{5} = \frac{13}{5}$$

$$3\frac{2}{5} = \frac{17}{5}$$

$$2\frac{3}{4} = \frac{11}{4}$$

$$\frac{8}{3} = 2\frac{2}{3}$$

$$\frac{12}{5} = 2\frac{2}{5}$$

$$\frac{29}{6} = 4\frac{5}{6}$$

C) Multiplying & Dividing Fractions

$$\frac{7}{12} \quad 2\frac{11}{12} \quad 1\frac{7}{8} \quad 9\frac{27}{32}$$

D) Adding & Subtracting Fractions

a) $1\frac{5}{8}$ b) $\frac{1}{24}$ c) $3\frac{19}{24}$

EXAM QUESTION

$$\frac{1}{18}$$

1.3 Recurring Decimals

A) Terminating and Recurring Decimals

| | | | |
|---|--------------------|---|------------------------|
| R | $0.\dot{1}\dot{2}$ | T | 0.2 |
| T | 0.375 | R | $0.\dot{4}\dot{5}$ |
| R | $0.\dot{8}\dot{7}$ | R | $0.\dot{2}8571\dot{4}$ |

B) Changing Recurring Decimals into Fractions

a) $\frac{2}{3}$

b) $\frac{25}{33}$

c) $\frac{29}{45}$

d) $\frac{613}{900}$

C) EXAM QUESTION

$$\frac{34}{165}$$

1.4 Power, Roots and Indices

A) Calculations

a) $3^3, 27$

b) $7^{-2}, \frac{1}{49}$

c) $15^2, 225$

d) $0.5^{-2}, 4$

e) $3^{-2}, \frac{1}{9}$

B) Negative Indices

a) $\frac{1}{7}$

b) $\frac{1}{16}$

c) $\frac{1}{25}$

d) $\frac{1}{27}$

e) $\frac{1}{64}$

C) Fractional Indices

a) 8

b) 5

c) $\frac{1}{3}$

d) 16

e) 8

D) Negative and Fractional Indices

a) $\frac{1}{4}$

b) $\frac{1}{8}$

c) $\frac{7}{2}$

d) 32

e) $\frac{64}{27}$

EXAM QUESTION

a) 4

b) $\frac{1}{9}$

c) $\frac{4}{9}$

1.5 Standard Form

A) Ordinary Numbers into Standard Form and Vice Versa

1. a) 1.4×10^{-2}
- b) 7.6×10^3
- c) 1.23×10^{-4}
- d) 1.5×10^5

2. a) 0.0065
- b) 2415
- c) 0.0000654
- d) 0.000212

B) Calculating with Standard Form

- a) 2.823×10^3
- b) 5.706×10^4
- c) 3.84×10^{-3}
- d) 3.2×10^{-3}
- e) 3×10^2
- f) 2×10^6

C) EXAM QUESTIONS

- 1.) 1.2915×10^{11}
- 2.) 6.6×10^2

1.6 Surds

A) Simplifying Surds

a) $4\sqrt{3}$

b) $5\sqrt{5}$

c) $5\sqrt{2}$

d) $6\sqrt{2}$

B) Manipulating Surds

$$\sqrt{a \times b}$$

$$\frac{\sqrt{a}}{\sqrt{b}}$$

do nothing!

$$a^2 + 2a\sqrt{b} + b$$

$$a^2 - b$$

$$\frac{a}{\sqrt{b}} \times \frac{\sqrt{b}}{\sqrt{b}}$$

$$\frac{a}{b \pm \sqrt{c}} \times \frac{b \mp \sqrt{c}}{b \mp \sqrt{c}}$$

Questions

a) 4

b) $\sqrt{5}$

c) $2\sqrt{2} - 2$

d) $4\sqrt{3} + 7$

EXAM QUESTIONS

1.) $8 + 2\sqrt{6}$

2.) a) $\frac{5\sqrt{2}}{2}$

b) $\frac{6 + 2\sqrt{2}}{7}$

2.1 Substitution into Formulae

A) Substitution into Circle Formulae

$$\text{Area of circle} = \pi r^2$$

$$\text{Circumference of circle} = \pi d$$

$$\text{Length of arc} = \frac{\theta}{360} \pi d$$

$$\text{Area of sector} = \frac{\theta}{360} \pi r^2$$

| | |
|---|-----------------------|
| Area of a circle, $r = 7$ | 47.0 cm |
| Perimeter of a Semi-Circle, $d = 10$ | 25.7 cm |
| Area of a Sector, $r = 14, \theta = 60^\circ$ | 153.9 cm ² |
| Perimeter of a Sector, $r = 11, \theta = 130^\circ$ | 102.6 cm ² |

