field is within the quadruped which supports the bearing.

In other cases the exciter may be belt-driven, in which case the power is taken off a pulley installed below the revolving field, as shown in Fig. 8. If the exciter is an ordinary horizontal shaft machine, the exciter belt is usually brought out through the base ring of the alternator, and over an idler pulley so as to permit of a quarter turn in the belt. However, a vertical shaft belted exciter may be used, and in that case no turning of the belt is needed.

Sometimes extra weight is wanted in the rotor, to increase the moment of inertia of the entire revolving element and improve the governing of the waterwheel. This extra material may be very easily and effectively added to the rotor rim.

MAGNETIC RELUCTIVITY

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This article is a review and discussion of magnetic reluctivity under the following headings: 1, Fröhlich's and Kennelly's laws; 2, The critical points or bends in the reluctivity line of commerical materials; 3, Unhomogenuity of the material as cause of the bends in the reluctivity line; 4, Reluctivity at low fields, the inward bend, and the rising magnetic characteristic as part of an unsymmetrical hysteresis cycle; 5, Indefiniteness of the B-H relation. The alternating magnetic characteristic. Instability and creepage; 6, The area of B-H relation. Instability of extreme values. Gradual approach to the stable magnetization curve; 7, Production of stable values by superposition of alternating field. The linear reluctivity law of the stable magnetic characteristic.—Editor.

Fröhlich's and Kennelly's Laws

1. Considering magnetism as the phenomena of a "magnetic circuit," the foremost differences between the characteristics of the magnetic circuit and the electric circuit are:

(*a*) The maintenance of an electric circuit requires the expenditure of energy, while the maintenance of a magnetic circuit does not require the expenditure of energy, though the starting of a magnetic circuit requires energy. A magnetic circuit therefore can remain "remanent" or "permanent."

(*b*) All materials are fairly good carriers of magnetic flux, and the range of magnetic permeabilities is therefore narrow, from 1 to a few thousands, while the range of electric conductivities covers a range of 1 to 10^{18}. The magnetic circuit thus is analogous to an uninsulated electric circuit immersed in a fairly good conductor, as salt water: the current or flux can not be carried to any distance, or constrained in a "conductor," but divides, "leaks" or "strays."

(*c*) In the electric circuit, current and e.m.f. are proportional in most cases; that is, the resistance is constant and the circuit can therefore be calculated theoretically. In the magnetic circuit, in the materials of high permeability, which are the most important carriers of the magnetic flux, the relation between flux, m.m.f. and energy is merely empirical; the "reluctance" or magnetic resistance is not constant, but varies with the flux density, the previous history, etc. In the absence of rational laws most of the magnetic calculations therefore have to be made by taking numerical values from curves or tables.

The only rational law of magnetic relation which has not been disproven is Fröhlich's (1882):

"The permeability is proportional to the magnetizability."

$$\mu = a\,(S-B) \qquad (1)$$

where B is the magnetic flux density, S the saturation density, and $S-B$ therefore the magnetizability, that is, the still available increase of flux density over that existing.

From (1) there follows, by substituting:

$$\mu = \frac{B}{H} \qquad (2)$$

and re-arranging:

$$B = \frac{H}{\alpha + \sigma H} \qquad (3)$$

where

$\sigma = \frac{1}{S} =$ *saturation coefficient*, that is, the reciprocal of the saturation value S of flux density B, and

$$\alpha = \frac{1}{a\,S} = \frac{\sigma}{a}$$

For $B=O$, equation (1) gives:

$$\mu_0 = a\,S = \frac{1}{\alpha};\ \alpha = \frac{1}{\mu_0} \qquad (4)$$

that is, α is the reciprocal of the magnetic permeability at zero flux density.

