

STEP Support Programme

Hints and Partial Solutions for Assignment 1

Warm-up

1 You can check many of your answers to this question by using *Wolfram Alpha* (Google it if you have not yet heard of it). Only use this as a check though — and if your answer was wrong try again!

- (i) The two things you need here are the fact that $\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$ (this will be of most help if a or b is a square number) and $5x + 3x = 8x$. Answer: $8\sqrt{2}$.
- (ii) You can use $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$, but if you set $y = 2\sqrt{5}$ you must remember that $y^2 = (2\sqrt{5})^2$ etc — those brackets are very important! Answer: $207 + 94\sqrt{5}$.
- (iii) This is a case of carefully expanding the brackets. I quite like to lay out this sort of thing as a table, I tend to find that I make fewer mistakes this way.
Answer: $9 - 4\sqrt{3} + 2\sqrt{6} - 2\sqrt{2}$
- (iv) “Hidden” quadratics crop up quite a lot. Just make sure that your answers make sense in the original equation. For example, if you let $y = x^2$ and you get (say) $y = -2$ as one possibility, you need to discard this solution if you want real values of x . Note that $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$ ¹.

When faced with $\sqrt{144 - 16}$ it will make life easier to take out a factor of 4, i.e. write it as $4\sqrt{9 - 1}$. To get the final answers in a neat form, use your answer to part (a).
Answers: (a) $3 + 2\sqrt{2}$ and (b) $\pm\sqrt{2} \pm 2$ (which represents 4 answers).

¹Your teachers have probably been going on about how $(a+b)^2 \neq a^2 + b^2$ (unless a and/or b is zero), and I am sure that you rarely make this mistake. For some reason students are far more likely to make the analogous mistake of writing $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$. Don't.



Preparation

- 2 (i) Substituting your answers back into the original equation to see if they work is **never** a bad idea. Answer: $x = 1, -2$.
- (ii) The word “value(s)” occurs in the question: this is just a way of not giving anything away about the number of solutions that are expected (unlike saying “value” or “values”). Answer: $b = \pm 12$.
- (iii) There are various ways to solve a quadratic inequality. I quite like sketching a graph, but you can also identify the “critical values” (where the expression is equal to zero) and then test values in each separate region.

Be **really** careful about using a standard expression such as $b^2 - 4ac$ when these letters already occur in the question. You can write it in inverted commas to remind yourself what you are doing: “Using ‘ $b^2 - 4ac$ ’ gives ...”. But it is probably better to write something like $B^2 - 4AC$ (provided these letters do not also occur in the question!) and then you can write $B = 5c$ etc without confusing yourself.

When you have an inequality, you must be very careful not to divide (or multiply) by something which may, or may not, be negative. For example, if you have $x^2 < 4x$ you should **not** divide by x to get $x < 4$ as this inequality is not equivalent to the previous one (if $x = -10$, the second equality holds but the first one does not). It is far better to rearrange to get $x^2 - 4x < 0$ and then sketch (or factorise the right hand side of) $y = x^2 - 4x$.

Answer: $0 < c < \frac{12}{25}$



The STEP question

3 Do remember that whenever you are asked to prove a given result then you must do so clearly with a full explanation.

- (i) The easiest approach is probably to solve the equation to get $x = \sqrt{ab}$ or $x = -\sqrt{ab}$, but you could also consider the discriminant.

You must clearly explain **why** there are two solutions if a and b are either both positive or both negative. It might seem obvious to you, but you need to **write it down!**

- (ii) For the first part, rearrange to get a quadratic equation and then set the discriminant equal to 0.

You were then asked to show something of the form: the condition $c^2 = A$ is equivalent to the condition $c^2 = B$. All you have to do is show, one way or another, that $A = B$.

You can go forwards or backwards. You can start with $c^2 = B$ and then work backwards to get $c^2 = A$ (for added Brownie points put a \iff between each bit of working to show that the argument can be followed both ways — provided you are sure that it really is \iff). Perfectionists can do the working from $c^2 = B$ in rough and then reverse the argument starting from $c^2 = A$ on neat paper. Please do not start with $A = B$ (because that is what you are trying to show); though you could start with $A - B = \dots$, hoping to show that $A - B = 0$.

For the last part, you do need to show that $c \neq 0$; i.e. you need to show that c^2 is **strictly** greater than zero (not just $c^2 \geq 0$). This is easiest if you use the first expression you have for c^2 and remember the information that you were given at the beginning of the question.

Warm down

4 As a starting point, write 2450 as a product of prime factors, and then carefully and logically write down all possible sets of three ages for the bellringers.

Don't forget that the Bishop knows how old he/she is — so if the Bishop is 42 then she/he is looking for the ages which sum to 84. The fact that the Bishop cannot immediately answer the question is important, and will enable you to work out how old the Bishop is. You then need to use the fact that knowing the Minister is older than anyone else in the room will lead to a unique solution to deduce the age of the Minister.

Final answer: Minister is 50 (and the Bishop is 32).