B) Equations of Motion

a) $v = -17$

b) $s = 260$

c) $u = \pm 12.5$ (1 dp)

C) EXAM QUESTION

£28,500

2.2 Algebraic Indices

A) Laws of Indices

$$b^3 \times b^{-4} \neq b^1$$

$$a^3 \div a^{-5} \neq a^{-2}$$

$$4^2 \neq 4$$

$$a^0 \neq 0$$

B) Simple Equations

| | |
|-----------------|---|
| $x^4 = 16$ | 4 |
| $3^x = 27$ | 3 |
| $2^{x+1} = 32$ | 1 |
| $2^{2x} = 1024$ | 2 |
| $x^4 = x^5$ | 5 |

EXAM QUESTION

$$\frac{16b^9}{3a^2}$$

C) Harder Equations

a) $x = -5$

b) $x = 2$

EXAM QUESTION

$$x = 3$$

2.3 Straight Line Graphs

A) Plotting Straight Line Graphs

$$y = 3x - 4$$

| | | | | |
|-----|----|----|--|---|
| -10 | -7 | -4 | | 2 |
|-----|----|----|--|---|

$$y = \frac{1}{2}x + 2$$

| | | | | |
|---|-----|---|-----|--|
| 3 | 2.5 | 2 | 1.5 | |
|---|-----|---|-----|--|

Correctly drawn graphs.

B) Finding the Equation of a Straight Line

Gradient line 1: $m = 1.5$

Gradient line 2: $m = -2$

Equation Line 1: $y = 1.5x - 2$

Equation Line 2: $y = -2x + 3$

C) Parallel and Perpendicular Lines

| |
|---------------|
| Neither |
| Perpendicular |
| Parallel |
| Perpendicular |

EXAM QUESTION

a) $y = 2x - 11$

b) $y = -\frac{1}{2}x - 3.5$

2.4 Simultaneous Equations – 1

A) Basic Simultaneous Equations

| | | |
|----------------------------------|-----------|-----------------|
| $x + y = 5$ $2x + y = 9$ | $3y = 15$ | $x = 4, y = 1$ |
| $2x + 2y = 14$ $x - 2y = -8$ | $x = 4$ | $x = 4, y = -1$ |
| $3x + 2y = 10$ $3x - 5y = 17$ | $7y = -7$ | $x = 2, y = 5$ |

B) Solving Simultaneous Equations Graphically

Correctly drawn graph.

$$x = -1, y = 3$$

C) Solving Harder Simultaneous Equations

a) $x = 3, y = 4$

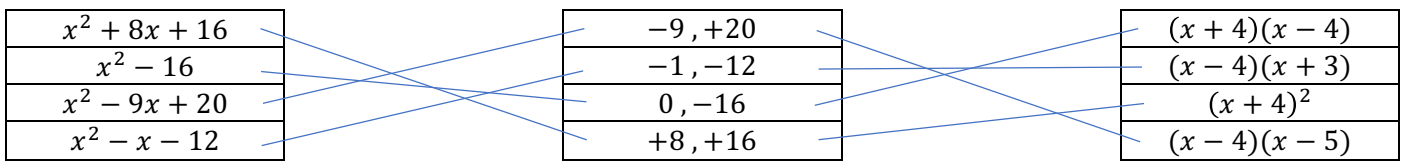
b) $x = -1, y = 3$

EXAM QUESTION

£2.30

2.5 Factorising and Solving Quadratic Equations

A) Factorising Quadratic Expressions



B) Solving Quadratic Equations

- a) $x = 7$ and -7
- b) $x = 8$ and 4
- c) $x = -3$ and 8

C) Solving Harder Quadratic Equations

$$2x^2 - 3x - 20 = 0 \quad \text{S D}$$
$$3x^2 + 14x + 8 = 4x \quad \text{I A}$$
$$4x^2 - 4x = 15 \quad \text{U R}$$

RADIUS

EXAM QUESTION

- a) Correctly shown.
- b) 64 cm^3

2.6 The Quadratic Formula

A) Substitution into the Quadratic Formula

- a) $x = -1.27$ and 6.27
- b) $x = 3.12$ and -1.12

B) The Discriminant and Roots

- a) One repeated root
- b) No real roots
- c) Two distinct roots

C) Common Errors

$$x = \frac{-5 \pm \sqrt{(-5)^2 - 4 \times 1 \times 4}}{2 \times 1}$$

First -5 should be 5 , $-(-5) = 5$

$$x = \frac{-3 \pm \sqrt{-3^2 - 4 \times 3 \times 5}}{2 \times 3}$$

Equation needs putting equal to zero before using quadratic formula to solve:

