ELECTRIC TRANSIENTS.

BY

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1. With the increasing size and importance of electrical systems, those phenomena, which temporarily occur in all electrical circuits, as oscillations, surges, electrical impulses and travelling waves, are becoming of increasing frequency and destructiveness, and a knowledge and understanding of their general nature and characteristics, and the cause of their origin, has become important and necessary for the electrical engineer.

While the theory of these electrical transients is still far from complete, and involves considerable mathematics, it is possible to gain an insight into their cause and origin, their nature and general characteristics, and even their most important numerical relations, without theoretical mathematics, and in view of the great and rapidly increasing importance of these phenomena, in the following an attempt is made to give without the use of mathematics a general discussion of them, and their bearing on the operation of electrical systems.*

2. In general, an electric circuit comprises a source or generator, a line and a load. In the generator, electric power is produced from some other form of power, as mechanical, and flows over the line into the load where it is used. Not all the electric power produced by the generator reaches the load, but a part is consumed throughout the entire circuit, dissipated as heat by the resistance of the conductor: ri^2.

This however does not yet constitute the entire phenomena which we will call the flow of electric power, but there also is something going on in the space outside of the conductor which directs the flow of electric power: there is a *magnetic field,* lines of magnetic force encircle the conductor; and there also is an *electrostatic* or *dielectric field,* lines of electrostatic or dielectric force radiate from the conductor, and the magnetic field, and the electrostatic or dielectric field in the space outside of the conductor are just as essential parts of the flow of the electric power, as the power dissipation in the resistance of the conductor.

The magnetic field is proportional to the current i, with a proportionality factor which is called the *inductance* L of the circuit, and thus is Li; the dielectric field is proportional to the voltage e, with a proportionality factor which is called the *capacity* C of the circuit, and thus is Ce.

The magnetic field and the dielectric field represent *stored energy*: the energy stored by the magnetic field is

$$\frac{Li^2}{2},$$

and that stored by the dielectric field is

$$\frac{Ce^2}{2}.$$

Before a steady current can flow in a circuit, energy must therefore be supplied in producing its magnetic field; before a steady voltage can be on the conductor, energy must be supplied in producing its dielectric field. This energy is supplied by a flow of power from the conductor into the surrounding space in which the field exists. As the magnetic field is proportional to the current, the power required to supply its energy is given by a voltage, the *inductance voltage;* as the dielectric field is proportional to the voltage, the power required to supply its energy is given by a current, the capacity current, and inductance voltage and capacity current thus are the phenomena, by which the stored energy of the magnetic and dielectric fields respectively is supplied to the space surrounding the conductors.

Whenever a change occurs in an electric circuit, which involves an increase of its stored magnetic or dielectric energy, as an increase of current or of voltage, power flows from the circuit into the surrounding space, until the stored energy has been supplied, and inversely, whenever a change occurs, which requires a decrease of the stored energy, power flows back from space into the circuit. This power flow is temporary, lasts only until the required stored energy has been supplied, or the surplus stored energy returned to the circuit, and thus it is called a *transient.*

Transients therefore are the phenomena, by which at a change of circuit condition the stored energy readjusts itself to the changed condition, and any change in an electric circuit, which requires a change of the stored energy, cannot occur instantly, but a transition period of the readjustment of the stored energy intervenes, before stationary conditions are reached, whether in a direct or alternating current circuit.

3. It is obvious, that transients are not a specifically electrical phenomenon, but occur in any system of forces, where energy storage occurs. Thus in a water power, energy is stored in the momentum

$$\frac{mv^2}{2}$$

of the moving mass of water in the supply pipe, etc., and any change of load, which requires a change of the flow of water and therefore of its stored energy, cannot be accomplished instantly, but a mechanical transient appears, and the problem of governing water powers, especially at high heads, is essentially the problem of taking care of the mechanical transient. In railroading, the periods of acceleration and of deceleration are nothing but the mechanical transients of the stored energy; the firing up and the cooling down of a steam boiler are thermal transients, etc.

