

# Laboratory Report

## Spinning Disk and Chladni Plates



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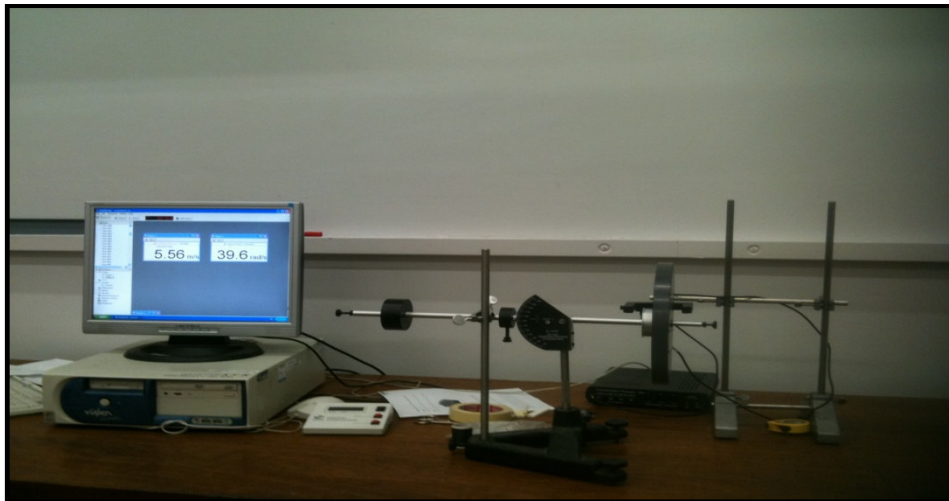
## Spinning Disk

### 1.0 Introduction:

This experiment is an introduction to some basic concepts of spinning disk. However, the purpose of this investigation is to measure the precession rate of a gyroscope and compare it to the theoretical value.

### 2.0 Description of Apparatus:

A gyroscope with super pulley and pulley mounting rod, mass and hanger set, balance, meter stick, table clamp for pulley, thread, stopwatch, photogate sensor computer interface, computer, smart pulley and smart pulley software.[2]



**Figure 1 : Experimental Apparatus for the Spinning Disk Experiment.**

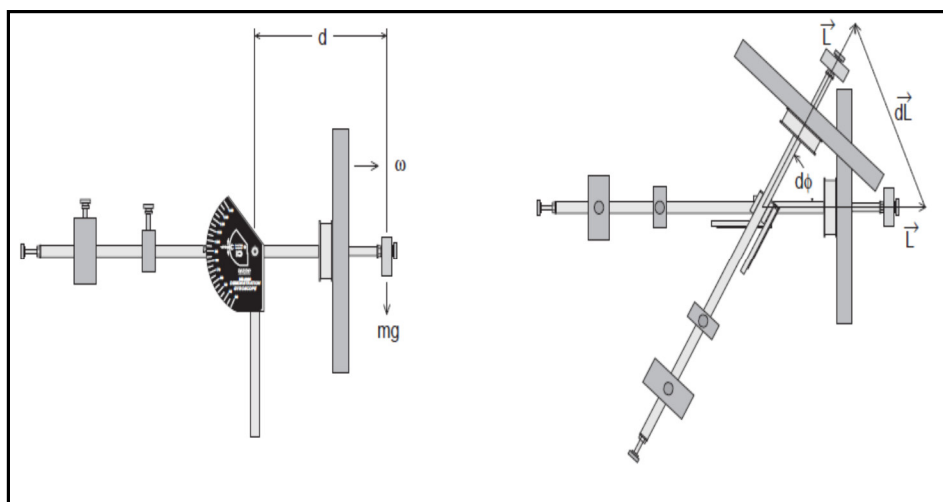
### 3.0 Theory:

A torque is applied to the gyroscope by hanging a mass on the end of the shaft, and this torque causes the gyroscope to precess at a certain angular speed,  $\Omega$ . However, assume that the gyroscope is primarily balanced in the horizontal position,  $\theta = 90^\circ$ . The disk is spun at an angular speed ( $\omega$ ) and then a mass,  $m$ , is attached to the end of the gyroscope shaft at a distance,  $d$ , from the axis of spinning. Hence we can write a torque:

$$\tau = mgd \quad (1)$$

Nevertheless the torque is also equal to  $\frac{dL}{dt}$ , where  $L$  [ $\text{kg} \frac{\text{m}^2}{\text{sec}}$ ] is the angular momentum of the disk. As we can see in figure 2, for small changes in angle,  $d\phi$ , and then the equation will be