A very convenient form of this law has been found by Kennelly (1893) by introducing the reciprocal of the permeability, as reluctivity ρ:

$$\rho = \frac{1}{\mu} = \frac{H}{B}$$

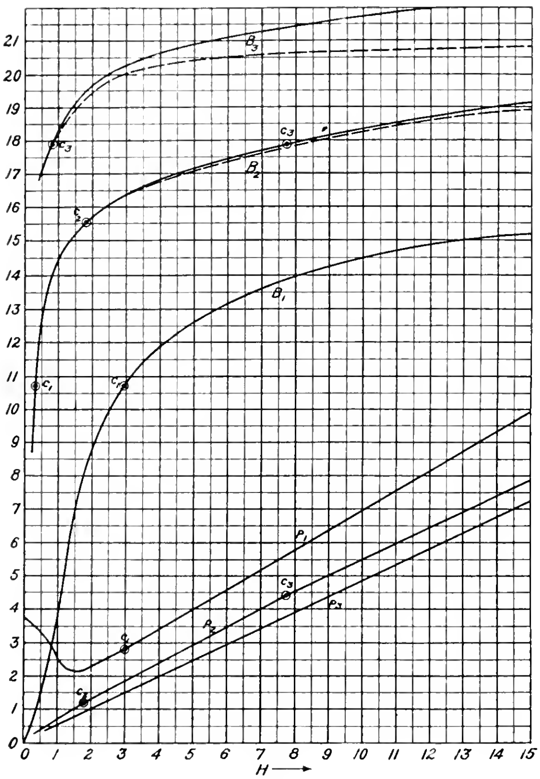


Fig. 1

in the form, which can be derived from (3) by transposition:

$$\rho = \alpha + \delta H \tag{5}$$

As α dominates the reluctivity at lower magnetizing forces, and thereby the initial rate of rise of the magnetization curve, which is characteristic of the "magnetic hardness" of the material, it is called the *coefficient of magnetic hardness.*

The Critical Points or Bends in the Reluctivity Line of Commercial Materials

2. When investigating flux densities B at very high field intensities H, it was found that B does not reach a finite saturation value but increases indefinitely; that, however,

$$B_0 = B - H \tag{6}$$

reaches a finite saturation value S, which with iron is usually not far from 20 kilolines per cm², and that therefore Fröhlich's and Kennelly's laws apply not to B but to B_0. The latter then is usually called the *metallic magnetic density* or *ferromagnetic density.*

B_0 may be considered as the magnetic flux carried by the molecules of the iron or other magnetic material, in addition to the space flux H, or flux carried by space independent of the material in space.

The best evidence seems to establish the fact that, with the exception of very low field intensities (where the customary magnetization curve usually has an inward bend, which will be discussed later) in perfectly pure magnetic materials, iron, nickel, cobalt, etc., the linear law of reluctivity (5) and (3) is rigidly obeyed by the metallic induction B_0.

In the more or less impure commercial materials, however, the $\rho - H$ relation, while a straight line, often has one and occasionally two points where its slope, and thus the values of α and σ, change.

Fig. 1 shows an average magnetization curve of good standard iron with field intensity H as abscissae and magnetic induction B as ordinates. The total induction is shown in drawn lines, and the metallic induction in dotted lines. The ordinates are given in kilolines per cm², the abscissae in units for B_1, in tens for B_2, and in hundreds for B_3.

The reluctivity curves for the three scales of abscissae are plotted as ρ_1, ρ_2, ρ_3, in tenths of milli-units, in milli-units, and in tens of milli-units.

Below $H = 3$, ρ is not a straight line, but is curved due to the inward bend of the magnetization curve B in this range. The straight line law is reached at the point c_1, at $H = 3$, and the reluctivity is then expressed by the linear law:

$$\rho_1 = 0.102 + 0.059\,H \tag{7}$$

for:

$$3 < H < 18$$

giving an apparent saturation value:

$$S_1 = 16{,}950$$

At $H = 18$, a bend occurs in the reluctivity line, marked by point c_2, and above this point the reluctivity follows the equation

$$\rho_2 = 0.18 + 0.0548\,H \tag{8}$$

for:

$$18 < H < 80$$

giving an apparent saturation value:

$$S_2 = 18{,}250$$

At $H=80$, another bend occurs in the reluctivity line, marked by point c_3, and above this point, up to saturation, the reluctivity follows the equation:

$$\rho_3=0.70+0.0477\,H \qquad (9)$$

for:

$$H>80$$

giving the true saturation value:

$$S=20{,}960$$

Point c_2 is frequently absent.