In the preceding cases, energy is stored in one form only; as kinetic mechanical energy in the pipe line and the railway train, as heat energy in the steam boiler, and the only change of the stored energy, which can occur, is an increase or a decrease. If, however, energy is stored in two or more forms, in addition to the increase or decrease of stored energy, a transformation of stored energy from one form to another may occur, and usually does occur. This for instance is the case in the pendulum. When the weight of the pendulum comes to rest, in its end position, it contains stored potential energy of gravitation, due to its elevation above the lowest position. When the weight reaches the lowest position, all this potential energy has vanished; some has been dissipated by friction, but most converted into kinetic energy of motion, and the weight now contains stored kinetic energy. During its further motion, this kinetic energy is again converted into potential energy, until the weight comes to rest in the next end position, and so on, and gradually, by a series of periodic transformations between potential energy of elevation and kinetic energy of motion, the stored energy of the pendulum is dissipated.

Two classes of transients thus exist: single energy transients, in which energy is stored in one form only, and the phenomenon is a steady increase or decrease of the stored energy, and double energy (or multiple energy) transients, in which in addition to an increase or decrease of the stored energy, a transformation of stored energy between its two forms occurs, which usually is periodic.

4. In electric distribution circuits of 110 to 220 volts, in 600 volt railway circuits, and even in 2200 volt primary distribution circuits, the delectric energy

$$\frac{Ce^2}{2}$$

is still so very small compared with the magnetic energy

$$\frac{Li^2}{2},$$

that the former can be neglected compared with the latter (except under special conditions, as with lightning discharges, etc.) and the circuit can be considered as storing energy in one form only, as magnetic energy. The transients of such circuits thus are single energy transients, that is, a steady increase or decrease of the stored magnetic energy during the transition period after a circuit change. The dielectric energy becomes comparable in magnitude with the magnetic energy, in transmission lines of 30,000 volts and over, and in underground cables of 10,000 volts and over, and such high voltage circuits then give double energy transients, that is, surges of the stored energy. This is the reason for the appearance in high voltage circuits, of electrical disturbances different from those met in low and medium voltage circuits.

The single energy transients of electric circuits, which result from the stored magnetic energy, are rarely of serious importance: as they are a gradual change from the previous to the after conditions of the electric circuit, their inductive effects are usually small; due to the limited extension of such low and medium voltage circuits, their energy is moderate, and due to the relatively large resistance of most such circuits, the energy is rapidly dissipated, that is, the duration of the transient short, and they usually require consideration only to avoid the annoyance which may be caused by them. For instance, when closing the field exciting circuit of a shunt motor, an appreciable time—frequently several seconds—elapses before the magnetic field has built up, and if then the armature is energized immediately and the armature rheostat cut out too rapidly, momentarily a big current may flow through the armature, without starting it, as the magnetic field is not yet there, and this current rush may blow the fuses.

Of serious moment such magnetic single energy transients become when of abnormally great energy, that is, in very extended systems or in highly inductive circuits, as motor fields, and especially, when forced to be of very short duration, as in the latter case, even with a moderate amount of energy, the power may be very great, since it is the energy divided by the time in which it is dissipated. The latter is the case with the opening transient of an inductive circuit. In opening an inductive circuit its stored magnetic energy must be dissipated. If then an inductive circuit were opened instantly, that is, its stored energy dissipated in an infinitely short time, this would represent infinite power, that is, infinite induced voltage, or in other words, the insulation of the circuit would be punctured. To a considerable extent, an inductive circuit protects itself against too rapid opening, by the arc following the switch blades: the more rapid the switch opens, the greater is the induced voltage, and the further the arc follows and maintains the circuit, that is, limits the rapidity of the circuit opening. The problem of switching inductive circuits thus is, to open the circuit sufficiently rapid to limit the burning of the switch contacts by the arc, but to open sufficiently slow to limit the induced voltage below the value which would seriously endanger the insulation. Inversely, if an inductive circuit is opened very rapidly, as by a powerful air-blast across the opening switch, or a small condenser shunted across the break, practically unlimited voltages are produced, as in the Ruhmkorff coil.

