The United Kingdom Mathematics Trust

Intermediate Mathematical Olympiad and Kangaroo (IMOK)

Olympiad Maclaurin Paper

Thursday 19th March 2015

All candidates must be in *School Year 11* (England and Wales), *S4* (Scotland), or *School Year 12* (Northern Ireland).

READ THESE INSTRUCTIONS CAREFULLY BEFORE STARTING

1. Time allowed: 2 hours.
2. **The use of calculators, protractors and squared paper is forbidden.** Rulers and compasses may be used.
3. Solutions must be written neatly on A4 paper. Sheets must be STAPLED together in the top left corner with the Cover Sheet on top.
4. Start each question on a fresh A4 sheet.

   You may wish to work in rough first, then set out your final solution with clear explanations and proofs.

   Do not hand in rough work.
5. Answers must be FULLY SIMPLIFIED, and EXACT. They may contain symbols such as π, fractions, or square roots, if appropriate, but NOT decimal approximations.
6. Give full written solutions, including mathematical reasons as to why your method is correct.

   Just stating an answer, even a correct one, will earn you very few marks; also, incomplete or poorly presented solutions will not receive full marks.
7. **These problems are meant to be challenging!** The earlier questions tend to be easier; the last two questions are the most demanding.
   Do not hurry, but spend time working carefully on one question before attempting another.
   Try to finish whole questions even if you cannot do many: you will have done well if you hand in full solutions to two or more questions.

DO NOT OPEN THE PAPER UNTIL INSTRUCTED BY THE INVIGILATOR TO DO SO!

The United Kingdom Mathematics Trust is a Registered Charity.
Enquiries should be sent to: Maths Challenges Office,
School of Maths Satellite, University of Leeds, Leeds, LS2 9JT.
(Tel. 0113 343 2339)
http://www.ukmt.org.uk

1. Consider the sequence 5, 55, 555, 5555, 55 555, ….

Are any of the numbers in this sequence divisible by 495; if so, what is the smallest such number?

2. Two real numbers x and y satisfy the equation $x^2 + y^2 + 3xy = 2015$.

What is the maximum possible value of xy ?

3. Two integers are *relatively prime* if their highest common factor is 1.

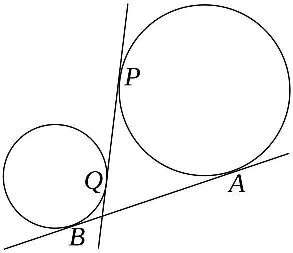
I choose six different integers between 90 and 99 inclusive.

(a) Prove that two of my chosen integers are relatively prime.

(b) Is it also true that two are *not* relatively prime?

4. The diagram shows two circles with radii a and b and two common tangents AB and PQ. The length of PQ is 14 and the length of AB is 16.

Prove that $ab = 15$.



5. Consider equations of the form $ax^2 + bx + c = 0$, where a, b, c are all single-digit prime numbers.

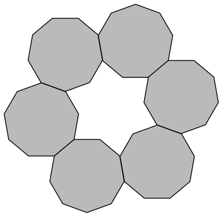
How many of these equations have at least one solution for x that is an integer?

6. A symmetrical ring of m identical regular n-sided polygons is formed according to the rules:

(i) each polygon in the ring meets exactly two others;

(ii) two adjacent polygons have only an edge in common; and

(iii) the perimeter of the inner region–enclosed by the ring–consists of exactly two edges of each polygon.



The example in the figure shows a ring with $m = 6$ and $n = 9$.

For how many different values of n is such a ring possible?