$$x^2 - 3x - 5 = 0$$

First -3 should be 3 , $-(-3) = 3$

-3^2 should be $(-3)^2$

5 should be -5

EXAM QUESTION

$$x = 0.17 \text{ and } -2.37$$

2.7 Completing The Square

A) Writing Expressions in Completed Square Form

| | |
|-----------------|-----------------------|
| $x^2 + 6x - 4$ | $(x - 9)^2 - 74$ |
| $x^2 - 6x + 12$ | $(x - 3)^2 + 3$ |
| $x^2 + 9x + 4$ | $(x + 3)^2 - 13$ |
| $x^2 - 18x + 7$ | $(x + 4.5)^2 - 16.25$ |

B) Finding the Minimum Point and Sketching

a) $(x - 1)^2 - 3 = 0$

b) $x = 1 \pm \sqrt{3}$

c) $(1, -3)$

d) -2

e) Correct sketch.

EXAM QUESTION

$$a = 8$$

$$b = -28$$

2.8 Rearranging Formulae

A) Multi-Step Formulae

| | |
|---------------------|----------------------------|
| $y = 2x - 4$ | $x = \sqrt{\frac{y-4}{2}}$ |
| $y = \sqrt{2x+4}$ | $x = \frac{y^2-4}{2}$ |
| $y = \sqrt{2x} + 4$ | $x = \frac{y+4}{2}$ |
| $y = 2x^2 + 4$ | $x = \frac{\sqrt{y-4}}{2}$ |
| $y = (2x+4)^2$ | $x = \frac{(y-4)^2}{2}$ |

B) Worded Problems Involving Equations of Motion

- a) 2 m/s
b) 112.5 m

C) Rearranging Formulae where the Subject Appears Twice

| | |
|---|---|
| Trash, should be $a = -\frac{3b}{5}$ | Trash, should be $a = \frac{3b-9}{6-b}$ |
| Trash, should be $a = \frac{4-3b}{b-1}$ | Tick |

EXAM QUESTION

$$b = \frac{2a-4}{a+3}$$

2.9 Non-Linear Graphs

A) Plotting Quadratic Graphs

| | | | | |
|---|--|----|--|---|
| 3 | | -5 | | 3 |
|---|--|----|--|---|

Correctly plotted curve.

$$x = -1.6 \text{ and } 1.6$$

B) Solving Quadratic Equations using Quadratic Graphs

Suitable lines drawn.

a) $x = -2.3$ and 1.3

b) $x = -2.2$ and 2.2

c) $x = -3.6$ and 1.6

C) Recognising Graphs

| | |
|-----------------------|---|
| $y = x^2 + 6x + 10$ | B |
| $y = -x^2 - 4x - 3$ | A |
| $y = \frac{2}{x}$ | D |
| $y = x^3 + 6x^2 + 8x$ | C |

EXAM QUESTION

$$p = -8$$

$$q = 22$$

2.10 Simultaneous Equations – 2

A) Algebraic Methods

$$\begin{aligned} \text{a) } x &= -1, y = 3 \\ x &= 2, y = 6 \end{aligned}$$

$$\begin{aligned} \text{b) } x &= 0, y = 5 \\ x &= -4, y = -3 \end{aligned}$$

B) Graphical Methods

$$\begin{aligned} x &= 2, y = 5 \\ x &= -2, y = 1 \end{aligned}$$

C) Solving Simultaneous Equations

$$\begin{aligned} x &= -2.2, y = 3.3 \\ x &= 3.2, y = -2.4 \end{aligned}$$

EXAM QUESTION

$$\begin{aligned} x &= 2, y = 12 \\ x &= -\frac{2}{3}, y = 4 \end{aligned}$$

2.11 The Equation of a Circle

A) Equation of a Circle

Correctly drawn circle with centre (0, 0) and radius 4

B) Tangents to a Circle

| | | |
|----------------|----------------|----------------------------------|
| $\frac{4}{3}$ | $-\frac{3}{4}$ | $y = -\frac{3}{4}x + 6.25$ |
| $-\frac{3}{4}$ | $\frac{4}{3}$ | $y = \frac{4}{3}x + \frac{7}{3}$ |
| $-\frac{4}{3}$ | $\frac{3}{4}$ | $y = \frac{3}{4}x - 6.25$ |

EXAM QUESTION

$$5x + 2y = 29$$

2.12 Algebraic Fractions

A) Simplifying Algebraic Fractions

| | |
|---------------------------------------|----------------------------|
| $\frac{3}{x-4} + \frac{2}{x+2}$ | $\frac{12}{(x-4)(x+2)}$ |
| $\frac{2}{x-4} - \frac{3}{x+2}$ | $\frac{12}{(4-x)(x+2)}$ |
| $\frac{-3}{x-4} \times \frac{4}{x+2}$ | $\frac{-x+16}{(x-4)(x+2)}$ |
| $\frac{6}{x-4} \div \frac{x+2}{2}$ | $\frac{5x-2}{(x-4)(x+2)}$ |