5. Occasionally magnetic transients are of importance not directly by their stored energy, but by the power production which they cause. A characteristic instance hereof is found in the momentary short circuit currents of alternators. The voltage of an alternator depends on the field excitation and the load, that is, the magnetizing or demagnetizing action of the armature current, which combines with the magnetomotive force of the field excit-

ing current to a resultant, and the latter produces the magnetic field flux. At open circuit, the magnetic field flux is that due to the field excitation alone. If now with the same field excitation, current is drawn from the armature, especially if it is a lagging current, the demagnetizing action of this current lowers the field flux and with it the voltage. At short circuit, the armature current—which then is lagging—rises to such a value, that its demagnetizing action lowers the field flux to a small fraction of the value which would exist with the same field excitation at open circuit: The value just sufficient to circulate the short circuit current in the armature. With modern high speed turbo alternators, the permanent short circuit current may be about 4 times full load current, and the field flux at short circuit may be as low as one-tenth of the field flux, which exists at open circuit with the same field excitation. In short circuiting the alternator, the field magnetic flux thus decreases to one-tenth its initial value. As it represents stored energy, it cannot do so instantly, but can decrease only gradually—though rapidly—from open circuit to short circuit value, and in the first moment after short circuit it thus still has practically full open circuit value, that is, a value, which may be ten times as great as the permanent short circuit value. As the latter gives a short circuit current in the armature equal to 4 times full load current, the former gives ten times as high a current, that is, a momentary short circuit current which may be as high as 40 times full load value. Thus, the effect of the magnetic field transient of the alternator is an excessive transient current rush in the armature at short circuit. Considering, that the permanent short circuit current of the armature, of 4 times full load value, demagnetizes the field so as to drop it to one-tenth of its value, the momentary short circuit current, of 40 times full load value, would demagnetize 10 times as much, and as at this moment of the beginning short circuit the field flux still has full value, the field current also must momentarily increase to 10 times its normal value, to maintain full field flux against the excessive demagnetizing action of the momentary short circuit current of the armature. That is, the magnetic field at the moment of short circuit begins to decrease at such a rate that by its decrease it induces in the field exciting winding an e.m.f., which raises the field current to the value required to hold the field flux against the armature reaction. Thus the momentary current rush in the armature at short circuit, is accompanied by a corresponding current rush in the field. With single phase short circuit, this field current is pulsating with double frequency, as the single phase armature reaction pulsates with double frequency; with polyphase short circuit, a transient full frequency pulsation appears in the field current rush, corresponding to the starting transient of the armature current.

6. The most important, because most dangerous electrical transients are the double energy transients of high voltage circuits. In these circuits, in which the stored dielectric energy

$$\frac{Ce^2}{2}$$

is of the same magnitude with the stored magnetic energy

$$\frac{Li^2}{2},$$

the gradual increase of the total stored energy, at a change of circuit conditions, is accompanied by a surge, or transformation of the stored energy between its two different forms, similar as was illustrated on the pendulum as a surge between potential energy of gravitation, and kinetic energy of motion.

During the periodic transformation between magnetic and dielectric stored energy, the magnetic energy is a maximum,

$$\frac{Li_0^2}{2},$$

and therefore the oscillating current i a maximum i_0, at that moment, where all the energy is magnetic, that is, the dielectric energy

$$\frac{Ce^2}{2}$$

and therefore the oscillating voltage e is zero. The dielectric energy is a maximum,

$$\frac{Ce_0^2}{2},$$

and therefore the oscillating voltage e a maximum e_0, when during the periodic transfer of stored energy all the energy has become dielectric, that is, the magnetic energy

$$\frac{Li^2}{2}$$

and therefore the oscillating current i is zero.