Fig. 2 gives once more the magnetization curve (metallic induction) as B, and the dotted curves B_1, B_2 and B_3 representing magnetization curves calculated from the three linear reluctivity equations (7), (8), (9). As seen, neither of the equations represents B even approximately over the entire range, but each represents it very accurately within its range. The first equation (7) probably covers the entire industrially important range.

Unhomogenuity of the Material as Cause of the Bends in the Reluctivity Line

3. As these critical points c_2 and c_3 do not seem to exist in perfectly pure materials, and as the change of direction of the reluctivity line is in general greater, with increase of impurity of the material, the cause seems to be lack of homogenuity of the material; that is, the presence, either on the surface as scale or in the body as inglomerate, of materials of different magnetic characteristics: magnetic, cementite, silicide. Such materials have a much greater hardness, that is, higher value of α, and would therefore give the observed effect. At low field intensities H the harder material carries practically no flux, all the flux being carried by the soft material. The flux density therefore rises rapidly, giving low α, but tends towards an apparent low saturation value, as the flux carrying material fills only part of the space. At higher field intensities the harder material begins to carry flux; and while in the softer material the flux increases at a lesser rate, the increase of flux in the harder material gives a greater increase of total flux density and a greater saturation value, but also a greater hardness as the resultant of both materials.

Thus, if the magnetic material is a conglomerate of fraction p of soft material of reluctivity ρ_1 (ferrite) and $q=1-p$ of hard material of reluctivity ρ_2 (cementite, silicide, magnetite):

$$\left.\begin{aligned}\rho_1&=\alpha_1+\sigma_1\,H\\ \rho_2&=\alpha_2+\sigma_2\,H\end{aligned}\right\} \qquad (10)$$

At low values of H the part p of the section carries flux by ρ_1 and the part q carries flux by ρ_2; but as ρ_2 is very high compared with ρ_1 the latter flux is negligible and is:

$$\rho'=\frac{\rho_1}{p}=\frac{\alpha_1}{p}+\frac{\sigma_1}{p}H \qquad (11)$$

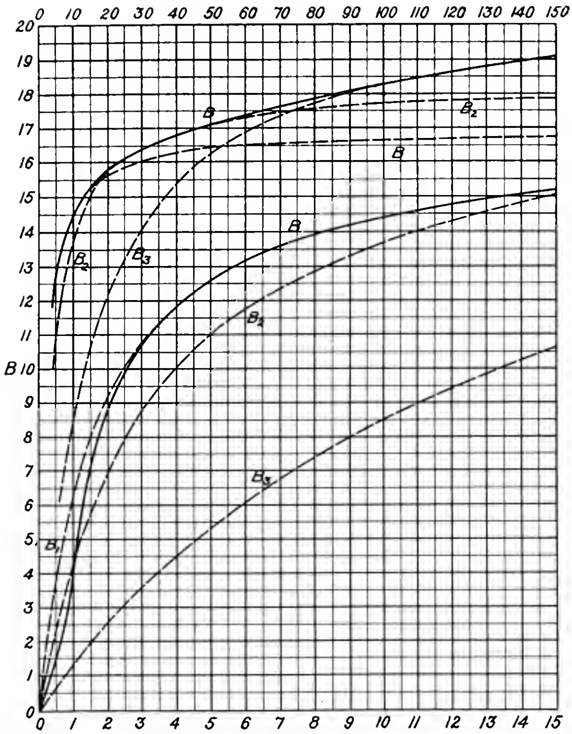


Fig. 2

At high values of H, the flux goes through both materials, more or less in series, and thus is:

$$\rho''=p\,\rho_1+q\,\rho_2=(p\alpha_1+q\alpha_2)+(p\,\sigma_1+q\,\sigma_2)\,H \qquad (12)$$