B) Solving Equations with Numerical Denominators

| | | |
|---|-------------------------|-------------------------|
| $\frac{15(2x+4)}{3} - \frac{15(3x-1)}{5} = 2$ | $5(2x+4) - 3(3x-1) = 2$ | $10x + 20 - 9x - 3 = 2$ |
| | Follow through | Follow through |
| | | $x + 17 = 2$ |
| | | $x = -15$ |

Correct solution $x = 7$

C) Solving Equations with Algebraic Denominators

- 1.) (i) Correctly shown.
(ii) $x = -3$ and 2
- 2.) (i) Correctly shown.
(ii) $x = 1.93$ and -2.76 (2 dp)

EXAM QUESTION

Correctly shown.

2.13 Transforming Graphs

A) Basic Transformations

1.)

| | | | | | |
|---|---|--|----|----|--|
| 6 | 1 | | -3 | -2 | |
|---|---|--|----|----|--|

Correctly drawn curve

2.) Correctly drawn curve

$(0, -5)$

3.) Correctly drawn curve

$(-1, -3)$

B) Transforming Graphs in the Form $y = f(x)$

1.) a) $(4, 3)$

b) $(2, 6)$

2.) Correctly drawn curve.

a) Correctly drawn curve (reflection of $y = f(x)$ in the x-axis).

b) Correctly drawn curve (reflection of $y = f(x)$ in the y-axis).

C) Recognising Transformations

1.) a) $\begin{pmatrix} 0 \\ 4 \end{pmatrix}$

b) $y = x^2 + 2x + 1$

2.) a) $y = 0$

b) $y = -x^2 - 2x + 3$

EXAM QUESTION

a) $y = (x - 4)^2 + 2$

b) $y = -x^2 + 8x - 16$ or $y = -(x - 4)^2$

3.1 Percentages – 1

B) Applying Percentage Change

Ben & Jerry's £4.40
Pot Noodle £1.04
Beef Mince £2.97
Chicken Thighs £2.24
Marmite £2.64
Shower Gel £1.14

C) Calculating Percentage Change

33.3%
increase

25%
decrease

| | | | |
|----------|----------|----------|----------|
| 45.45% | 83.3% | 20% | 25% |
| decrease | increase | decrease | increase |

9.09%
decrease

10%
increase

D) Finding Profit and Loss

20.0% profit 48.5% profit 28.7% loss

EXAM QUESTION

1.) £200

2.) £12.51

3.2 Percentages – 2

A) Compound Interest

Multiplier Match-Up

| | |
|-----------------|------|
| Increase by 6% | 1.94 |
| Decrease by 6% | 1.06 |
| Increase by 94% | 0.94 |
| Decrease by 94% | 0.06 |

Question

- (a) i. £77416.04
ii. £155776.28
iii. £66921.60
(b) 7 years

B) Growth & Decay

- (a) i. $C_1 = 202.5$
ii. $C_4 = 107.6$
(b) $k = 7$ minutes

EXAM QUESTION

Account A £1966.91
Account B £1941.86
Account A gives best return

3.3 Speed, Density and Pressure

B) Speed, Distance and Time: The Planets

| | | |
|-------|-----|--------|
| | | 27,462 |
| | 687 | |
| | | 34,720 |
| 4,497 | | |

EXAM QUESTION

1.) Lower bound calculation

$$\frac{147.5}{67.5} = 2.185 = 2 \text{ hours } 11 \text{ mins}$$

$$7:50 \text{ am} + 2 \text{ hours } 11 \text{ mins} = 10:01 \text{ am}$$

Upper bound calculation

$$\frac{152.5}{62.5} = 2.44 = 2 \text{ hours } 26 \text{ mins}$$

$$7:50 \text{ am} + 2 \text{ hours } 26 \text{ mins} = 10:16 \text{ am}$$

Luke will definitely be late, his earliest possible arrival time is 10:01 am. His latest possible arrival time is 10:16 am.