It is however the same amount of stored energy (neglecting the gradual and slow decrease of energy by the power dissipation), which at one moment appears as magnetic, at the next moment as dielectric energy. That is, it is:

$$\frac{Li_0^2}{2} = \frac{Ce_0^2}{2}.$$

This *relation between the maximum value of the oscillating current, i_0, and the maximum value of the oscillating voltage, e_0,* is of the greatest industrial importance. It applies to all double energy transients, that is, surges of stored energy. Transposed, it gives:

$$\frac{e_0}{i_0} = \sqrt{\frac{L}{C}}$$

$\sqrt{\frac{L}{C}}$ is of the nature of an impedance, and is called the *natural impedance of the circuit,* and may be denoted by z_0:

$$z_0 = \sqrt{\frac{L}{C}}$$

in transmission lines, z_0 usually is between 200 and 600 ohms; it is must lower in cables, due to their high capacity C and low inductance L, and it is much higher in coiled circuits, as the high potential windings of high voltage transformers.

The great industrial importance of this relation between the maximum oscillating voltage and the maximum oscillating current is, that it allows the calculation of the one from the other. In most cases of electric surges of energy, one of the two quantities, the current or the voltage, is known, or can be estimated approximately, from the cause of the disturbance, and above relation allows the calculation of the other.

7. Consider for instance the switching on of a transmission line of 30,000 volts. With the switches open, the voltage on the line is zero; with the switches closed, the voltage has an effective value of 30,000, and thus a maximum value of $30{,}000\sqrt{2} = 42{,}400$ volts. That is, at the moment of closing the circuit, a sudden change of voltage occurs on the line which as maximum can amount to: $e_0 = 42{,}400$.

If then the natural impedance of the line, as calculated from the equations or from the tables of L and C of line conductors is: $z_0 = 400$ ohms, the maximum oscillating current, which can be produced in the line by the operation of switching the dead line onto the live transformer is limited to:

$$i_0 = \frac{e_0}{z_0}$$

$$= \frac{42{,}400}{400}$$

$$= 106 \text{ amperes.}$$

Inversely, when a line carrying 30,000 volts is suddenly grounded, or short circuited, that is, its voltage changed from 30,000 volts effective, to zero, 106 amperes is the maximum current oscillation which may occur, and which, in case of short circuit, superimposes on the wave of the permanent short circuit current.

An interesting application of this relation is for the calculation of the maximum *lightning discharge* from a transmission line, against which the stations have to be protected. While the voltage of lightning may be practically unlimited, the maximum voltage which lightning can produce on the line obviously is limited by the disruptive strength of the line insulation against momentary voltages. If a higher voltage is impressed on the line by a lightning stroke (as frequently the case), it spills over at the next insulator, and if of sufficient energy, may puncture insulators, shatter cross arms and poles locally on the line, but the lightning discharge current, which rushes along the line towards the station, obviously is limited to the voltage which can actually exist, at least momentarily, on the line as result of the lightning stroke.

Assuming a 30,000 volt line: the insulators must be expected to stand continuously double voltage, or 60,000, and momentarily at least twice as much, or 120,000 to 150,000. Thus the lightning discharge current is limited to that given by the oscillating voltage of a maximum: $e_0 = 150{,}000$.

At a natural impedance of the line, of $z_0 = 400$ ohms, this gives:

$$i_0 = \frac{e_0}{z_0} = 375 \text{ amperes.}$$

Thus, 375 amperes is the maximum discharge current, which lightning could produce in this line, and thus the maximum current, against which the lightning arresters in the station have to protect in case of a lightning stroke on the line near a station. If the lightning strikes at a considerable distance from the station, obviously the discharge current at the station is much less, as the energy of the oscillation is rapidly dissipated in its passage along the line.