If we assume the same saturation value σ for both materials, and neglect α_1 compared with α_2, it is:

$$\rho''=q\,\alpha_2+\sigma\,H \qquad (13)$$

Substituting, for instance (7) and (9) into (11) and (13) respectively, gives:

$$\frac{\alpha_1}{p}=0.102$$

$$\frac{\sigma}{p}=0.059$$

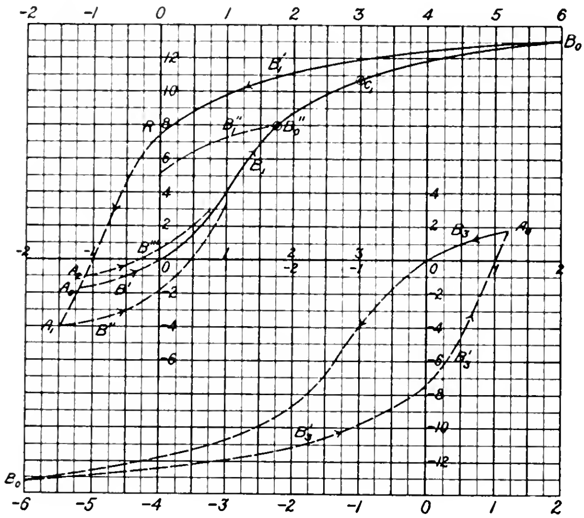
$$q\,\alpha_2=0.70$$

$$\sigma=0.0477$$

Hence:

$$p=0.80:\ \rho_1=0.082+0.0477\ H$$
$$q=0.20:\ \rho_2=3.5+0.0477\ H$$

However, the saturation coefficients σ of the two materials probably are usually not equal.



Figs. 3 and 4

The deviation of the reluctivity equation from a straight line, by the change of slope at the critical points c_2 and c_3 thus probably is only apparent, and is the outward appearance of a change of the flux carrier in an unhomogeneous material; that is, the result of a second and magnetically harder material beginning to carry flux.

Such bends in the reluctivity line have been artificially produced by Mr. John D. Ball in combining by superposition two different materials which separately gave straight line ρ curves, but which when combined gave a curve showing the characteristic bend.

Very impure materials, like cast-iron, may give throughout a curved reluctivity line.

Reluctivity at Low Fields, the Inward Bend, and the Rising Magnetic Characteristic as Part of an Unsymmetrical Hysteresis Cycle

4. For very low values of field intensity ($H<3$), however, the straight line law of reluctivity apparently fails, and the magnetization curve in Fig. 1 has an inward bend which gives a rise of ρ with decreasing H.

This curve is taken by ballistic galvanometer, by the step-by-step method; that is, H is increased in successive steps, and the increase of B observed by the throw of the galvanometer needle. It is thus a "rising magnetization curve."

The first part of this curve is reproduced as B_1 in Fig. 3, in twice the abscissae and half the ordinates so as to give it an average slope of 45 degrees, as with this slope curve shapes such as the inward bend of B_1 below $H=2$, are best shown ("Engineering Mathematics," 2d edition, p. 286).

Suppose now that at some point $B_0=13.15$ we stop the increase of H, and decrease again to 0. We do not return on the same magnetization curve B_1, but on another curve B_1', the "decreasing magnetic characteristic," and at $H=0$ we are not back to $B=0$, as residual or remanent flux is left, (Fig. 3: R=7.4).

Where the magnetic circuit contains an air gap, as in the field circuits of electrical machinery, the decreasing magnetic characteristic B_1' is very much nearer to the increasing one B_1 than in the closed magnetic circuit of Fig. 3, and practically coincides for higher values of H.