C) Density, Mass and Volume

| | | | | |
|----------------------|------------------------|-----------------------|---------|------------------------|
| | 3.6 kg/cm ³ | | 1.4 kg | |
| 0.70 cm ³ | | | | 300 cm ³ |
| | | 360 m ³ | | |
| 32 g | 710 kg | | | 0.83 g/cm ³ |
| | | 850 kg/m ³ | 1100 kg | |
| | | | | |

EXAM QUESTION

2.) Cuboid

Density 1.2 g/cm³

Volume 480 cm³

Mass 576 g

Cylinder

Mass 920 g

Volume 502.7 cm³ (1dp)

Density 1.8 g/cm³ (1dp)

John is correct. Mass of cylinder (920 g) is greater than mass of cuboid (576 g).

Becki is incorrect. Density of cuboid (1.2 g/cm³) is less than density of cylinder (1.8 g/cm³).

D) Pressure, Mass and Area

a) 3.3 N

b) 141 N/m²

EXAM QUESTION

3.) 1200 N/m²

3.4 Direct and Inverse Proportion

A) Direct or Inverse

| | |
|------------|------------|
| a) Inverse | d) Inverse |
| b) Direct | e) Direct |
| c) Inverse | f) Direct |

B) Proportionality Statements

| | |
|---------------------------|---------------------|
| $r \propto y^2$ | $r = ky^2$ |
| $r \propto \sqrt{y}$ | $r = k\sqrt{y}$ |
| $r \propto \frac{1}{y^3}$ | $r = \frac{k}{y^3}$ |
| $r \propto \frac{1}{y^2}$ | $r = \frac{k}{y^2}$ |

C) Graphs

A Direct

B Direct

C Inverse

D) Ordering Activity

| |
|---|
| P and Q are directly proportional. If P = 30 when Q = 6, find P when Q = 8 |
| $P \propto Q$ |
| $P = kQ$ |
| $30 = k \times 6$ |
| $k = 5$ |
| $P = 5Q$ |
| $P = 5 \times 8$ |
| $P = 40$ |

y is inversely proportional to x.
y = 6 when x = 1.5, find x when y = 2

$$y \propto \frac{1}{x}$$

$$y = \frac{k}{x}$$

$$6 = \frac{k}{1.5}$$

$$k = 9$$

$$y = \frac{9}{x}$$

$$2 = \frac{9}{x}$$

$$x = 4.5$$

Practice Questions

- 1.) a) $r = 63$
b) $s = 10$

- 2.) a) $y = 0.16$
b) $x = 0.89$ (2 dp)

EXAM QUESTION

6 minutes

3.5 Real Life Graphs

A) Distance-Time Graphs

- a) Correctly sketched.
- b) 9 km

B) Velocity-Time Graphs

- a) Correctly drawn.
- b) 138 m
- c) 0.25 m/s^2

EXAM QUESTION

- 1.) No, she travelled 300 m

C) Rates of Change

- 1. $\dot{6} \text{ cm/s}$

Areas under Curves

- a) 19.5
- b) Underestimate. Most of area calculated lies under the curve.

EXAM QUESTION

- 2.) a) Steepest gradient between 55 and 60 seconds.
- b) $0.104 \text{ m}^3/\text{s}$ per second ± 0.004

4.1 Congruence and Similarity

A) Congruent Triangles

ASA SSS RHS SAS AAS

a and i ASA
b and j SSS
c and h ASA
d and f SAS
e and g RHS

B) Similar Triangles

a) i) LSF = 2.5 or 0.4 ii) $x = 20$ cm $y = 7.2$ cm
b) i) LSF = 0.75 or $\frac{4}{3}$ ii) $x = 12$ m $y = 6.75$ m
c) i) LSF = 1.4 or $\frac{5}{7}$ ii) $x = 17.64$ cm $y = 10$ cm

C) Reasoning and Proof

STEP 1: Angles in a triangle sum to 180°
STEP 2: $70^\circ + 30^\circ = 100^\circ$
STEP 3: Angle CDF = $180^\circ - 100^\circ = 80^\circ$
STEP 4: Angle EDF = $80^\circ - 50^\circ = 30^\circ$
STEP 5: $\triangle DEF$ is isosceles
STEP 6: $DE = EF = 8$ cm
STEP 7: $LSF = \frac{10}{8} = 1.25$
STEP 8: $x = 7.37 \times 1.25 = 9.21$ cm (2 dp)
STEP 9: $y = 8.15 \div 1.25 = 6.52$ cm

EXAM QUESTION

a) Angle BDA = XDC = 90° as DA is a perpendicular bisector.