$$dL = L d\phi \quad (2)$$



**Figure 2 : Torque Applied to Horizontal Gyroscope. [2]**

Substituting Eq no (1) and (2) for  $dL$

$$\tau = mgd = \frac{dL}{dt} = L \frac{d\phi}{dt} \quad (3)$$

Since  $\Omega = \frac{d\phi}{dt}$ , the precession speed,

$$mgd = L \Omega \quad (4)$$

and the precession rate is given by

$$\Omega = \frac{mgd}{I\omega} \quad (5)$$

Where  $I$  [kg m<sup>2</sup>] is a the rotational inertia of the disk and  $\omega$  [rads/sec ] is the angular speed of the disk.

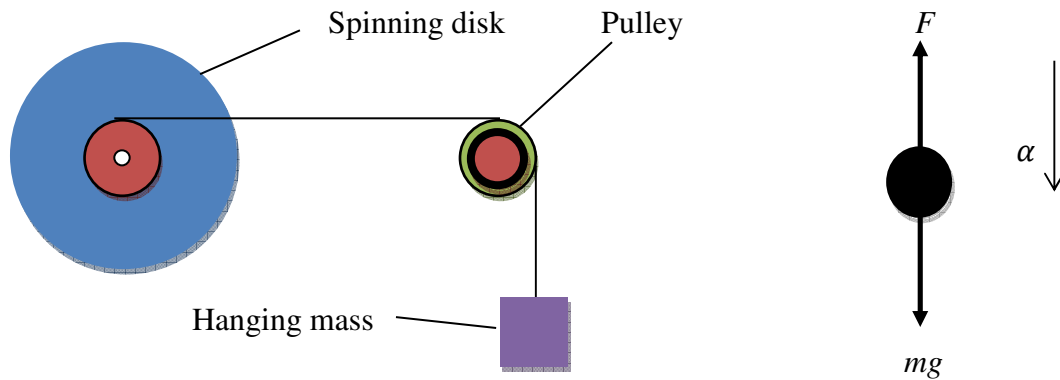
To find rotational inertia of the disk experimentally, a known torque is applied to the disk and resulting angular acceleration is measured. Since  $\tau = I\alpha$ ,

$$I = \frac{\tau}{\alpha} \quad (6)$$

Where  $\alpha$  [rads/ sec<sup>2</sup>] is the angular acceleration which is equal to  $a/r$  and  $\tau$  is the torque caused by the weight hanging from the thread which is wrapped around the pulley on the disk.

$$\tau = rF \quad (7)$$

Where  $r$  is the radius of the pulley about which is the thread is wound and  $F$  is the tension in the thread when the disk is spinning.



**Figure 3 : Spinning Disk and Free-Body Diagram.**

In figure 3 by applying Newton's 2<sup>nd</sup> Law for the hanging mass,  $m$ , gives

$$\sum F = ma$$

$$mg - F = ma \quad (8)$$

Solving equation No. (8) for the tension in the thread gives

$$F = m(g - a) \quad (9)$$

Therefore, once the linear acceleration of the mass ( $m$ ) and gravitational acceleration ( $g = 9.81 \text{ m/s}^2$ ) is determined, the torque and the angular acceleration can be obtained for the calculation of the rotational inertia. Conversely, the acceleration is achieved by timing the fall of the hanging mass as it falls from rest a certain distance ( $y$ ). [2] Finally the acceleration is given by

$$a = \frac{2y}{t^2} \quad (10)$$

#### 4.0 Experimental Procedure:

**Step 1:** Attached the add-on mass (50 g, 100g and 150 g) to the end of the shaft and measured the distance ( $d$ ) from the axis of rotation to the center of the add-on mass; recorded this distance in table 1.

**Step 2:** Gripped the gyroscope so it cannot precess, spun the disk at about one revolution per seconds. However, time one revolution of the disk to determine the angular speed ( $\omega$ ) of the disk and recorded in table 1.

**Step 3:** Let the gyroscope precess and time one revolution to find the precession rate. Hold the photogate sensor for a second or so such that the paper piece attached to the disk blocks the photo gate as it passes through it (Figure 1) and recorded in table 1.

**Step 4:** Repeated the measurement of one revolution of the disk. The before-and-after data used to find the average angular speed of the disk during the precession.

**Step 5:** Found the friction mass  $m_f = 10\text{g}$  that just makes the disk rotate.

**Step 6:** To find acceleration, we put about 30 g over the pulley. Wind the thread up and let the mass dropped from the table to the floor, timing the dropped. However, repeated this for a total of 5 times, always beginning the hanging mass in the same position.