Allowing a momentary rise of voltage at the station of 50 per cent., the maximum voltage drop across the lightning arresters should be limited to half the maximum line voltage, or 21,400 volts. At 375 amperes discharge, this allows an impedance of the lightning arrester and its discharge path, of

$$z = \frac{21,400}{375} = 57 \text{ ohms.}$$

To afford effective lightning protection, the resistance of the lightning arrester and impedance of its connection to line and ground should therefore not exceed 57 ohms in this case.

If however the natural impedance z_0 were much lower, due to the use of large conductors and relatively short distances between the conductors, for instance, $z_0 = 200$, and two parallel lines led from the station, then, if both lines were struck simultaneously, the natural impedance of the discharge would only be:

$$\frac{z_0}{2} = 100 \text{ ohms,}$$

and the maximum discharge current thus 1500 amperes, and thus would limit the permissable impedance of the lightning arrester circuit to 14 ohms. That is, either a very low resistance arrester and very low impedance discharge path becomes necessary, or the use of several lightning arresters and several line and ground connections in multiple, to afford protection.

8. As seen, without going into detail study of the lightning transients, considerable information can be derived from the above fundamental relation of current and voltage of the double energy transient.

Inversely, this relation may be applied to determine the oscillating voltage, which may be produced by a sudden change of current in the line. Assuming that the full load current of the 30,000 volts line is 200 amperes effective, that is, has a maximum value of $200\sqrt{2} = 283$ amperes. If then, in switching off full load, the circuit would be broken at the maximum value of the current, the oscillation, which results, would be that due to a sudden change of current from 283 amperes to zero, that is:

$$i_0 = 283 \text{ amperes.}$$

At a natural impedance of: $z_0 = 400$ ohms, this gives a maximum oscillating voltage of:

$$\begin{aligned} e_0 &= i_0 z_0 \\ &= 283 \times 400 \\ &= 113{,}000 \text{ volts.} \end{aligned}$$

This is more than double voltage, that is, more than the voltage which the line can stand continuously, and already a moderate overload, as 50 per cent. would give an oscillating voltage higher than which the line could stand even momentarily, that is, would cause a flashover or a puncture of insulation.

Switching at or near maximum current thus is dangerous and frequently destructive, and the problem of controlling transmission lines is to devise a switching mechanism, which can be relied upon always to open the circuit at zero current. This is the reason which has lead to the development of the modern oil circuit breaker, as this type has the characteristic to open the circuit at the zero value of the current. It also has led to the abandonment of all types of air circuit breakers as dangerous on high voltage circuits, since the air circuit breaker cannot be relied upon to open always at zero current, but occasionally opens at or near maximum current, and then is liable to be destructive to the circuit insulation. The operation of the present extended high voltage systems thus has became possible only by the development of the modern oil circuit breaker.

As already the opening of full load current, when taking place at the wrong point of the wave, leads to dangerous voltages, it is interesting to see, what would be the result of opening a *short circuit* at or near maximum current.

Assuming the line impedance to be 20 per cent. at full load. That is, at full load current, the impedance voltage of the line is 20 per cent or one-fifth of impressed voltage. At short circuit, the total impressed voltage is consumed by the line impedance, and the short circuit current thus would be 5 times full load current, provided that the generating system can maintain full voltage on the line at least momentarily, in case of short circuit at the end of the line. With a full load current of 200 amperes effective or 283 amperes maximum, the maximum short circuit current would thus be $5 \times 283 = 1415$ amperes, and if interrupted at the maximum value, would give an oscillation of maximum current: $i_0 = 1415$ amperes, thus, with a natural impedance of the line of $z_0 = 400$ ohms, on oscillating voltage of maximum value:

$$\begin{aligned} e_0 &= i_0 z_0 \\ &= 1415 \times 400 \\ &= 566{,}000 \text{ volts,} \end{aligned}$$

that is, far beyond the possible insulation strength of the line, or in other words, the line insulation, no matter how good, would be broken down in case of a short circuit being interrupted at or near the maximum value of current.