Angle DCX = 30° as CX is an angle bisector and angles in an equilateral triangle are each 60° , angles in a triangle sum to 180° hence angle CXD = $180^\circ - 90^\circ - 30^\circ = 60^\circ$

Angle ABD = 60° (equilateral triangle) hence angle DAB = $180^\circ - 90^\circ - 60^\circ = 30^\circ$ (angles in a triangle)

So, angle DCX = angle DAB = 30° , angle CXD = angle ABD = 60° , and angle BDA = angle XDC = 90° , hence by the AAA rule, the two triangles are similar.

b) AD and CD are corresponding sides

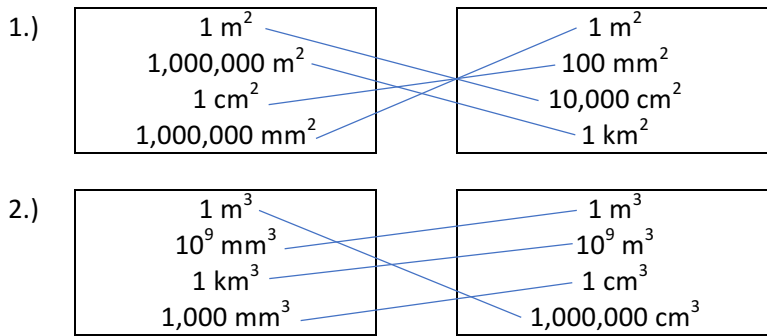
DC = 1 cm (D is mid-point of BC and BC = AB = 2 cm, isosceles triangle)

AD = $\sqrt{3}$, hence the LSF = $\sqrt{3} \div 1 = \sqrt{3}$

BD and XD are corresponding sides. BD = DC = 1 cm. hence $XD = \frac{1}{\sqrt{3}}$

4.2 Length, Area and Volume Scale Factors

A) Area and Volume Unit Conversions



B) LSF, ASF and VSF

- a) i) 0.33 m^2 ii) 3300 cm^2
b) 128 cm^3

EXAM QUESTIONS

- 1.) a) 12 cm
 b) $150\pi \text{ cm}^3$
- 2.) 2500 g

4.3 Circle Theorems

A) Recalling and Naming Theorems

- 1.) The angle at the centre of a circle is twice the angle at the circumference
- 2.) Angles in the same segment are equal
- 3.) The angle in a semi-circle is 90 degrees
- 4.) Opposite angles in a cyclic quadrilateral add up to 180 degrees
- 5.) The perpendicular bisector of a chord passes through the centre of the circle
- 6.) A tangent and radius meet at 90 degrees
- 7.) Tangents from the same point are the same length
- 8.) The alternate segment theorem

B) Seeing the Wood for the Trees

- | | | | |
|-----|---|-----|---|
| 1.) | • Angles in the same segment are equal | 2.) | • The angle in a semi-circle is 90 degrees |
| | | | • Opposite angles in a cyclic quadrilateral add up to 180 degrees |
| | | | • The angle at the centre of a circle is twice the angle at the circumference |
| 3.) | • The angle at the centre of a circle is twice the angle at the circumference | 4.) | • Angles in the same segment are equal |
| | • A tangent and radius meet at 90 degrees | | • The angle at the centre of a circle is twice the angle at the circumference |
| | • Tangents from the same point are the same length | | |
| 5.) | • Tangents from the same point are the same length | 6.) | • The perpendicular bisector of a chord passes through the centre of the circle |
| | • The alternate segment theorem | | |

EXAM QUESTIONS

- 1.) a) i) 40° ii) The angle at the centre of a circle is twice the angle at the circumference
b) i) 10° ii) Opposite angles in a cyclic quadrilateral add up to 180° , base angles in an isosceles triangle are equal
- 2.) a) 35°
b) $\angle WXY = 110^\circ$ due to the alternate segment theorem. Triangle WXY is isosceles, base angles are equal and the total angle sum is 180° .

4.4 Pythagoras' Theorem and Trigonometry

A) Pythagoras' Theorem

- 1) **10 cm**
- 2) **No** ($\sqrt{85} \neq \sqrt{81}$)
- 3) **a) 5.7 cm b) $4\sqrt{2}$**
- 4) **24.1 m (1dp)**

B) Trigonometry

- 1) 36.3°
- 2) 6.8 m
- 3) 11.4 m
- 4) a) 33.6° b) 17 m

C) Common Trigonometric Values

| | 0° | 30° | 45° | 60° | 90° |
|----------|-----------|----------------------|----------------------|----------------------|------------|
| $\sin x$ | 0 | $\frac{1}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{3}}{2}$ | 1 |
| $\cos x$ | 1 | $\frac{\sqrt{3}}{2}$ | $\frac{1}{\sqrt{2}}$ | $\frac{1}{2}$ | 0 |
| $\tan x$ | 0 | $\frac{1}{\sqrt{3}}$ | 1 | $\sqrt{3}$ | ∞ |