**Step 7:** Measured the height that the mass dropped and recorded this height in table 2.

## 5.0 Experimental Results:

**Table 1: Angular Speed Measurements and Experimental Precession Rate**

Add-On Mass <i>Kg</i>	Distance <i>d</i> <i>m</i>	Time for one Revolution (initial) <i>rads/sec</i>	Time for one Revolution (final) <i>rads/sec</i>	Average Angular Speed of Disk $\omega$ <i>rads/ sec</i>	Time for Precession <i>T</i> <i>sec</i>	Experimental Precession Rate $\Omega = \frac{2\pi}{T}$ <i>rads/sec</i>
<b>0.05</b>	<b>0.204</b>	<b>23.2</b>	<b>21</b>	<b>22.10</b>	<b>18</b>	<b>0.35</b>
<b>0.10</b>	<b>0.205</b>	<b>24.3</b>	<b>19.4</b>	<b>21.85</b>	<b>9.51</b>	<b>0.66</b>
<b>0.15</b>	<b>0.208</b>	<b>46.5</b>	<b>39.6</b>	<b>43.05</b>	<b>10.15</b>	<b>0.62</b>

**Table 2 : Experimental Rotational Inertia Data**

Friction Mass <i>m<sub>f</sub></i> <i>Kg</i>	Hanging Mass <i>m<sub>h</sub></i> <i>Kg</i>	Original Mass <i>m =</i> <i>m<sub>h</sub> - m<sub>f</sub></i> <i>Kg</i>	Height Mass Falls <i>m</i>	Radius of Pulley <i>r</i> <i>m</i>	Average Times <i>sec</i>	Linear Acceleration $a = \frac{2y}{t^2}$ <i>m/ sec<sup>2</sup></i>	Tension $F = m \times$ $(g - a)$ <i>N</i>	Torque $\tau = rF$ <i>Nm</i>	Angular Acceleration $\alpha = a/r$ <i>rads/ sec<sup>2</sup></i>	Experimental Rotational Inertia $I = \tau / \alpha$ <i>Kg m<sup>2</sup></i>
<b>0.01</b>	<b>0.03</b>	<b>0.02</b>	<b>0.915</b>	<b>0.029</b>	<b>9.106</b>	<b>0.022</b>	<b>0.19576</b>	<b>0.0057</b>	<b>0.7610</b>	<b>0.0075</b>

**Table 3 : Theoretical Rotational Inertia Data and Precession Rate**

Solid Disk Mass <i>M</i> <i>Kg</i>	Solid Disk Radius <i>K</i> <i>m</i>	Theoretical Rotational Inertia $I = \frac{1}{2} MK^2$ <i>Kg m<sup>2</sup></i>	Add-On Mass <i>m</i> <i>Kg</i>	Distance <i>d</i> <i>m</i>	Gravitational Acceleration <i>g</i> <i>m/ sec<sup>2</sup></i>	Average Angular Speed of Disk $\omega$ <i>rads/sec</i>	Theoretical Precession Rate $\Omega = \frac{mgd}{I\omega}$ <i>rads/sec</i>	Experimental Precession Rate $\Omega = \frac{2\pi}{T}$ <i>rads/sec</i>	Precession Rate Difference <i>%</i>
<b>1.72</b>	<b>0.125</b>	<b>0.0134</b>	<b>0.05</b>	<b>0.204</b>	<b>9.81</b>	<b>22.10</b>	<b>0.34</b>	<b>0.35</b>	<b>1</b>
			<b>0.10</b>	<b>0.205</b>		<b>21.85</b>	<b>0.69</b>	<b>0.66</b>	<b>3</b>
			<b>0.15</b>	<b>0.208</b>		<b>43.05</b>	<b>0.53</b>	<b>0.62</b>	<b>9</b>

## 5.0 Discussion of Results:

The precession rate of a gyroscope and the rotational inertia was calculated experimentally compare it to theoretical value. Here, the gyroscope precession rate difference was 1% for 50 g, 3% for 100 g and 9% for 150 g add-on mass compare to theoretical value with experimental results. Consequently, the experimental rotational inertia result was 0.0075 Kg m<sup>2</sup>, but 'theoretical value' was 0.0134 Kg m<sup>2</sup>. However, errors could arise in calculations due to incorrect procedures and critical mass taken, e.g. - when the mass is released; it might be pushed with a force thus taking less time to reach the floor. Also, the reaction time of the group when the mass is released and when it hits the floor might vary. There were 5 sets of data taken to minimise this error. Other discrepancies could arise from error in calculations, or incorrect readings of experiment apparatus.