There appears no theoretical reason why the rising characteristic B_1 should be selected as the representative magnetization curve, rather than the decreasing characteristic B_1', except the incident that B_1 passes through zero. In many engineering applications, for instance, the calculation of the regulation of a generator (the decrease of voltage under increase of load), it is obviously the decreasing characteristic, B_1', which is determining.

Suppose we continue B_1' into negative values of H, to the point A_1, where $H=1.5$ and $B=-4$, and then again reverse. We get a rising magnetization curve B'', which passes $H=0$ at a negative remanent magnetism. Suppose we stop at point A_2, where $H=-1.12$ and $B=-1.0$; the rising magnetization curve B''' then passes $H=0$ at a positive remanent magnetism. There must thus be a point A_0, between A_1 and A_2, such that the rising magnetization curve B', starting from A_0, passes through the 0 point $H=0$, $B=0$, and thereby runs into the curve B_1.

The rising magnetization curve, or standard magnetic characteristic determined by the step-by-step method, B_1, is thus nothing but the rising branch of an unsymmetrical hysteresis cycle, traversed between such limits $+B_0$ and $-A_0$ that the rising branch of the hysteresis cycle passes through the zero point.

Indefiniteness of the B-H Relation. The Alternating Magnetic Characteristic. Instability and Creepage

5. The characteristic shape of a hysteresis cycle is a loop, pointed at either end and thereby having an inflexion point about the middle of either branch. In the unsymmetrical loop $+B_0$, $-A_0$ of Fig. 3, the zero point is fairly close to one extreme, A_0, and the inflexion point, characteristic of the hysteresis loop, thus lies between 0 and B_0; that is, on that part of the rising branch which is used as the "magnetic characteristic" B_1, thereby producing the inward bend in the magnetization curve at low fields, which has always been so puzzling.

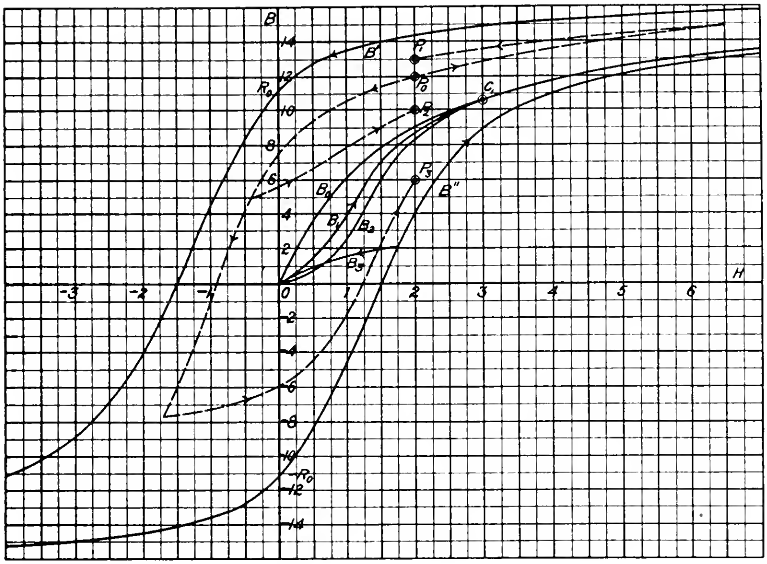


Fig. 5

If, however, we should stop the increase of H at B_0'', we would get the decreasing magnetization curve B_1'', and still other curves for other starting points of the decreasing characteristic.

Thus, the relation between magnetic flux density B and magnetic field intensity H is not definite, but any point between the various rising and decreasing characteristics B'', B_1, B''', B_1'', B_1', and for some distance outside thereof, is a possible $B-H$ relation. B_1 has the characteristic that it passes through the zero point, but it is not the only characteristic which does this. If we traverse the hysteresis cycle between the unsymmetrical limits $+A_0$ and $-B_0$, as shown in Fig. 4, we see that its decreasing branch B_3 passes through the zero point, that is, has the same feature as B_1. It is interesting to note that B_3 does not show an inward bend, and the reluctivity curve of B_3, given as ρ_3 in Fig. 6, apparently is a straight line.