EXAM QUESTIONS

- 1) 8.5 m
- 2) a) 6.1 km b) 059°

4.5 The Sine & Cosine Rules and Using $Area = \frac{1}{2}ab \sin C$

A) The Sine Rule

- a) 4.2 cm
- b) 39.3°

B) The Cosine Rule

- a) 13.6 m
- b) 78.5°

C) $Area = \frac{1}{2}ab \sin C$

- a) 143.3 m^2
- b) 30°

EXAM QUESTION

- a) 8.51 cm
- b) 12.4 cm

4.6 Vectors

A) Parallel Vectors

- a) \parallel b) \parallel c) \nparallel
d) \parallel e) \parallel f) \nparallel

B) Finding Vectors

- a) $\mathbf{a} + 2\mathbf{b}$ b) $\mathbf{a} + 2\mathbf{b}$ c) $2\mathbf{b} - \mathbf{a}$ d) $\mathbf{b} - 3\mathbf{a}$
e) $\mathbf{b} - 2\mathbf{a}$ f) $2\mathbf{a} + 4\mathbf{b}$ g) $\mathbf{a} + 2\mathbf{b}$ h) 0

C) Question

$$\overrightarrow{XY} = \mathbf{a} + \mathbf{b}$$

$$\overrightarrow{BC} = 3(\mathbf{a} + \mathbf{b})$$

The vectors are parallel, $\overrightarrow{BC} = 3\overrightarrow{XY}$

EXAM QUESTION

a) i) $\overrightarrow{OP} = \frac{1}{2}\mathbf{a}$ ii) $\overrightarrow{QX} = \frac{1}{3}\mathbf{a} - \frac{1}{6}\mathbf{b}$

b) $\overrightarrow{QA} = \mathbf{a} - \frac{1}{2}\mathbf{b}$

$$\overrightarrow{QX} = \frac{1}{3}\overrightarrow{QA}$$

QX and QA are parallel. Q is common point. Therefore, Q, X and A lie on the same line.

5.1 Sets and Venn Diagrams

A) Notation

- 1.) 2 3 5 7
- 2.) 1 3 5 7 9
- 3.) 2
- 4.) 3 5 7

B) Shading Regions on Venn Diagrams

Correctly shaded regions.

EXAM QUESTION

a) 2 11 10 2

b) (i) $\frac{11}{25}$

(ii) $\frac{11}{13} > \frac{11}{21}$ (1) a pupil who likes marmite also liking marmalade $\left(\frac{11}{13}\right)$ is more likely than (2) a pupil who likes marmalade also liking marmite $\left(\frac{11}{21}\right)$

5.2 Tree Diagrams and Frequency Trees

A) Tree Diagrams

a) Correctly completed tree diagrams with branches as follows:

0.7

0.3 0.7
0.3

b) 0.49

c) 0.91

EXAM QUESTION

1. a) Correctly completed tree diagrams with branches as follows:

$\frac{1}{3}$
 $\frac{2}{7}$ $\frac{2}{3}$
 $\frac{5}{7}$ $\frac{1}{3}$
 $\frac{2}{3}$

b) (i) $\frac{2}{21}$

(ii) $\frac{19}{21}$

B) Frequency Trees

a) 2
12 10

1
73 72

b) $\frac{1}{6}$

c) $\frac{2}{12} \div \frac{1}{73} \approx 12$

EXAM QUESTION

2. a) 12

60 48

100 3

40 37

b) 7.5%

5.3 Conditional Probability

A) With Tree Diagrams

a)

| | |
|---------------|---------------|
| | $\frac{3}{8}$ |
| $\frac{4}{9}$ | $\frac{5}{8}$ |
| | $\frac{4}{8}$ |
| $\frac{5}{9}$ | $\frac{4}{8}$ |

b) (i) $\frac{4}{9}$

(ii) $\frac{1}{2}$

(iii) $\frac{13}{18}$

B) Without Tree Diagrams

1. $\frac{4}{35}$

2. $\frac{4}{15}$

EXAM QUESTION

a) Correctly drawn tree diagram with branches as follows:

| Getting up late | Arrive at work late |
|-----------------|---------------------|
| $\frac{1}{3}$ | $\frac{9}{10}$ |
| | $\frac{1}{10}$ |
| $\frac{2}{3}$ | $\frac{1}{5}$ |
| | $\frac{4}{5}$ |

b) (i) $\frac{1}{10}$

(ii) $\frac{17}{30}$

6.1 Stratified Sampling

A) Sampling Question

- a) Not a fair / representative sample.
- b) Short 8
 - Medium 17
 - Long 5

EXAM QUESTION

15

6.2 Frequency Tables – Averages and Range

A) Frequency Tables

| | |
|----|-----|
| | 64 |
| | 45 |
| | 60 |
| | 11 |
| 20 | 180 |

- 1.) 20
- 2.) 8
- 3.) 9
- 4.) 9
- 5.) 3

B) Grouped Frequency Tables

| | | |
|----|------|------|
| | 37.5 | 300 |
| | 55 | 330 |
| | 75 | 300 |
| 30 | | 1080 |