The most destructive transient of a high voltage circuit thus is the short circuit oscillation.

Occasional short circuits however are unavoidable in transmission lines, either due to accidents in apparatus, or due to flashovers on the line, at defective insulators, by atmospheric electrical disturbances, by heavy winds swinging the wires together, etc. As far as the short circuits are opened by the oil circuit breakers in the system, they are relatively harmless, due to the nature of the oil circuit breaker to open at zero current. More dangerous are flashovers on the line, resulting in flaring arcs, which lengthen and then blow themselves out. Fortunately, such flaring arcs only occasionally develop their destructive possibilities, but usually go out at the zero of the current wave: at the end of one half wave, the arc vapor stream cools sufficiently so that at the beginning of the next half wave the arc does not start again. Occasionally however such arcs blow themselves out explosively, by the rapid expansion of the air due to the heat of the arc, and as the heating effect is a maximum at or near maximum current, this explosive rupture of a flaring arc tends to occur near maximum current, and then produces a destructive oscillation. Especially is this the case in low reactance high capacity circuits, as cables, where the current and therefore the heating effect in the flaring arc is very large and the arc inherently more unstable. Such flaring arcs thus constitute the most dangerous and destructive phenomenon of high voltage circuits.

It is interesting to note, that the surges resulting from a current transient are usually much greater in magnitude and thus in destructiveness, than those resulting from a voltage transient.

9. We have seen that in the surge of stored magnetic and dielectric energy, the current is a maximum when the voltage is zero, and inversely, that is, the oscillating current and the oscillating voltage are in quadrature, and the power thus is reactive, that is, surges to and fro, but there is no flow of power along the circuit. Such a surge of energy is called a *stationary oscillation, standing wave,* or *surge.*

Oscillating current and voltage may also be in phase with each other. Then there is a flow of power along the circuit, and the phenomenon is called a *travelling wave,* or an electric *impulse.* Travelling waves of high frequency frequently precede standing waves. Travelling waves have been observed of frequencies from a few hundred cycles, to millions of cycles; the former by oscillograph, the latter by spark gap across a small reactance.

In circuits comprising sections of different character, as transmission line, transformer, load, etc., that is, in most industrially important cases, the energy surge is a combination of standing waves and travelling waves. The energy surge obviously must die down simultaneously in all circuit sections. In some circuit sections however, as the transmission line, energy dissipation occurs at a greater rate than in other circuit sections, as transformer coils, and left separate, the surge would die down in the line much more rapidly than in the transformer. Thus, when line and transformer are connected together energy must be supplied to the line, from the stored energy of the transformer, by a travelling wave. Thus in the energy surge, or the transient of such a compound circuit, a flow of oscillating power occurs between the circuit sections. This results in a transformation of the oscillating voltage and current at the transition point between the successive circuit sections.

For instance, if a travelling wave passes from a transmission line into the high potential winding of the step-down transformer, and e_1 is the maximum voltage, i_1 the maximum current hence

$$p_1 = \frac{e_1 i_1}{2}$$

the power of the wave in the transmission line, e_2 the maximum voltage and i_2 the maximum current hence

$$p_2 = \frac{e_2 i_2}{2}$$

the power of the wave in the transformer; as the same power leaves the line which enters the transformer, it must be

$$\frac{e_1 i_1}{2} = \frac{e_2 i_2}{2}$$

e and i are however related by the natural impedance of the circuit section, that is:

$$\left. \begin{aligned} i_1 &= \frac{e_1}{z_1} \\ i_2 &= \frac{e_2}{z_2} \end{aligned} \right\} \qquad (2)$$

where z_1 is the natural impedance of the line, and z_2 the natural impedance of the transformer,

Substituting (2) in (1) gives:

$$\frac{e_2}{e_1} = \sqrt{\frac{z_2}{z_1}}$$

That is, *at the transition point between two circuit sections, a transformation of voltage occurs, with a transformation ratio, which is the square root of the ratio of the natural impedances of the two circuit sections, and a transformation of currents by the inverse ratio:*

$$\frac{i_2}{i_1} = \sqrt{\frac{z_1}{z_3}}.$$

This relation between the oscillating voltages and currents in different circuit sections is of fundamental importance in tracing their origin and destructiveness.