Magnetic characteristics are frequently determined by the method of reversals, by reversing the field intensity H and observing the voltage induced thereby by a ballistic galvanometer, or by using an alternating current for field excitation and observing the induced alternating voltage, preferably by the oscillograph to eliminate wave shape error.

This "alternating magnetic characteristic" is the one which is of consequence in the design of alternating current apparatus. It differs from the "rising magnetic characteristic" B_1 in giving lower values of B for the same H—materially so at low values of H. It shows the inward bend at low fields still more pronounced than B_1 does. It is shown as curve B_2 in Fig. 5, and its reluctivity line is given as ρ_2 in Fig. 6. At higher values of H (from $H=3$ upwards) B_1 and B_2 both coincide with the curve B_0 representing the straight line reluctivity law.

The alternating characteristic B_2 is not a branch of any hysteresis cycle. It is reproducible and independent of the previous history of the magnetic circuit, except perhaps at extremely low values of H, and in view of its engineering importance as representing the conditions in the alternating magnetic field, it would appear the most representative magnetic characteristic and is commonly used as such.

It has, however, the disadvantage that it represents an unstable condition. Thus in Fig. 5, an alternating field $H=1$ gives an alternating flux density $B_2=2.6$. If, however, this field strength $H=1$ is left on the magnetic circuit the flux does not remain at $B_2=2.6$, but gradually creeps up to higher values, especially in the presence of mechanical vibrations or slight pulsations of the magnetizing current. To a lesser extent the same thing occurs with the values of curve B_1, and to a greater extent with B_3. At very low densities this creepage due to instability of the $B-H$ relation may amount to hundreds of per cent and continue to an appreciable extent for minutes, and with magnetically hard materials for many years. Thus steel

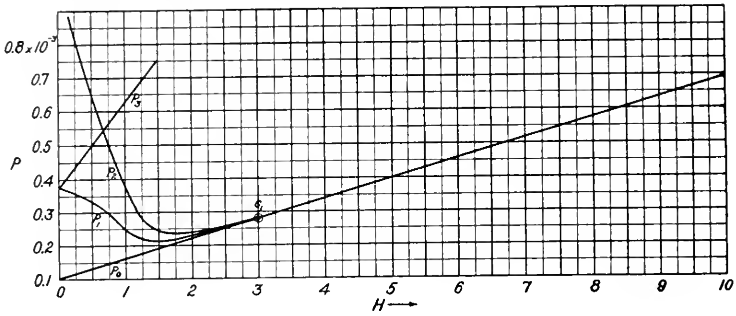


Fig. 6

structures in the terrestrial magnetic field show immediately after erection only a small part of the magnetization which they finally assume after many years.

Thus the alternating characteristic B_2, however important in electrical engineering, cannot, owing to its instability, be considered as representing the true physical relation between B and H, any more than can the branches of hysteresis cycles B_1 and B_3.

The Area of B-H Relation. Instability of Extreme Values. Gradual Approach to the Stable Magnetization Curve

6. Correctly, the relation between B and H thus cannot be expressed by a curve, but by an area.

Suppose a hysteresis cycle is performed between infinite values of field intensity: $H = \pm\infty$, that is, practically, between very high values, for instance such as are given by the isthmus method of magnetic testing (where values of H of over 40,000 have been reached, and where very much lower values probably give practically the same curve). This gives the magnetic cycle shown in Fig. 5 as $B'-B''$. Any point H, B within the area of this loop between B' and B'' of Fig. 5 then represents a possible condition of the magnetic circuit, and can be reached by starting from any other point H_0, B_0, such as the zero point, by gradual change of H.

Thus, for instance from point P_0, the points P_1, P_2, P_3, etc., are reached on the curves shown in the dotted lines in Fig. 5.