- 1.) a) 36
b) Mid-points used as estimate
- 2.) a) $0 \leq t < 25$
b) $25 \leq t < 50$

EXAM QUESTION

- a) £207.0 (1 dp)
- b) Joseph is incorrect. 20 out of 28 coats cost more than £100, which is 71%

6.3 Cumulative Frequency Graphs and Box Plots

A) Cumulative Frequency Graphs

a)

| |
|-----|
| 112 |
| 148 |
| 178 |
| 194 |
| 200 |

Correctly drawn cumulative frequency curve.

- b) (i) 37 years ± 1
(ii) 25 years ± 2

B) Box Plots

Box plot drawn correctly at the following points:

| Min | LQ | Median | UQ | Max |
|-----|----|--------|----|-----|
| 10 | 14 | 16 | 20 | 29 |

EXAM QUESTIONS

1. a) 36%
b) Yes, 90% were at least 20 grams

2. a)

| Girls Cumulative Frequency | Boys Cumulative Frequency |
|----------------------------|---------------------------|
| 4 | 8 |
| 12 | 20 |
| 32 | 52 |
| 52 | 56 |
| 60 | 60 |

Both cumulative frequency curves drawn correctly.

- b) On average, boys did better as indicated by their lower median time of 44 seconds (compared to girls 50 seconds). Boys were also more consistent at estimating the right time, as indicated by their lower interquartile range of 10 compared to girls 14.

| | LQ | Median | UQ | IQR |
|-------|----|--------|----|-----|
| Girls | 42 | 50 | 56 | 14 |
| Boys | 38 | 44 | 48 | 10 |

6.4 Using Box Plots

Different averages and measures of spread: Mean, mode, median, range, interquartile range

A) Comparing Box Plots

- a) On average, boys scored better than girls as indicated by their higher median of 10 out of 20 compared to girls 8 out of 20.
b) Girls IQR = 10
Boys IQR = 7
Girls were less consistent as indicated by their interquartile range of 10 compared with boys 7.

EXAM QUESTION

The tickets were more expensive in May as indicated by the median of £26 compared with £21 in April. There was a greater spread of prices in April as indicated by the interquartile range of £14 compared with £12 in May.

| | Min | LQ | Median | UQ | Max |
|-------|-----|----|--------|----|-----|
| April | 3 | 15 | 21 | 29 | 50 |
| May | 7 | 18 | 26 | 30 | 42 |

6.5 Histograms

A) Frequency Density

| | | |
|---|----|-----|
| | 3 | 20 |
| | 5 | 4 |
| | 5 | 2.4 |
| 5 | 10 | |
| 1 | 10 | |

B) Drawing Histograms

a) Correctly drawn histogram using the following data:

| Time (t minutes) | Frequency Density |
|---------------------|-------------------|
| $0 \leq t < 8$ | 9 |
| $8 \leq t < 12$ | 21 |
| $12 \leq t < 16$ | 14 |
| $16 \leq t < 20$ | 9 |

b) Find the total area of the bars.

C) Completing Histograms

a)

| |
|----|
| 48 |
| |
| |
| 20 |

b) Correctly drawn histogram using the following data:

| Wage, w (£) | Frequency Density |
|------------------|-------------------|
| $5 \leq w < 8$ | 16 |
| $8 \leq w < 10$ | 17 |
| $10 \leq w < 14$ | 6 |
| $14 \leq w < 24$ | 2 |

EXAM QUESTION

a) Correctly drawn histogram using the following data:

| Age (a years) | Frequency |
|------------------|-----------|
| $15 \leq a < 25$ | 1.4 |
| $25 \leq a < 30$ | 3 |
| $30 \leq a < 40$ | 1.3 |
| $40 \leq a < 60$ | 0.4 |

b) 28.8 years

7 Problem Solving

EXAM QUESTIONS

1.) £4.80 profit

2.) Madeleine is correct.

Fuel poured 60 m^3 ($1000 \text{ L} = 1 \text{ m}^3$)

Volume cylinder 68.4 m^3 (1dp)

Maximum capacity cylinder 61.6 m^3 (1dp)

Mass fuel poured $51,000 \text{ kg} = 51 \text{ tonnes}$ ($1000 \text{ kg} = 1 \text{ tonne}$)

It is safe because less fuel is poured than the maximum permitted capacity and weight.