## 6.0 References:

1. Goss, D.G. (2012) *Dynamics and system modeling 3: Introduction to gyroscope* [Class handout]. LSBU, 10<sup>th</sup> February.
2. Workshop Manual Model ME-8960 (1994) *Demonstration Gyroscope: 012-05327B*.USA. PASCC scientific.



## Chladni Plate

### 8.0 Introduction:

This experiment is an introduction to some basic concepts of Chladni Plates. However, Chladni Pattern is the nodal pattern formed on a plate when an external driving source is given to make the plate vibrate. On a circular plate we put some fine sand, and then give the plate a certain frequency of vibration at some point. Resonance arises when the vibration frequency reached the plate's natural frequency. As a response, all the points on the plate start to vibrate but each point vibrates with different amplitude. Particularly there are some points that always stand still. Therefore, the sand at the neighbourhood moves towards the "silent" points, which is how the sand pattern is formed. For the same plate, the pattern varies with the frequency of the driving source.[2]

### 9.0 Description of Apparatus:

The model WA-9607 Chladni Plates kits includes square Chladni Plate, circular Chladni Plate, sand, sand shaker.



Figure 4 : Experimental Apparatus for Chladni Plate investigation.

### 10.0 Theory:

Chladni's Law established Ernst F. F. Chladni (1756- 1827), and who was a physicist and musician born in Wittenberg in Germany. The law relates the frequency of modes of vibration for flat circular surfaces with fixed centre as a function of the numbers  $m$  of diametric (linear) nodes and  $n$  of radial (circular) nodes. The equation is a function of two dependent variables,  $m$  and  $n$  which change with frequency  $f$ . It is stated as the equation, where  $C$ ,  $b$  and  $p$  are coefficients which depend on the properties of the plate. The harmonics of musical instruments such as bells and cymbals can be described by this law [2].

$$f = C(m + bn)^p \quad (11)$$

### 11.0 Experimental Procedure:

**Step 1:** Placed the mechanical drive on the tray and connected Chladni Plate to the drive shaft. However, the banana plug mates directly with the hole in the drive shaft.






**Step 2:** Placed Sprinkle sand on top of the plate and unlocked the drive shaft of the Wave Driver. Then connected the Wave Driver to our student function generator (PI-9598), and we tried to vibrate the plate over a range of frequencies about 60 Hz to 1 KHz. Slowly vary the frequency of vibration. At frequencies which were too low to produce a nodal figure the plate was seen to move up and down. When a frequency was reached that produced a nodal pattern, the plate was no longer seen to move up and down. When a Chladni figure was generated, the frequency was noted in the logbook and a photograph was taken of the pattern.

**Step 3:** Repeated step 1 and 2 until the maximum frequency was reached, and the above procedure was repeated for all plates.[1]

### 12.0 Experimental Results Analysis:

The experimental results are verified Chladni’s Law and table 2 illustrates the clearest and finest nodal patterns. In fact at low value of the voltage output the pattern was unclear and the nodal lines were thick and maximum voltage output was created the finest pattern. However, vibrating circular plate, which is fixed at centre first proper picture was taken with frequency  $f=86.32$  Hz, which only showed sand forming two parabola linear nodal around the corner of the plate. After frequency gets higher than 100 Hz, first proper circular nodal line forms at  $f=110.5$  Hz . At about 339.7 Hz second circle starts forming from the centre of the plate and with increasing the frequency, circular nodal lines move towards the edge of the plate. On the other hand, for square plates with frequency  $f=314.6$  Hz, which were showed sand forming a circular nodal at center and eight linear nodal around the corner of the plate.

Table 4 : Experimental Results for Circular and Square plates

Circular and Square plates fixed at center				
$f=86.32$ Hz, $m=2$ ; $n=0$	$f=110.5$ Hz, $m=0$ ; $n=1$	$f=339.7$ Hz, $m=0$ ; $n=2$	$f=831.9$ Hz, $m=0$ ; $n=3$	$f=314.6$ Hz, $m=8$ ; $n=1$
				

### 13.0 References:

1. Workshop Manual Model WA-9607 (1994) *Chladni Plates kit: 012-03283D*.USA. PASCSC scientific.
2. Mary D. Waller. (1961) *Chladni Figures: A Study in Symmetry*. London: G. Bell and Sons LTD