Hence, a travelling wave, coming from one circuit section into another circuit section of higher natural impedance, increases in voltage and thereby in destructiveness, and inversely, when entering a section of lower natural impedance, it decreases in voltage.

Thus, if a travelling wave of $e_1 = 30{,}000$ volts comes from a line of natural impedance $z_1 = 400$ ohms, into the high potential winding of a transformer, of natural impedance $z_2 = 3600$ ohms, it rises in voltage to:

$$e_2 = e_1\sqrt{\frac{z_2}{z_1}}$$

$$= 30{,}000\sqrt{\frac{3600}{400}}$$

$$= 90{,}000 \text{ volts.}$$

10. Electrical disturbances, originating in one circuit section, frequently develop their destructiveness in adjacent circuit sections, in which due to the higher natural impedance of the latter section they have risen to destructive voltages. The result, and the indication of a transmission line disturbance thus often is the break down of the transformers connected to the line, not because the transformers are weaker in insulation, but because the disturbance, while of harmless voltage in the line, has in entering the transformer risen in voltage to destructive values.

When speaking of voltage transformation at the transition point between two circuit sections, it is obvious that at the transition point the voltage is the same in both circuits, but a change of phase occurs, which results in a different maximum voltage some distance further on in the circuit, at the point where the voltage maximum occurs. With a voltage increase, this maximum is approximately a quarter wave distant from the transition point, and depending on the frequency of the wave, this point may be very near the transition point, with very high frequency waves, or far distant, with lower frequencies. Thus transformers may break down in the middle of their winding, as result of disturbances in the transmission line to which they are connected, if the disturbance is of moderate frequency, while the break down would occur in the end turns, if the disturbance in the line is of very high frequency. That is, the location of a break down in a transformer does not indicate, whether the disturbance originated in the transformer, or entered from the outside.

To conclude then, the two most important numerical relations, which govern the electrical disturbances in high voltage systems, are:

a. The relation between the maximum current i_0 and the maximum voltage e_0 of the oscillation, standing wave, impulse or travelling wave:

$$\frac{e_0}{i_0} = z_0$$

where:

$$z_0 = \sqrt{\frac{L}{C}}$$

is the natural impedance of the circuit.

b. The transformation ratio of voltages e_1 and e_2, and currents i_1 and i_2, at the transition point between two adjoining circuit sections, of the natural impedances z_1 and z_2 respectively:

$$\frac{e_2}{e_1} = \sqrt{\frac{z_2}{z_1}} = \frac{i_1}{i_2}$$

and from these two relations, very considerable information on the nature, cause and effect of transients can be derived.

"Brown" Gold or "Dark" Gold. M. HANRIOT. (*Compt. rend.*, clii, 138.)—"Dark" or "brown" gold is the name given by the author to the very finely divided gold obtained by the action of an acid on an alloy of gold and silver. When exposed to heat the color changes and it shrinks considerably. The shrinkage is slow and reaches its limit at the end of 5 to 6 hours. The shrinkage of the sheet, when exposed to nitric acid, depends only on the composition of the alloy and not on the temperature. The thickness has no effect, except that the thicker the sheet the longer it must be heated with nitric acid to completely remove the silver.

New American Lathes. ANON. (*Iron Age*, lxxxvii, 546.)—The American Tool Works Co., Cincinnati, O., have introduced two new types of high-duty lathes, said to be the latest and highest developed types, and to surpass in power, range, and convenience any previously designed lathes of similar sizes. Full descriptions and drawings are given in this article, which will repay those interested for its perusal.