As seen from Fig. 5, a given value of field intensity, such as $H=1$, may give any value of flux density between $B=-4.6$ and $B=+13.6$, and a given value of flux density, such as $B=10$, may result from any value of field intensity between $H=-0.25$ and $H=+3.4$.

The different values of B corresponding to the same value of H in the magnetic area, Fig. 5, are not equally stable, but the values near the limits B' and B'' are very unstable and become more stable towards the interior of the area. Thus the relation of point P_1, Fig. 5, $H=2$, $B=13$, would rapidly change with a decrease in flux density to P_0, slower to P_2, and then still slower; while from point P_3 the flux density would gradually creep up.

It thus follows that somewhere between the extremes B' and B'', which are most unstable, there must be a value of B which is stable, that is, represents the stationary and permanent relation between B and H, and towards this stable value B_0 all other values would gradually approach. This then would give the true magnetic characteristic—the stable physical relation between B and H.

At higher field intensities, beyond the first critical point c_1, this stable condition is rapidly reached, and therefore is given by all the methods of determining magnetic characteristics. Hence the curves B_1, B_2, B_0 coincide there, and the linear law of reluctivity applies. Below c_1, however, the range of possible B values is so large and the final approach to the stable value so slow as to make it difficult of determination.

Production of Stable Values by Superposition of Alternating Field. The Linear Reluctivity Law of the Stable Magnetic Characteristic

7. For $H=0$, the magnetic range is from $-R_0=-11.2$ to $+R_0=11.2$; the permanent value is zero. The method of reaching the permanent value, whatever may be the remanent magnetism, is well known. It is by "demagnetizing," that is, placing the material into a powerful alternating field, a demagnetizing coil, and gradually reducing this field to zero; that is, describing a large number of cycles with gradually decreasing amplitude.

The same procedure can be applied to any other point of the magnetization curve. Thus for $H=1$, to reach permanent condition, an alternating m.m.f. is superimposed upon $H=1$ and gradually decreased to zero, and during these successive cycles of decreasing amplitude, with $H=1$ as mean value, the flux density gradually approaches its permanent or stable value. (The only requirement is, that the initial alternating field must be higher than any unidirectional field to which the magnetic circuit had been exposed.)

This seems to be the value given by curve B_0, that is, by the straight line law of reluctivity. In other words, it is probable that:

Fröhlich's equation, or Kennelly's linear law of reluctivity, represents the permanent or stable relation between B and H, that is, the true magnetic characteristic of the material over the entire range down to $H=0$, and the inward bend of the magnetic characteristic for low field intensities and corresponding increase of reluctivity ρ is the persistance of a condition of magnetic instability, just as are remanent and permanent magnetism.

In approaching stable conditions by the superposition of an alternating field, this field can be applied at right angles to the unidirectional field, as by passing an alternating current lengthwise, that is, in the direction of the lines of magnetic force, through the material of the magnetic circuit. This superimposes a circular alternating flux upon the continuous length flux and permits observations while the circular alternating flux exists, since the latter does not induce in the exploring circuit of the former. Some twenty years ago Ewing showed that under these conditions the hysteresis loop collapses, the inward bend of the magnetic characteristic practically vanishes, and the magnetic characteristic assumes a shape like curve B_0.

To conclude, it is probable that in pure homogeneous magnetic materials the stable relation between field intensity H and flux density B is expressed, over the entire range from zero to infinity, by the linear equation of reluctivity:

$$\rho=\alpha+\sigma H$$

where ρ applies to the metallic magnetic induction, $B-H$.

In unhomogeneous materials, the slope of the reluctivity line changes at one or more critical points, at which the flux path changes by reason of a material of greater magnetic hardness beginning to carry flux.

At low field intensities the range of unstable values of B is very great, and the approach to stability so slow that considerable deviation of B from its stable value can persist, some times for years, in the form of remanent or permanent magnetism, the inward bend of the magnetic characteristics, etc.